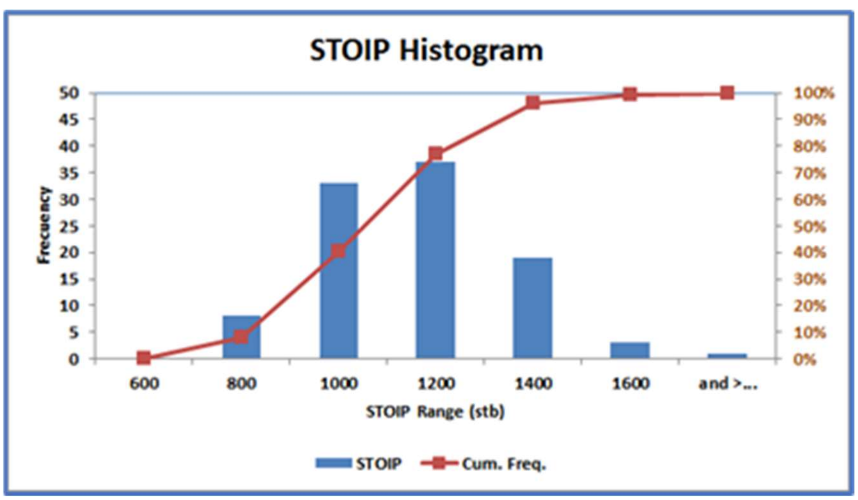
CHAPTER 7 RISK MANAGEMENT

Engineering Economics Topics in Hydrocarbon Subsurface_ a Bibliographical Review Handbook



1/11/2022 ROLANDO GARCÍA LUGO CARACAS VENEZUELA

Engineering Economics Topics in Hydrocarbon Subsurface _a Bibliographical Review Handbook

Chapter 7 Risk Management

Learning Objectives Chapter 7 Risk Management

Having worked through this chapter the reader will be able to:

- Risk Reduction
  - Identify sources of information to reduce uncertainty
  - List important methods of transferring risk to others
  - Explain the use of financial instruments for transferring risk
  - Explain the use of commodity trading for transferring risk
- Diversification
  - List important methods of diversification
  - Explain the benefits of joint ventures
  - Explain vertical integration and conglomeration
  - Explain the purpose of scenario planning
- Decisions
  - Explain the concept of relevant information in the context of decision-making
  - Simple Decision Methods
  - Review simple methods of dealing with risk
  - Explain discount rate adjustment
- Base Case Methods
  - Explain the process of sensitivity analysis
  - Construct a spider diagram
  - Describe the process of random number simulation
  - Differentiate between Monte Carlo and Latin Hypercube sampling
  - Explain the problem of parameter dependence
  - Mathematical Expectation
  - Define and explain the significance of expected value
  - Use expected value to rank investments
  - Use the Binomial probability Function to calculate expected value
- Preference Theory
  - Explain the construction and application of utility functions
  - Decision Trees
  - Construct a tree from given logical information and data
  - Derive and apply Bayes Theorem to decision tree problems
  - Solve decision trees and use to derive the value of information
- Probability concepts
  - Describe the concepts of randomness
  - Understand the probability distributions concepts
  - Use the concepts of expected monetary value
  - Evaluate projects using the probability distribution of the NPV

Introduction

Having described in Chapter 5 the sources of uncertainty and risk, and in Chapter 6 the use of economic indicators to perform feasibility analysis, in this chapter the goal is to describe the tools used to partially offset one of the main sources of danger in the hydrocarbon industry: the risk.

Two factors that affect most financial and economic decisions are the time value of money and risk. The time value was considered in previous chapters and its application to the valuation of projects. Also, the definition of risk and its classification were already described, in addition to consideration of risk as part of the interest rate to be applied to a project.

This chapter is about the valuation of risk and how corporations value risk and how managers trade off risk against expected returns.

Risk is a complex topic that cannot be captured in a series of formulas. Therefore, only some general rules are provided as to how rational managers should approach projects valuations. Any financial and production forecasts including estimations of hydrocarbon price and production are subject to uncertainty. Therefore, in any explicit consideration of risk it is required to express uncertain predictions in terms of probabilities.

Management of risk requires:

- Risk reduction by understanding the sources of risk
- Quantifying for risk
- Scenario planning by looking for opportunity to reduce risk
- Methods for incorporating risk into the decision process

Risk Reduction

Companies have access to a range of procedures for risk reduction, some of which relate to specific investments or to strategy. The most common procedures in the hydrocarbon business to achieve risk reduction are:

- Acquisition of information
- Transfer to another party
- Diversification of activity

Acquisition or Collect Information

As indicated in Chapter 5, uncertainty and risk usually result from incomplete information. Some of the information relates to uncertain events or conditions in the future, which may damage or compromise individual components or system integrity. In these circumstances, both the system design and the environment require attention.

Oilfield production facilities are subjected to variable use, abuse and exposure to the elements. Engineered systems must be durable, reliable and, preferably flexible, to ensure continued operation under tough and unexpected conditions. Engineering analysis will highlight areas, which are critical or vulnerable and consequently targets for appropriate built-in redundancy and upgrading.

Many of the factors, which may create a threat in the future, exist in the present. Corrosion is a time-dependent process, which can be modelled. Ocean energy can be modelled and fatigue processes investigated. Systems may be introduced or improved to reduce vulnerability to human intervention.

Some of the information relates to uncertain events in the future, over which the company has limited or no control. Markets for oil and gas, for example may fluctuate daily and government policy is always subject to change.

Strategy must be to ensure that relevant parameters are known and mechanisms for change are understood, so that the surprise element is reduced. Economic and political scouting are required to keep the company up-to-date with activities of the rest of the hydrocarbon's world actors.

Risk Reduction by Transferring to Another Party

Several mechanisms are generally available to enable one company to pass to another, some elements of risk. Almost inevitably, this is at a price. An upstream hydrocarbon investment might include one or several of these items:

- Insurance policy
- Farm-out agreement
- Construction contract
- Lease finance contract
- Project finance contract
- Commodity trading

Insurance Policy

Insurance is a familiar concept. Relatively small payments are made on a regular basis, to avoid a larger loss, which may or may not happen. The decision to invest in insurance may be dependent on several factors, including:

- Legal requirements
- Size of potential loss in relation to assets of the individual (company)
- Cost of the insurance contract
- Perceived probability of loss occurrence
- Availability of a suitable contract
- Attitude of the company towards risk

There is commonly a legal requirement to provide insurance cover against possible harm or loss caused to other people or organizations. The individual car owner must insure against damaging other persons and property; the tanker operator must insure against polluting beaches and damaging wild life. After the Exxon Valdez disaster, the US Government tightened up on oil-spill insurance, costing the industry $17 billion over ten years.

The larger the potential loss, the more likely that insurance cover would be required. Many small risks are self-insured by organizations, on the idea that it is more economical an occasional payment to replace a damaged or stolen asset, rather than paying insurance premiums every year. The issue is perhaps whether such loss recovery could be achieved without disrupting the normal business activity of the company.

The cost of insurance is an important element of corporate cash flow. Rates relate to perception of risk and cost to the insurance company. Annual cost may be reduced by limiting both minimum (excess) and maximum payments in the event of a claim. The insurance industry has had rough times, with major petroleum accidents such as Piper Alpha and Exxon Valdez, natural disasters, such as hurricanes. Furthermore, with falling stock markets and low rates of interest, companies find it difficult to earn a good return on their financial investments. Consequently, there is strong upward pressure on all insurance rates.

Insured losses normally have a relatively low probability of occurrence. The “standard” rate of 0.1% per year implies a maximum loss event once every 1000 years or longer, or one loss event per year for every 1000 similar policies. At 2%, this changes to 50 years. These are break-even numbers, which do not allow for profit.

An important aspect of insurance business is actuarial study of previous statistics, where they exist and assessment of other indicators of risk, when they do not. Like the oil business, insurance companies succeed or fail on their ability to understand risk.

In assessing whether to accept a specific insurance contract, the insurer may consider the following:

- Randomness of loss occurrence
- Definability of maximum possible loss
- Accessibility of the probability and severity of loss
- Independence of loss occurrences
- Size of maximum possible loss

Very large and not well-defined losses, which are systematic or non-independent must be avoided as unacceptably high or non-quantifiable risk.

Over time, attitude to specific types of risk may change as data accumulates.

Each company must make its own decisions about the risks to be covered by insurance. This will depend partly on the corporate attitude towards risk.

The whole industry would be better off without insurance under normal circumstances. However, in the real-world insurance is a must.

FARM-OUT

A farmout is the assignment of part or all an oil, natural gas, or mineral interest to a third party for development. The interest may be in any agreed-upon form, such as exploration blocks or drilling acreage.

The third-party, called the "***farmee***," pays the "***farmor***" a sum of money upfront for the interest and commits to spending money to perform a specific activity related to the interest, such as operating oil exploration blocks, funding expenditures, testing or drilling.

Income generated from the farmee's activities will go partly to the farmor as a royalty payment and partly to the farmee in percentages determined by the agreement.

A farm-out is an agreement, which enables a third party to earn an interest in a property by contributing part of the risk investment. Commonly, the investment is the drilling of an exploration well.

For example, a small company has successfully developed an exploration prospect, through licensing, seismic investigation and preliminary drilling; consequently, the small company has no budget but does not want to desist.

Farming-out provides the opportunity to retain some equity involvement, without further financial commitment, by releasing part of the equity to the other company

CONSTRUCTION CONTRACT

Construction of production facilities is a critical phase for a project. It is normally the time of major capital investment, and the economics of the project may be dependent on delivery of the facility on budget and on time. A delay in the start-up date normally means that every barrel will be produced and sold later than planned. Additional delay between production (positive cash flow) and development (negative cash flow) has a negative impact on discounted parameters.

For one a year delay, at 10% discount, the proportional reduction in discounted revenue is:

$$P = F/(1+i)^n = F/(1+0.1) = F*0.909$$

The operator in such circumstances will include a penalty clause in the agreement with the contractor. This would trigger a substantial payment if the facility was not delivered by an agreed date. The contractor may in his turn negotiate an appropriate insurance policy and charge it to the operator.

Increasingly, operators can subcontract the construction of an entire system to a single contractor, who may be responsible for:

- Design
- Acquisition
- Fabrication
- Installation and
- Commissioning

Such a contract may be called "***turnkey***", in the sense that all the operator has to do is to "turn the key" and the facility will become productive.

This type of arrangement protects the operator from the uncertainty of development cost.

Lease Contract

Some Financial leases are negotiated on the basis that the lessee pays according to system performance.

- For example, a company leases an FPSO unit (Floating Production Storage and Offloading, this is a floating vessel used by the offshore oil and gas industry for the production and processing of hydrocarbons, and for the storage of oil)
- The lease includes a payment based on production
- At US$ 2 per barrel, if the reservoir performs as expected, the lessor will recover its investment
- If the field performance varies from expectation, the lessor will receive more money or less money
- The lessee has transferred part of the reservoir risk to the lessor

In a general sense, whenever a financier or a contractor agrees to accept payment, which is linked to field performance, that company has agreed to accept part of the risk of the project.

Project Finance

Project finance is a form of long-term debt finance, in which the debt is secured against the cash flow stream from a particular project. In this arrangement the lender has no recourse to the general assets of the borrower. Hence, project finance is sometimes called non-recourse finance. This restriction is to the disadvantage of the lender, with recovery of the debt dependent on the performance of the project. By this mechanism, the borrower transfers some measure of project risk to the lender.

Project finance is particularly useful in a situation, where a corporation has an investment opportunity, which is beyond its capacity to finance by conventional methods.

- Perhaps the project is very large in relation to the balance sheets of the investors
- Perhaps their freedom to take on new debt has been constrained by gearing limits on existing debt contracts

Gearing refers to the ratio of a company's debt-to-equity (D/E).

- Gearing shows the extent to which a firm's operations are funded by lenders versus shareholders
- This is a measure of a company's financial leverage
- When the proportion of debt-to-equity is large, then a business may be thought of as being highly geared, or highly leveraged

- In general, a company with excessive leverage (high gearing ratio) could be more vulnerable to economic downturns than a company that is not as leveraged

Project finance from the viewpoint of the lender it is important to ensure that the project is robust and that the companies involved can deliver. The successful project financier must be able to reproduce and to verify the project modelling of the field operator. Most of the institutions involved in this type of finance form themselves into large groups or consortia to spread the risk.

- Not all projects are amenable to this type of funding
- The ideal projects are separable from other corporate activity and have an identifiable cash flow
    - Since project cash flow is the basis of security, it must be definable without ambiguity

Commodity Trading

Volatility in the price of oil has produced a wide range of procedures, which are designed to offer varying degrees of protection against price risk. Many of these involve the transfer of risk from one party to another, inevitably at a price. The following are considered:

- Forward contracts
- Future markets contracts
- Options
- Swaps
- Netback pricing
- Price escalation formulae

Forward Contracts

Most oil trade is forward, in the sense that the oil is delivered to the buyer sometime after the paper contract has been signed. Such forward trade is predominantly SPOT, with price linked to a relevant marker and where contracts are short term based on a current price. Both buyers and sellers may wish to sign contracts, for delivery several months ahead, to protect themselves against price variation. If sellers are unable to identify an appropriate buyer, they may sell to a trader, who is prepared to speculate on the market. The speculator, in turn, may hold onto the oil in expectation of a price increase, or sell some or all of it on to another trader, or to a refiner. The speculator would expect to receive the oil at a discount, in return for accepting price risk.

Futures Markets Contract

The futures markets are a more formalized mechanism for companies to establish a price for a consignment of oil, at some time in the future. Whereas forward contracts may be concluded in secret, futures trade is open and transparent.

Trading pits for petroleum are to be found at NYMEX in New York, IPE in London and SIMEX in Singapore. All market transactions take place openly, in public and are recorded, and price finds its own level, dictated by the interaction between supply and demand.

Price set in this fashion is widely used as a basis for SPOT trade elsewhere. IPE claims that Brent is the basis of 65% of the world pricing of oil.

The total volume of trade is typically several times world production, so clearly most of such transactions are confined to paper and are being used as a basis for risk management, and not for the physical disposal of liquid oil. To buy or sell oil at a specific price, on some specific date in the future is basically a gamble on the direction of movement of the oil price.

The futures market is used as the basis for ***hedging*** against future price fluctuation. The term "hedging" has a general, financial meaning, which concerns the provision of some protection against some form of uncertainty. Here, hedging is used to imply the use of futures contracts to reduce exposure to risk resulting from variation in oil price.

The conventional hedge is to match one's position in the SPOT market, with an opposite action in the Futures market. For example, a production oil company has oil to sell later and wishes to protect against unfavorable movement in the oil price. The company sells an equivalent volume on the Futures market. This contract is dependent on finding a trader, who is prepared to buy at the specific time. The price will reflect current perception of future trends, discounted for risk.

Once the Future contract is signed, the company's selling price is fixed, no matter what happens to the market price, before the company sells its own oil on the SPOT market. If SPOT market price falls, the company will have made a profit on the futures market to offset his loss on the SPOT market. The company has covered its downside price risk, but at the expense of missing any increase in price.

Options

An ***option*** is a contractual right (not an obligation) to buy or to sell a specific quantity of a commodity at a specific price over a specific period. The option enables the company to protect against price volatility, without having to make a major trading commitment ahead of time.

- An option to buy is a "**call**" contract
- An option to sell is a "**put**" contract
- The transaction price is the "**strike price**"
- The buyer of such a contract pays a "**premium**" to the seller to cover his risk

The size of the premium will depend on:

- The quantity involved
- The length of time
- The perceived volatility of the market

The buyer's cost is limited to the premium paid and this is independent of whether the buyer exercises the option.

The premium is also the maximum revenue that the seller can earn from the arrangement. In a falling market, "put" contracts (option to sell) will be exercised and the holder of the option will benefit by realizing a higher than market price for the sale.

In a rising market, the holders of "call" options (option to buy) will benefit from buying below market price. The potential loss of the option seller is limited, only by the feasible range of market prices. The holders may well seek to pass on all or part of the risk.

Swaps

A ***swap*** is a mechanism, whereby two organizations make an exchange, to mutual advantage. Commonly, this is a means of reducing exchange rate risk. An oil company, as supplier might enter into a swap agreement with an airline as buyer, to agree a fixed price for a hydrocarbon product. The oil company is required to give up the benefits of a high price, in favor of stability; the airline is required to give up the benefits of a low price in favor of stability. Normally, an intermediary, such as a merchant bank negotiates terms and prices with each party separately. If the bank can arrange differing prices for each, that is reward for the risk that they accept.

Netback Pricing

Netback pricing focuses on the refinery. Under normal circumstances, the refinery buys crude oil from the upstream market and sells its refined products into other markets. Refining margins depend on the behavior of both markets, neither of which is controllable by the refiner.

In some cases, the netback pricing could be used to establish upstream prices.

A netback price is one, which is designed to allow the refiner to make a reasonable profit in the current downstream market.

It is based on a formula that includes product prices, product yields, refinery costs, transportation costs and profit margins.

A netback price guarantees refiners a profit and transfers downstream risk back to the upstream producer.

Price Escalation Formulae

Price formulae are commonly included in gas contracts to protect both seller and buyer from problems caused by price variation in competing products.

Diversification

In business diversification translates to spreading the investments, to ensure that all these investments are not harmed by a single negative event. Investments may be diversified by:

- Joint ventures
- Geographical spread

- Vertical Integration
- Conglomeration

Joint Ventures

A joint venture is a form of alliance between two or more companies. It is a mechanism for sharing expertise and resources, and for spreading investment and risk.

Companies may consider establishing a joint venture to enter new markets or areas of business because sometimes, it is a precondition of gaining an exploration license that an oil company forms a joint venture with a local or national oil company or because the companies need to form voluntarily a joint venture.

An important and highly risky area of "new business" for the oil industry is exploration, and companies usually form joint venture type alliances to spread exploration risk. If a joint venture explores and discovers, the same group of companies assumes ownership of any potential field development.

Exploration alliances allow to spread the risk. For example, if 15% of exploration wells lead to commercial development (85% of failure) and based on the intuitive probability:

$p_{(fail)} = 1 - p_{(success)}$

For the first well

$p_{1\ (fail)} = 1 - 0.15 = 0.85$

For the second well is the combined probability

$p_{1\ (fail)}, p_{2\ (fail)} = 0.85 * 0.85$

$p_{2\ (fail)} = 0.723$

For the nth well

$p_{n\ (failures)} = 0.85^n$

If five wells are drilled, the probability of all five being commercial failures is

$p_{5\ (failures)} = 0.85^5$

$p_{5\ (failures)} = 0.444$

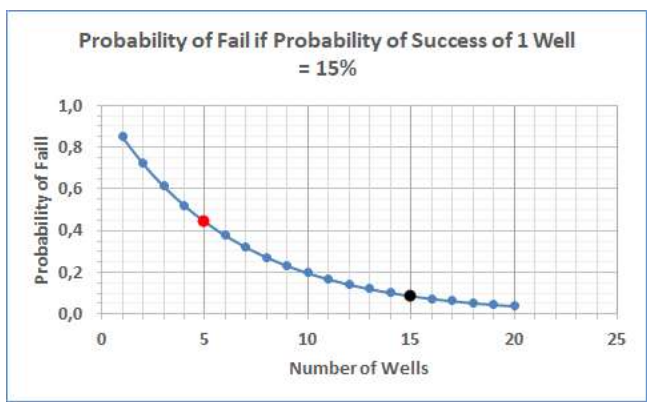


FIGURE 36 PROBABILITY OF FAILURE IF PROBABILITY OF SUCCESS OF ONE WELL = 15%

The topic of probability will be expanded later; however, for now accept that the probability of successive failure is the same as probability of zero success following a series of trials.

The mathematical relationship is sometimes called "gamblers' ruin", being the process whereby a gambler may run out of money, before encountering success.

In the example above (Figure 36), with p $_{(fail)}$ at 0.85, the company would have to drill at least 5 wells (red dot) to bring the probability of failure below 50% (0.444 or 44.4%). However, by joining a joint venture with two compatible companies and pooling exploration budgets, 15 wells could be drilled (black dot in Figure 36). This investment would have a probability of failure of:

$$p_{(fail)} = (0.85)^{15} = 0.087\ (8.7\%)$$

The benefits from any discovery must be shared with three partners in this example. However, 1/3 or 33% of something is better than 100% of zero.

Geographical Spread

Geographical spread ensures that a company is exposed to a range of geological, developmental and political environments. It also increases marketing opportunities. Concentration of effort in one area leaves a company vulnerable to geological risk, environmental problems and political change. Large hydrocarbon companies are example of a successful international business, since the geographical spread allows production of oil and gas from diverse countries.

Vertical Integration

The path from hydrocarbon reservoirs to customers includes several stages:

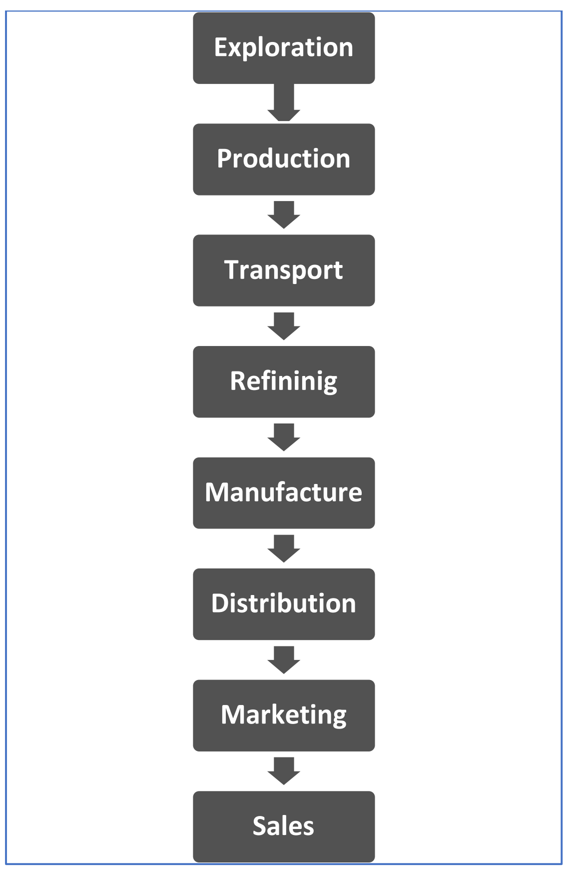


FIGURE 37 PATH FROM HYDROCARBON RESERVOIRS TO CUSTOMER

Vertical integration implies that a company operates in more than one of above stages:

- An upstream company might specialize in exploration and production
- A downstream company might focus on refining and manufacture

Many of the larger oil companies have successfully integrated the whole value chain from exploration through to sales, providing exposure to a range of markets and spreading risk. If the price of crude oil is low, the upstream end of the business has low income. However, the downstream, with access to low price oil, may perform better.

TRANSFER PRICING

Integration may provide a company with the opportunity to use transfer pricing to alleviate taxation problems.

- Transfer pricing is the process of charging, when a commodity is traded from one subsidiary to another of the same parent
- Adjustment in the price will shift profit from one to the other, thereby gaining tax advantage, if the assets are taxed differently

Government may of course impose a market reference price for tax purposes, when transactions are judged not to be at "arm's length". This refers to a business deal in which buyers and sellers act independently without one party influencing the other. This type of sales assert that both parties act in their own self-interest and are not subject to pressure from the other party; furthermore, it assures others that there is no collusion between the buyer and seller. In the interest of fairness, both parties usually have equal access to information related to the deal.

CONGLOMERATION

A conglomerate is something composed of heterogeneous elements. As a sedimentary rock, it consists of clasts of various rock types; as a company, it consists of diverse business activities. Diversification by this means implies investing outside the core business area of the company, thereby reducing exposure to risk from that core market.

Historically, larger oil companies considered diversifying into other business areas. There was often a reason to these moves, involving mining or energy, to benefit from perceived cooperation with the core business. Renewable energies, especially wind and solar, are taking a role of increasing importance in the energy industry. Therefore, oil majors are progressively positioning themselves for the proclaimed energy transition.

Quantifying for Risk

Sound judgement in any capital investment decision is a must. This includes that the decision maker should minimize the associated uncertainty by quantifying the risks. As it has been indicated in previous chapters some of the common traditional methods to take uncertainty into account are as follows:

- Risk Adjusted Payout Period
    - Not recommended for capital projects
    - This method uses a shorter payout period to compensate for risk
- Risk Adjusted Discount Rate
    - It uses higher rates of return on investment for "risky" projects
    - The major drawback is the use of subjective risk-adjusted rates of return
- Risk Adjusted Input Parameters
    - Using conservative values for input parameters
    - The drawback is that being too conservative can lead to all projects being unjustifiably rejected
- Probability Distribution
    - Quantify or estimate the magnitude of the risk by quantifying the "chances of achieving a given level of profitability" in a specific project
    - This is a common or standard analysis used by management to evaluate possible trade-offs between risk and expected profits

Scenario Planning

Scenario planning prepares a company for an uncertain future. Scenarios are models of, or stories about the future, which are plausible and internally consistent. Several scenarios may be constructed as alternate views of the future. The timescale may be 5 or 50 years and the subject matter is chosen to be relevant to the specific organization.

Unlike conventional forecasters, who start in the present and extrapolate forward, using historical trends to estimate the most likely future, scenario planners are more interested in ideas than probabilities.

Scenario planning is a testing ground for current strategy, to ascertain how company assets, organization and management might survive and perform across a range of possible futures. It is back to the idea of eggs and baskets, where the different baskets are alternate futures, rather than alternate reservoirs, countries or industries.

Scenario planning permits the exploration of possible futures, without having to admit to them being probable.

The fact that a possibility is identified and considered, will facilitate rational decision-making. Recent long-term scenarios explore the future of energy, ranging from a gradual shift from oil to renewables, to a more rapid shift to fuel-cell technology.

Methods to Incorporate Risk into the Decision Process

Decisions are the basis of successful business particularly those pertaining to project investment. Poor investment decisions may lead to poor financial performance or to failure. One of the best supports for a good decision is information.

Relevant Information

Investment decisions require consideration of critical factors, For example:

- Subsurface geosciences (geology, geophysics, reservoir engineering, petrophysics)
- The biological environment
- Production technology
- Government policy
- Human resources
- Sources of finance
- Project economics
- Uncertainty

The first and basic task is to determine which are the data relevant to the decision. For the economic analysis the focus is the cash flow. If the data are irrelevant, the decision is unlikely to be optimum. To be relevant, cash flow must result from, or be changed by the decision. Unless there are cumulative taxation issues, a historical cash flow will not have any relevance to a current decision. Neither will a cash flow in the future, which was previously committed or anticipated.

Figure 38 indicates the usual stages in capital investment for a large upstream project. Each is dependent on data generated by the previous stage.

- ***Appraisal*** does not start, for example, without an exploration discovery
- Development does not start, without a successful appraisal
- Adjustments may be made to the production system depending on performance
- ***Abandonment*** will be initiated, once production declines to an uneconomic level
- These decisions are all independent of each other and independent of previously committed cash flows
- The development decision is dependent on future cost and revenue expectation and not on appraisal cost

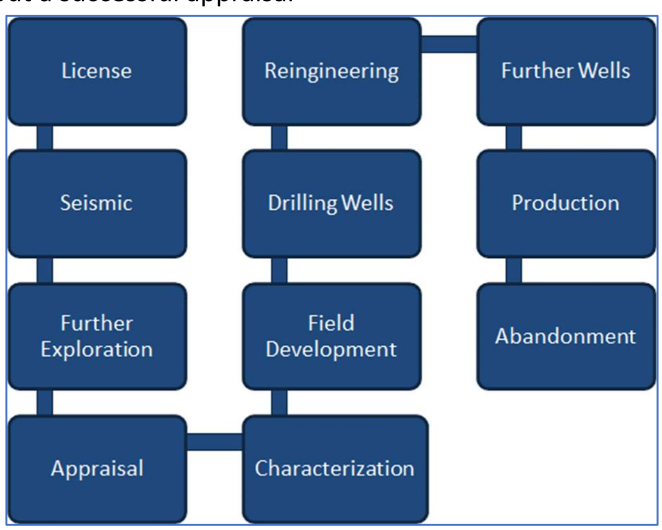


FIGURE 38 STAGES IN CAPITAL INVESTMENT FOR A LARGE UPSTREAM PROJECT

Whether appraisal costs $1 million or $100 million it is irrelevant to this decision. Figure 38 indicates ***Further Wells*** after the start of ***Development***. If this expenditure was previously committed, it is irrelevant to the development decision, even though it is in the future, with respect to that decision.

Once the ***Development*** is ongoing a range of expenditure and revenue streams are initiated. Any subsequent investment in wells or topside engineering must be analyzed in relation to changes made to these streams. A new investment may change rates of production; it may also change operating expenditures.

The Production Development Phase

Previous sections have described the basic concepts and fundamentals pertinent to the evaluation of any investment proposition in the hydrocarbon-industry environment. The need to "review the options" or "qualify an investment proposition (corporate or otherwise)" as desirable-acceptable, meeting required standards or discardable, will be required by the investor, a lender, controlling authorities and any interested party (such as an owner of surface rights, a government approving a producing license and many others). Thus, the "Investment Decision Analysis" as a procedure, must be formally addressed under a significant number of real conditions.

Having addressed the concepts and fundamentals associated with project valuation, the risks involved in various critical parameters, the tools to handle such risks (including the probabilistic approaches), and finally the most common yardsticks supporting the acceptance or not of the "investment Project" as a general proposition, we now turn to the most common example of income generating option available in the "investment possibilities" market. This is related, basically, to investment in ***production phase projects***.

In general, although investment opportunities do exist in different segments such as refineries, oil and gas transmission systems, gas compression facilities, oil-water treating facilities, and many others, the most common investment option available is that of the generation of production potential (oil and gas) or more generically called Production Development Projects.

Regardless of size, all these types of Production Development Projects (large or as small as drilling a single new well) all need to follow a very standard project evaluation activities flow-chart and basically a common procedure of evaluation.

The core of all this evaluation effort is production forecasting for a single well or a group of wells. Without the production, contributed by a well or a group of wells, any such project will not have the expected income to develop facilities and in the end net a profit for the investors. It is for this very reason it has been included a Figure 38 as a Flow Diagram of the procedure. Since the revenue from the ultimate sale of such production is the main generator of income to justify the investment and its payback, with all its risks involved, the profits (after return or payback of the capital invested) must meet the expectations of the investors.

The following discussion only intends to cover the simplest procedures usually used in the investment proposal received or developed by smaller producers, of limited volumes, also requiring limited funding. The much larger projects will require the use of much more sophisticated approaches (as complex as three-dimensional three phases modelling) completely out of scope of this review. Thus, the following discussion will be restricted to the simplest considerations tools for the investment decisions.

State of Nature Concept

Decision analysis, also called statistical decision theory, involves procedures for choosing optimal decisions in the face of uncertainty. In the simplest situation, a decision maker must choose the best decision from a finite set of alternatives when there are two or more possible future events, called states of nature, that might occur. The list of possible states of nature includes everything that can happen, and the states of nature are defined so that only one of the states will occur.

Decisions and Risk

The nature of hydrocarbon business requires the reduction of risk, either by acquiring information or by transferring to a third party. Therefore, oil and gas companies will make investment decisions, facing significant uncertainty regarding a range of project parameters.

Depending on the size of the investment and on the perceived nature and extent of the risk, management has access to a range of procedures, which incorporate this risk into the decision process. These include the following:

- Simple Methods
  - Discount rate adjustment
  - IRR
  - NPV
  - Discounted Payback Period
  - PIR and other economic hurdles
- Base-Case Methods
  - Sensitivity analysis
  - Numerical Simulation
- Expected value Methods
  - Preference Theory
  - Decision Trees
  - Probabilities
- Portfolio Analysis

When discounting of cash flow is part of the decision process, screening and ranking parameters are required.

Simple Methods

A range of simplistic methods are available as an aid to decision making. They are relatively basic and some of these should be applied with caution as it was presented in previous chapters.

The Payoff Matrix

A payoff matrix is a simple method of representing the key elements of a decision.:

- Actions available to the decision maker
- Possible states of nature, as perceived by the decision maker
- Monetary value of possible outcomes of the decision

This illustrates a payoff matrix for a simple exploration problem, where there are two possible actions:

- "Drill"
- "Relinquish"

Two assumed states of nature:

- "Producer"
- "Dry Hole"

The well costs MMUS$ **C** and the present value of success is MMUS$ **R-C**. A decision criterion is required to convert this matrix into a decision.

Action	**Drilling Results**		**Minimum**
	Producer	**Dry Hole**	
Drill a Wildcat (Exploration Well)	R - C	-C	-C
Relinquish the License /Concession	0	0	0

TABLE 137 Payoff Matrix

The Pessimist Rule is based on the premise that whatever course of action is taken, the worst possible outcome will happen. In other words, the decision maker assumes the worst possible payoff for each "Action".

Therefore, the decision maker anticipates scoring minus MMUS$ C if the well is drilled. The strategy is to select the action with the best "Minimum" payoff. The decision maker therefore chooses zero and decides not to drill. Exploration is then not a business for pessimists.

The pessimist rule is simplistic, but does model one aspect of human behavior; people do not like to lose money. Perhaps they set a limit on possible loss to reflect their resources and other commitments.

The Matrix may be modified to include consideration of "***Regret***" or earning potential. "Regret" is defined as the difference between a matrix element and the best possible outcome (payoff) for that state of nature.

- If there is oil present and the well is drilled
  - The highest possible payoff is scored, for that state of nature
  - The regret is zero
- If there is oil present and the well is **not** drilled
  - The profit has been missed out
  - The regret is R - C
- If there is no oil present and the well is drilled
  - The regret is C, the cost that could have been avoided
- If there is no oil present and the well is **not** drilled
  - There is zero regret

Action	Drilling Results		Maximum
	Producer	Dry Hole	
Drill Wildcat (Exploration Well)	0	C	C
Relinquish the License/Concession	R - C	0	R - C

TABLE 138 Regret Matrix

The solution of the regret matrix requires identifying the maximum regret for each action (a pessimistic assumption) and then choosing the action to minimize this regret. The criterion is sometimes called "Mini-Max Regret". In this example, the choice is between a regret of "C" and a regret of "R - C".

The "Drill" decision is optimum if the maximum regret is less than for the alternative decision "Relinquish". Therefore, drill if $C < R - C$, or if $R > 2C$. Such simplistic comparison between cost and potential benefit can form the basis of a decision.

Payout or Payback Period as Mitigating Risk

As already explained in Chapter 4, payback, or "Payout" is the time taken to recover invested capital from a project. It may be derived from undiscounted or, preferably from discounted cash flows.

As a measure of time, during which the project is in a negative, cumulative cash position, it is an indicator of investment risk; the longer the payback, the higher the perceived risk.

If risk is perceived to be a problem, payback may be used as a means of screening or ranking of projects. For example, a maximum Payback Period of 5 years might be used as a screening criterion.

Some risks are time related and can logically be managed in this way. Political (Government) related risk, is particularly time dependent. The longer the time, the greater the chance that government or regulations may change. Other issues, such as product prices are also time dependent, in the sense that they do vary over time and the longer the time, the more likely that market fundamentals will change and that current forecasts become irrelevant.

Some risks are not time dependent. Drilling a vertical well at location Z will either intercept oil or not, depending on what has happened in geological history. Timing and speed of penetration will not alter the chances of success.

Variants of Payback are sometimes used, including ***Payback Volume*** (minimum volume of produced oil to recoup investment) and ***Bailout Time*** (minimum time to reach cumulative zero, if salvage income and abandonment charges are included).

Discount Rate Adjustment as Mitigating Risk

As already discussed in Chapter 4, one of the most common and effective methods of handling risk in an investment decision is to adjust the discount rate. The important issue here is that a risky investment must offer a higher potential return, to become more attractive than a safer investment.

A company with a high debt ratio and/or a poor cash flow record, must offer a higher rate of interest than a company with lower debt and better performance, to attract further debt.

An oil company will expect a higher return from its upstream investments than from its downstream investments, because the upstream investments are considered riskier. A company may apply a range of discount rates as screening criteria, to match its perception of risk.

Risky projects not always offer a higher return. It is only those, which have been screened and accepted, which offer that higher return. Many high-risk projects offer average or poor returns and are consequently rejected.

Methods of Describing Project Risk Based on Cash Flow

Cash flow models are commonly constructed from best estimates of all the important project parameters such as CAPEX, OPEX and Revenue. These combine to produce a Net Cash Flow (**NCF**) profile, from which a series of measures of value may be derived. This is a form of Base Case where methods of incorporating risk, focus on a base case as a central value and review the impact of parameter variation around this central value

Usually, the first step in analyzing project risk is by first determining the uncertainty inherent in a project´s cash flow. The analysis can be done in several ways, ranging from making informal judgements to performing complex economic and statistical analysis. The most common methods of describing project risk are:

- Sensitivity Analysis
- Breakeven Analysis
- Scenario Analysis

Sensitivity Analysis (What-if)

Sensitivity analysis involves varying several parameters individually; simulation techniques incorporate simultaneous variation of uncertain variables.

Sensitivity analysis is a review of the impact of changes on a project. In the context of project economics, this normally involves representing a project by its cash flow and investigating the implications of changing one or more of the input parameters. The output is commonly measured by calculating a measure of value such as NPV.

A characteristic of conventional sensitivity analysis is that input parameters are varied individually, while maintaining the others as constants.

For example, a project NPV has been plotted as a function of variations in revenues (mainly due variations on oil price). The Base Case plots MMUS$ 321,1. The graph assumes a range of oil price from US$/bl 0.5 to 2 only to assess its impact on NPV.

n	Sensitivity		i =	10,0%
	Año	NCF (MMUS)	$1/(1+i)^n$	$A_j/(1+i)^n$
0	1999	-23,6	1,000	-23,6
1	2000	-102,5	0,909	-93,2
2	2001	-315,8	0,826	-261,0
3	2002	-320,8	0,751	-241,0
4	2003	10,4	0,683	7,1
5	2004	244,9	0,621	152,1
6	2005	447,7	0,564	252,7
7	2006	113,0	0,513	58,0
8	2007	222,5	0,467	103,8
9	2008	197,3	0,424	83,7
10	2009	151,6	0,386	58,4
11	2010	146,2	0,350	51,2
12	2011	126,4	0,319	40,3
13	2012	113,9	0,290	33,0
14	2013	101,4	0,263	26,7
15	2014	93,6	0,239	22,4
16	2015	84,5	0,218	18,4
17	2016	76,7	0,198	15,2
18	2017	68,9	0,180	12,4
19	2018	71,8	0,164	11,7
20	2019	-56,3	0,149	-8,4
21	2020	8,7	0,135	1,2
	Total	1460,5		321,1

Revenue Sensitivity	NPV (MMUS$)
0,0	321,1
0,5	-148,8
0,7	0,0
1,1	415,1
1,5	791,1
2,0	1261,0

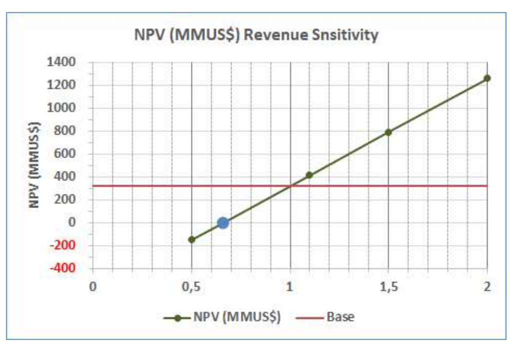


FIGURE 39 NPV AS A FUNCTION OF VARIATIONS IN REVENUES

In Figure 39 the blue dot represents a variation of the oil price around 0.7 of the Base Case. This equates with an NPV (@i = 0.10) of zero. This may be called the “breakeven” price, i.e., the price at which the project is on the borderline between being profitable and not. This value must become a focus of attention. If the oil price were to remain below this value for an extended period, the viability of the project would be at risk.

In summary, this analysis determines the effect on NPV of variations in the input variables used to estimate after-tax cash flows, such as:

- Revenues
- OPEX or CAPEX
- Salvage value

A sensitivity analysis reveals how much the NPV will change in response to a given change in an input variable.

In a calculation of cash flows, some items have a greater influence on the results than others. In some problems it is easy to identify the most significant items. For example, the estimated volume of oil and its price are often a major factor in a problem in which the quantity produced and sold varies among the options.

In other problems the goal is to identify the items that have an important influence on the results, so that they can be subjected to special scrutiny.

Sensitivity analysis is sometimes called "***what-if***" analysis because it answers questions such as: What if incremental production is 1000 stbpd, rather than 2000 stbpd? Then what will the NPV be?

Sensitivity analysis begins with Base-Case situation, which is developed using the most-likely values for each input.

Then, specific variables of interest are changed by several specific percentages above and below the most-likely value, holding other variables constant. Next, the new NPV is calculated for each of these values.

Spider Graphs

A convenient and useful way to present the results of a sensitivity analysis is to plot a sensitivity graph (***spider graphs***). The slopes of the lines show how sensitive the NPV is to change in each of the inputs. The steeper the slope the more sensitive the NPV is to change in the variable. It identifies the main variables that have a larger impact on the outcome. For example, based on the following income statement a NCF was generated and presented in Table 139:

Income Statement		Years					
		0	**1**	**2**	**3**	**4**	**5**
Production (bpd) (I)	2000		2000	2000	2000	2000	2000
Barrel Price (US$) (II)	50						
Variable Cost/Barrel (US$) (III)	15						
Income (US$) (IV)	100000		100000	100000	100000	100000	100000
Total Variable Cost (US$) (V)	30000		30000	30000	30000	30000	30000
Fixed Costs (US$) (VI)	10000		10000	10000	10000	10000	10000
Investment (US$) (VI)		125000					
Depreciation (US$) (VIII)	30%		18750	31875	22313	15619	10933
Income Tax (US$) (IX)			41250	28125	37688	44381	49067
Tax rate (X)	40%		16500	11250	15075	17753	19627

Net Cash Flow		**Years**					
		0	**1**	**2**	**3**	**4**	**5**
Net Income (MMUS$) (IX)–(X)		0	24750	16875	22613	26629	29440
Depreciation (US$) (VIII)		0	18750	31875	22313	15619	10933
Investment (US$) (XI)		-125000	0	0	0	0	0
Salvage Value (US$) (XII)	40%	0	0	0	0	0	50000
Effect of the Abandonment Tax	40%	0	0	0	0	0	-9796
NCF (US$) (Σ)		-125000	43500	48750	44925	42248	80577

TABLE 139 Income Statement and NCF Example

As a reminder, a Net Cash Flow is defined as the net income after taxes, plus depreciation, minus capital expenditures and working capital. The net income after tax represents the return of the investment while depreciation represent the recovery of the investment.

Results

The sensitivity analysis begins with a "Base-Case" situation, which reflects the best estimate (expected value) for each input variable.

	Cash Flow Economic Analysis					**Option 1**			
Input						Use as equivalent rate:		**15,00%**	**i = i'+f+i'*f**
Base Case NPV			Clear Input			**Option 2**			
Project Life (N, yrs)	5		*Max 40 Yrs*			15,0% is corrected to			
Initial Investment (P_{oINF}, $)	125000						**To be added to i**		
Interest (I)	15,0%		inflation (f)	0,0%	r corected by **f =**		**15,00%**	**r = (i - f)/(1 + f)**	
Fill Rows 8 & 10		*Yr*	*0*	*1*	*2*	*3*	*4*	*5*	
Annty Outflow (C, $ per year)		**Ci**		0,0	0,0	0,0	0,0	0,0	
Annty Inflow (A, $ per year)		**Ai**		43500	48750	44925	42248	40373	
Sunk Cost @ N (S, $)	9796								
Salvage Value @ N (Sv, $)	50000								

Results	
Profit to Investment Ratio (PIR)	**0,348**
Discounted Payout (Payback yrs)	**3,84**
NPVI	**0,348**
Net Future Worth (FW_{PC}, $)	**87380**
Net Present Value (NPV, $)	**43443**
Benefit/Cost Ratio (PI)	**1,335**
Internal Rate of Return (IRR,%)	**27,76%**
	Calc IRR
Eqvlnt Total Annual Cost (A_{PC}, $)	**12960**

Discount Rate	15.0%
NPV (US$)	43443
AEW (US$)	12960
IRR	27.8%

Then a given variable is changed by 20% below and above the Base-Case value and new NPV is calculated, holding other variables constant.

The Base-Case NPV was calculated as US 43443. Then, a series of "what-if" questions. What if oil rate is 20% below the expected level? What if operation cost rises? etc.

Production	-20%	0%	20%
NPV (US$)	15285	43443	71601
Price	-20%	0%	20%
NPV (US$)	3217	43443	83669
Variable Cost	-20%	0%	20%
NPV (US$)	55511	43443	31376
Fix Cost	-20%	0%	20%
NPV (US$)	47466	43443	39421
Salvage	-20%	0%	20%
NPV (US$)	40460	43443	46426

Figure 40 is the ***sensitivity graph or spider graph***. It shows the sensitivity for the five identified key input variables. The base-case NPV is plotted on the Y axis (ordinate) of the graph at the value of 0% on the X axis (abscissa). To superimpose the plots on the same diagram, parameters must be logged as a proportional or percentage change, rather than as absolute values. The central point in the diagram represents an NPV of MUS$ 43.443 for a proportional change factor of zero. This is the base case and all plots emanate from this point. Then, the values of above table are plotted.

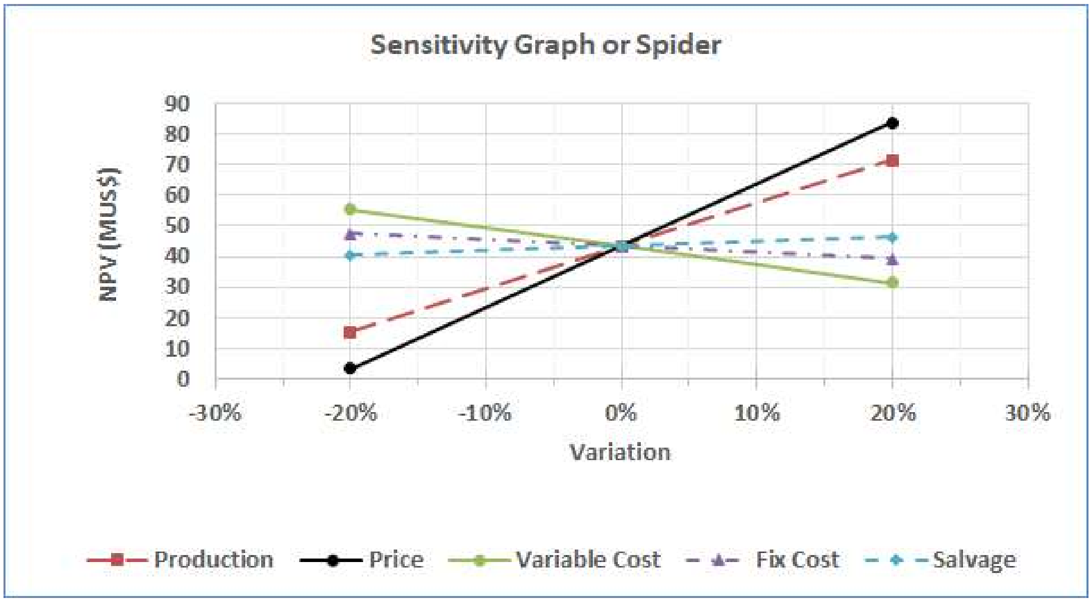


FIGURE 40 SENSITIVITY GRAPH OR SPIDER

From Figure 40 the conclusion is that project's NPV is sensitive to change in production and oil price.

NPV is less sensitive to changes in the variable cost and it is insensitive to changes in the fixed cost and salvage value.

These types of graphs are a useful means to communicate the relative impact of the different variables on the corresponding NPV value. However, the sensitivity graph does not explain any interactions among the variables or the likelihood of realizing any specific deviation from the Base Case. It is possible that answers might not be very sensitive to changes in either of two items, but very sensitive to combined changes in them.

Different parameters may have different ranges of value represented. The price of oil, for example is varied from $25 to $100 per barrel, a range from 0.50 to 2.00 as proportions of the 50$ Base Case. As part of the analysis, consider the range of values that may be appropriate for each parameter.

The method helps to highlight those parameters, which may undermine the economic performance of an investment. Blindly mass-producing plots of parameter values plus and minus X% will not contribute much to this process. It appears from this analysis that the project is more at risk from a collapse of oil price than from any other single factor. Variation of oil price seems to be the only likely parameter change, which on its own could reduce project NPV to zero.

Tornado Chart

As an alternative, the values used for the spider graphs are now plotted as a modified ***tornado chart***. Figure 41 represents the range of NPV values resulting from variation in each of the parameters. It is a plot of each pair of extreme values of NPV. These are arranged, so that the smallest range is at the bottom and the largest at the top. This creates an image like a tornado.

The X axis through the center of the diagram represents Base Case NPV (US$ 43443), with reduced values to the left and increased values to the right.

- The upper group is at +20% variation
- The lower group is at -20% variation

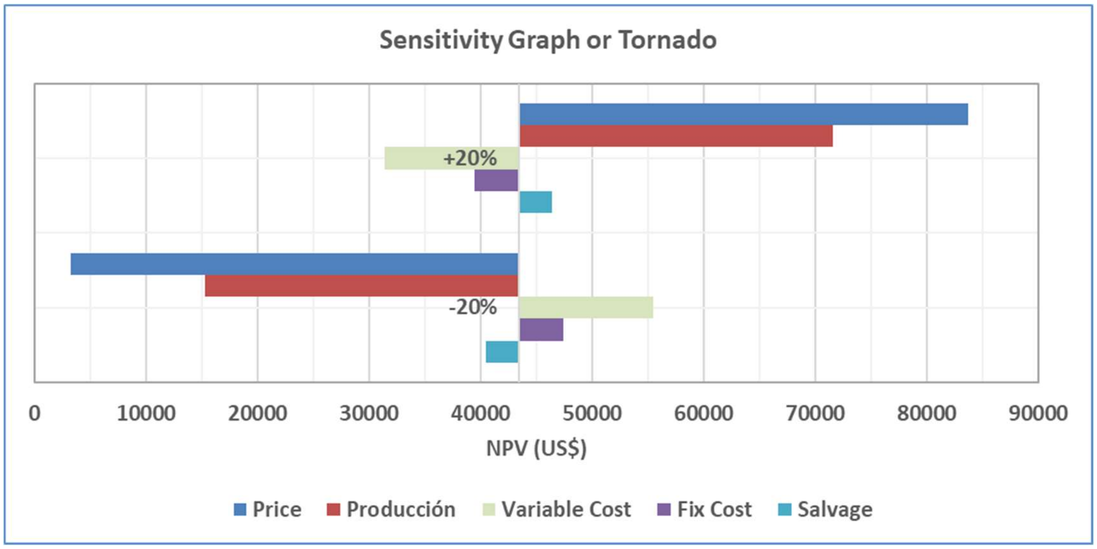


FIGURE 41 SENSITIVITY GRAPH OR TORNADO

This form of analysis is constrained because project parameters are varied individually, whereas in practice, they can change independently or all together. Varying CAPEX independently does not create a project with a negative NPV. It only means that increased CAPEX is a risk.

Assume that the base parameters from Table 139 are shifted by 10% in the negative direction

Parameter	Base Case	Adjusted
Production (stbpd)	2000	1800
Barrel Price (US$)	50	45
Variable Cost per Barrel (US$)	15.0	16.5
Fixed Costs (US$)	10000	11000

Income Statement		Years					
		0	**1**	**2**	**3**	**4**	**5**
Production (stbpd)	1800		1800	1800	1800	1800	1800
Barrel Price (US$)	45						
Variable Cost per Barrel (US$)	16,5						
Income (US$)	81000		81000	81000	81000	81000	81000
Total Variable Cost (US$)	29700		29700	29700	29700	29700	29700
Fixed Costs (US$)	11000		11000	11000	11000	11000	11000
Investment (US$)		125000					
Depreciation	30%		18750	31875	22313	15619	10933
Income Tax (US$)			21550	8425	17988	24681	29367
Tax rate	40%		8620	3370	7195	9873	11747

Net Cash Flow		**Years**					
		0	**1**	**2**	**3**	**4**	**5**
Net Income (MMUS$)		0	12930	5055	10793	14809	17620
Depreciation (US$)		0	18750	31875	22313	15619	10933
Investment (US$)		-125000	0	0	0	0	0
Salvage Value (US$)	40%	0	0	0	0	0	45000
Effect of the Abandonment Tax	40%	0	0	0	0	0	-7796
NCF		-125000	31680	36930	33105	30428	65757
NPV (US$)	**2329**	-125000	27548	27924	21767	17397	32693

Economic Indices	Base Case	Change of 10% in the Negative Direction
NPV (US$)	43443	2329
AEW (US$)	12960	695
IRR	27.8%	15.7%
PIR	0.348	0.019

TABLE 140 Base Case Shifting Parameters by 10% In the Unfavorable Direction

Shifting the uncertain parameters by 10% in the unfavorable direction, generates a cash flow with an NPV of US$ 2329 compared to the base case of US$ 43443. The NPV is still positive, so the project may be accepted.

Breakeven Analysis

When a sensitivity analysis of a project is performed, a common question is how severe the effect of lower revenues or higher cost will be. Managers sometimes prefer to ask how much production can decrease below forecast before the project begins to lose money. This type of analysis is called ***breakeven analysis***.

Specifically, breakeven analysis is a technique for studying the effect of variations in output on the NPV of a project. The breakeven value calculation is an analog of the internal rate of return where the goal is to find the interest rate that makes the NPV equal zero. The direct way is using computational capabilities such as solver in Excel to find the answer.

For the same example of the Sensitivity Analysis (Table 139) the production level that makes NPV equal zero is 1383 stbpd

Income Statement		Years					
		0	**1**	**2**	**3**	**4**	**5**
Production (stbpd)	1383		1383	1383	1383	1383	1383
Barrel Price (US$)	50						
Variable Cost per Barrel (US$)	15						
Income (US$)	69143		69143	69143	69143	69143	69143
Total Variable Cost (US$)	20743		20743	20743	20743	20743	20743
Fixed Costs (US$)	10000		10000	10000	10000	10000	10000
Investment (US$)		125000					
Depreciation (US$)	30%		18750	31875	22313	15619	10933
Income Tax (US$)			19650	6525	16088	22782	27467
Tax rate	40%		7860	2610	6435	9113	10987

Net Cash Flow		**Years**					
		0	**1**	**2**	**3**	**4**	**5**
Net Income (MMUS$)		0	11790	3915	9653	13669	16480
Depreciation (US$)		0	18750	31875	22313	15619	10933
Investment (US$)		-125000	0	0	0	0	0
Salvage Value (US$)	40%	0	0	0	0	0	50000
Effect of the Abandonment Tax	40%	0	0	0	0	0	-9796
NCF		-125000	30540	35790	31965	29288	67617

Discount Rate	15.0%
NPV (US$)	0.0

TABLE 141 Production Level that Makes NPV Equal Zero

Another option is using the graphical solution based on the production levels used before.

Production	-20%	0%	20%
Oil Rate (stbpd)	1600	2000	2400
NPV (US$)	15285	43443	71601

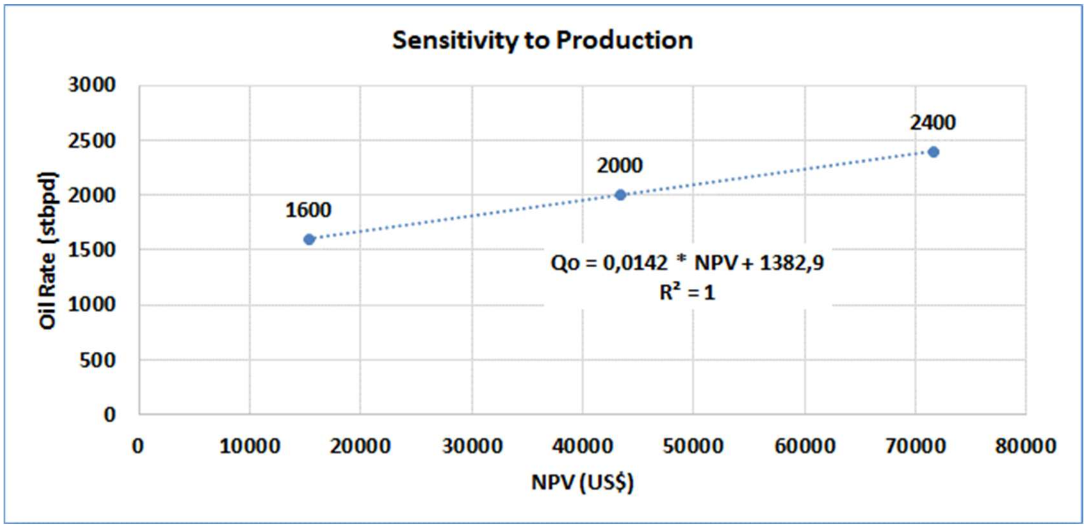


FIGURE 42 OIL PRODUCTION SENSITIVITY THAT MAKES NPV EQUAL ZERO

Using the graphical solution based on sensitivity. This is -20% equals 1600 stbpd, 0% equals 2000 stbpd, and +20% equals 2400 stbpd. The intercept (1383) is the value that makes NPV equal zero.

Scenario Analysis

Although sensitivity and breakeven analysis are useful, they do have limitations. It is not easy to specify precisely the relationship between a particular variable and the NPV. The relationship is complicated by interdependencies among the variables. Keeping operating costs constant while varying production rate may ease the analysis; however, operating costs do not behave in this way. The analysis is more complicated if the movement is in more than one variable at a time.

A scenario analysis is a technique that considers the sensitivity of NPV (or any other economic hurdle) to changes in key variables and to the range of likely variable values. For example, the decision maker may consider two extreme cases, a “best case” scenario and a “worst-case” scenario. For example:

Variable	Worst-Case Scenario	Best-Case Scenario
Production rate	Low	High
Oil price	Low	High

Variable	Worst-Case Scenario	Best-Case Scenario
Variable cost per barrel	High	Low
Fixed cost	High	Low

The NPV under the worst and best conditions are then calculated and compared to the expected, or Base-Case, NPV.

For example, using the same income statement as in Table 139 and assuming that managers are confident of their estimates of all the project's cash flow variables except for the production rate. Also, if they regard a decline in oil rate below 1600 stbpd or a rise above 2200 stbpd as unlikely. Thus, decremental annual rate of 400 stbpd defines the lower bound, or the worst-case scenario, whereas incremental annual rate of 200 stbpd defines the upper bound, or the best-case scenario, compared to the most likely value of 2000 stbpd in annual production rate.

To perform the scenario analysis the corresponding teams, provide their optimistic (Best-Case) and pessimistic (Worst-Case) estimates for the key variables. Then, the pessimistic variable values are used to obtain the Worst-Case NPV and the optimistic variable values to obtain the Best-Base NPV. The results using these combinations of parameters are as follows:

Income Statement	Worst Case	Base Case	Best Case
Production (stbpd)	1600	2000	2200
Barrel Price (US$)	48	50	53
Variable Cost per Barrel (US$)	17	15	12
Income (US$)	76800	100000	116600
Total Variable Cost (US$)	27200	30000	26400
Fixed Costs (US$)	11000	10000	8000
Investment (US$)	125000	125000	125000
Depreciation	30%	30%	30%
Tax rate	40%	40%	40%
Net Cash Flow	**Worst Case**	**Base Case**	**Best Case**
Salvage Value (US$)	30000	50000	60000
Effect of the Abandonment Tax	-1796	-9796	-13796
Discount Rate	15.0%	15.0%	15.0%
NPV (US$)	-5565	43443	91077
AEW (US$)	-1660	12960	27170
IRR	13.2%	15.0%	40.7%
PIR	-0,045	0,348	0,729

TABLE 142 Optimistic (Best), Pessimistic (Worst) and Base Case Estimates for the Key Variables

The Best-Case produces a positive NPV (MUS$ 91.08), the Worst-Case generates a negative NPV (MUS$ -5.57), and the Base-Case has a relative high value NPV (MUS$ 43.44).

From the results of Table 142 it is tough to interpret scenario analysis or to make a decision based on it. It can be said that there is a chance of losing money on the project; though, without a specific probability for this outcome.

Therefore, it is needed estimates of the probabilities of occurrence for the three cases considered and all the other possibilities. This leads to the next step: developing a probability distribution. The goal is to predict the effects on the NPV of variations in the parameters and to assign a probability distribution to the possible outcomes of each parameter and combine these distributions in a way to produce a probability distribution for the possible outcomes of the NPV.

Probability Concepts and Economic Evaluation

Following is a condensed review of different concepts which will be used in the rest of the chapters, about economic evaluation of projects in the hydrocarbon-related industry.

Stochastic Risk Analysis Models

Stochastic modeling forecasts the probability of various outcomes under different conditions, using random variables and it presents data and predicts outcomes that account for certain levels of unpredictability or randomness.

The models are used when the level and magnitude of risk associated with capital projects (e. g. hydrocarbon-industry related projects) is determined by establishing probability distributions for the input variables, rather than treating each of these parameters as being known with certainty. The appropriate definition of probability in this case is “the likelihood of occurrence” and not the “long-run relative frequency” as considered in a time-scale.

The opposite of stochastic modeling is deterministic modeling, which gives you the same exact results every time for a particular set of inputs.

The Monte Carlo simulation is one example of a stochastic model; it can simulate how a project may perform based on the probability distributions of individual input parameters.

Random Variable

This is a parameter that can have more than one possible value. Thus, for each possible or conceivable value of the random variable, there is associated a likelihood or probability of its occurrence.

In probability and statistics, random variables are used to quantify outcomes of a random occurrence, and therefore, can take on many values. Random variables are required to be measurable and are typically real numbers.

For example, the letter X may be designated to represent the sum of the resulting numbers after three dice are rolled. In this case, X could be 3 (1 + 1+ 1), 18 (6 + 6 + 6), or somewhere between 3 and 18, since the highest number of a die is 6 and the lowest number is 1.

In the corporate world, random variables can be assigned to properties such as the average price of oil over a given period, the return on investment after a specified number of years, the estimated OPEX within the following six years, etc.

Risk analysts assign random variables to risk models when they want to estimate the probability of an adverse event occurring. These variables are presented using tools such as scenario and sensitivity analysis tables which risk managers use to make decisions concerning risk mitigation.

Probability Distribution

For a discrete random variable, it is needed to assess the probability values for each random event.

For a continuous random variable, it is required to assess the applicable probability function, as the event takes place over a continuous domain.

In either case (discrete variable or continuous random variable) there is a range of possibilities for each feasible outcome, which (considered as a continuum) make up the corresponding probability distribution.

Continuous Probability Distribution

When input variables are described by discrete probability functions, there are a finite number of possible outcomes, each of which can be calculated along with its respective probability of occurrence.

If an input variable is allowed to be continuous rather than discrete, probability functions are used to describe these inputs parameters, and there is no longer a finite set of possible outcomes. Each random variable can assume an infinite number of values within specific ranges or boundaries.

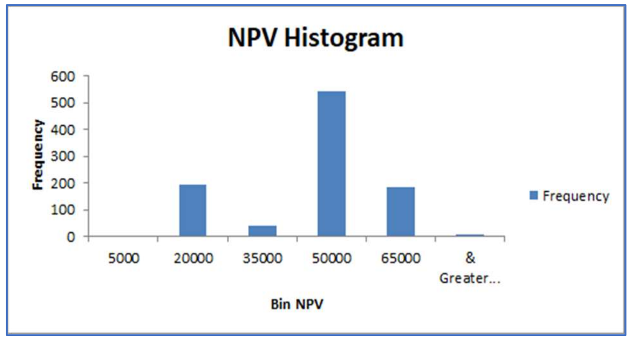
Histogram

Histograms are a way to display counts of data. They are a graphical representation that organizes a group of data points into user-specified ranges. Similar in appearance to a bar graph, the histogram condenses a data series into an easily interpreted visual by taking many data points and grouping them into logical ranges or bins. Histograms are commonly used in statistics to demonstrate how many of a certain type of variable occurs within a specific range.

A histogram is a representation of data that falls into a range of outcomes into columns along the x-axis.

The y-axis represents the number count or percentage of occurrences in the data for each column and can be used to review and analyze data distributions.

A bar graph charts actual counts against categories; The height of the bar indicates the number of items in that category. A histogram displays the same categorical variables in “bins”.



A bin (ΔX) shows how many data points are within a range (an interval). Normally, the analyst chooses the range that best fits the data. There are no set rules about how many bins we can have, but the rule of thumb is 5-20 bins. Any more than 20 bins and the graph will be hard to read. Fewer than 5 bins and the graph will have little (if any) meaning. Most histograms in elementary statistics will have about 5 to 7 bins. A rule of thumb is:

Bin NPV	*Frequency*	*% Cummulative*
5000	0	0,00%
20000	195	20,27%
35000	39	24,32%
50000	540	80,46%
65000	182	99,38%
& Greater...	6	100,00%

$$\Delta X = 5 * \frac{(X_n - X_1)}{n}$$

Simulation Studies

Simulation is the process of building a physical or theoretical model of a system, to reproduce significant elements of condition or behavior. Experimentation with such a model may improve understanding of the likely character and behavior of the real system. Physical simulation may take the form of an aircraft flight deck or drilling rig floor; numerical simulation may include development scenarios for a hydrocarbon reservoir.

A cash flow model is a form of theoretical simulation model, representing the financial logic. Altering the input data to this model enables the analyst to investigate the likely performance of the project and to identify situations, which might lead to a financial loss. The sensitivity study described earlier (Table 142) identified a situation leading to a loss, namely the worst-case scenario. This is a rudimentary form of simulation study.

Income Statement	Worst Case	Base Case	Best Case
NPV (US$)	-5565	43443	91077

All uncertain or stochastic, project parameters (or variables) are subject to deviation from their Base-Case values. Consequently, sensitivity analysis, involving change in individual parameter value, has limited application. The alternative is to consider the impact on project economics of allowing all uncertain parameters to vary.

Standard Deviation

The standard deviation is a statistic parameter that measures the dispersion of a dataset relative to its mean. If the data points are further from the mean, there is a higher deviation within the data set; thus, the more spread out the data, the higher the standard deviation.

The general formula used for computing Standard Deviation is:

$$S = \sqrt{\frac{\sum_{i=1}^{n}(x_i - X)^2}{n-1}}$$

Where:

x_i = Value of each data point

X = Mean

n = Number of data points

The mean value is calculated by adding all the data points and dividing them by the number of data points.

The ***variance*** is the square of the standard deviation. Later, in the chapter Measure of Variance these topics will be expanded.

Cash Flow Logic

This means that the project cash flow model is constructed in the usual way, to include all the appropriate computational logic and values for all relevant parameters.

Previous example has some ***deterministic*** data or parameters known with reasonable certainty, whereas some could be ***stochastic***, this is involving a random variable. Stochastic parameters will form the basis of the simulation study and therefore require further investigation.

Random Number Samples

The probabilistic calculation method requires individual values of each stochastic parameter to be chosen to create multiple “states of nature,” all of which are feasible and the spread of which reflect the underlying probability distributions.

Two methods are commonly applied, Monte Carlo Sampling (**MCS**) and Latin Hypercube Sampling (LHS). The latter is sometimes called Stratified Sampling. Random number generation is the basis of both methods and both works best with cumulative probability distributions.

Cumulative Distribution Function (F(X))

This representation shows the probability that a random variable will have a value "smaller than or equal to" some value ***x***.

Discrete Random Variables (DRV)

This name DRV applies to any random variables that take only isolated values. Discrete random variables take on a countable number of distinct values.

Consider an experiment where a coin is tossed three times. If X represents the number of times that the coin comes up heads, then X is a discrete random variable that can only have the values 0, 1, 2, 3 (from no heads in three successive coin tosses to all heads). No other value is possible for X.

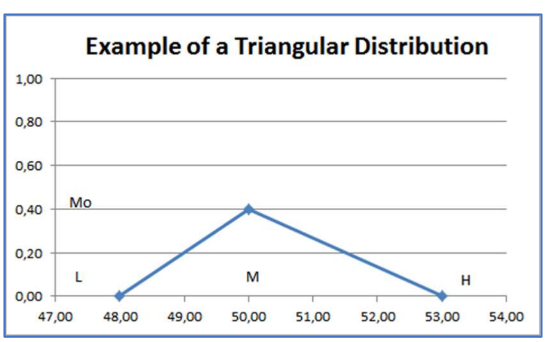
Continuous Random Variables (CRV)

They can represent any value within a specified range or interval and can take on an infinite number of possible values. An example of a continuous random variable would be an experiment that involves measuring the oil price over a year or the average porosity of a random group of 25 cores.

Continuous Random Variables (CRV) Description

TRIANGULAR DISTRIBUTION

This representation corresponds to variables that may have any value in certain interval. For a CRV, a minimum value (**L**) and a maximum value (**H**) are determined. Then, a most likely value (**Mo**) is calculated. If there is a Mode or Modal value, the variable is represented by a triangular distribution.



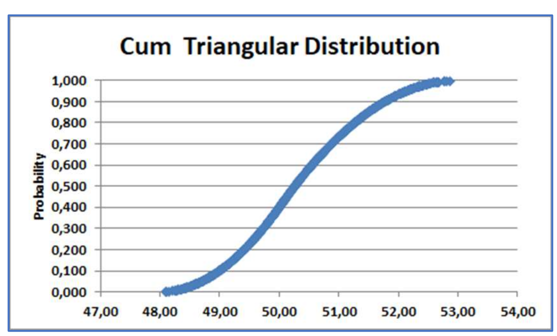


FIGURE 43 EXAMPLE OF A TRIANGULAR DISTRIBUTION

All probability distribution functions have the property that the area under the function is equal to 1.0. For the triangular distribution, this property implies that the maximum value (Mo) of the probability distribution function is (p_{Mo}-L)/(H-L). It occurs at the peak value of Mo.

The probability density function of a triangular distribution is zero for values below L and values above H. It is linear, it rises from zero at L to Mo at M, then dropping down to zero at H.

The formula for the probability density function or cumulative distribution is as follows:

($P_{(x)}$)	**When**
0	x < L
$\frac{2\,(x-L)}{(H-L)(M-L)}$	H <= x <= M
$\frac{2\,(H-x)}{(H-L)(H-L)}$	when M <= x <= H
0	x >= H

And the distribution function is

For L ≤ x ≤ M

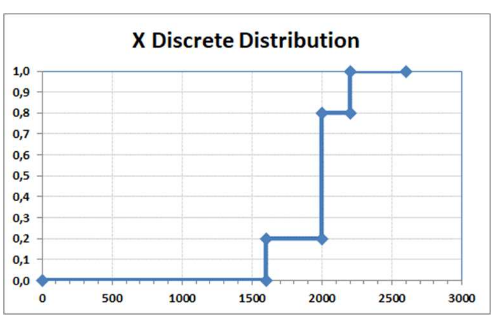
$$D(x) = \frac{(x-L)^2}{(H-L)*(M-L)}$$

For M < x ≤ H

$$D(x) = 1 - \frac{(H-x)^2}{(H-L)*(H-M)}$$

Discrete Distribution

If there is no Mode, a discrete distribution can be used



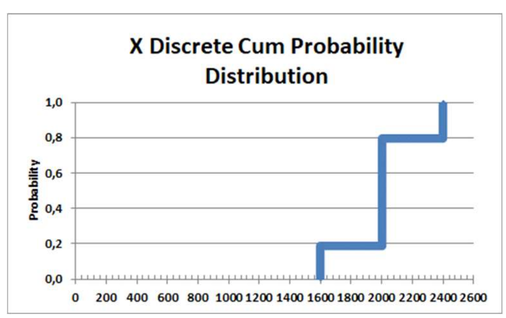


FIGURE 44 EXAMPLE OF A DISCRETE DISTRIBUTION

UNIFORM DISTRIBUTION

If there is no reason to assume that one value is more likely to occur than any other value, a uniform distribution is usually used.

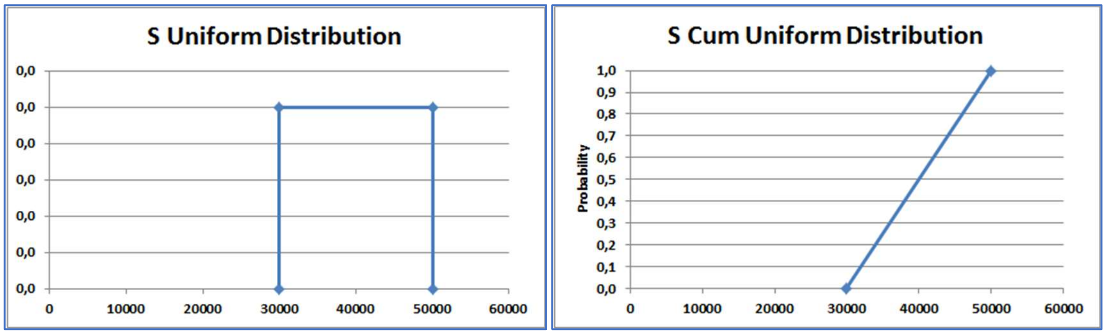


FIGURE 45 EXAMPLE OF A UNIFORM DISTRIBUTION

Simulation Analysis

If for example NPV can be associated with continuous random variables, it is possible to use probabilities to run several cases. As an example, using the values from Table 142, if NPV is a function of the three variables indicated in Table 143 (X, Y, S):

$$Mo = (P_{Mo} - L)/(H-L)$$

Discrete		Triangular		Uniform	
P_x	X_f	P_y	Y_f	P_s	S_f
0.2	1600	0.0	48	0.00	30000
0.6	2000	0.4	50	1.00	50000
0.2	2200	1.0	53		

TABLE 143 Probabilities for Discrete, Triangular, and Uniform Parameters

Where **X** is the oil rate, **Y** is the oil price and **S** is the salvage value and if the NPV, for example, has the form of:

$$NPV\ (15\%) = -112263 + 2.0113*X*(Y-15) + 0.2983*S$$

It is possible to run several cases to obtain the following results:

		PW(15%) = -112263+2.0113*X*(Y-15)+0.2983*S				
		Count	Std Dev NPV	Min NPV	Maz NPV	Average NPV
		962	14932	5138	67582	39132
Random P_x	X	Random P_y	Y	Random P_s	S	NPV
0,07	1600	0,10	49,00	0,90	47988	11474
0,93	2200	0,66	50,75	0,89	47822	60204
0,74	2000	0,55	50,40	0,26	35194	40648
0,96	2200	0,21	49,45	0,15	33088	50059
0,00	1600	0,09	48,96	0,43	38603	8551
0,32	2000	0,83	51,42	0,44	38787	45808
0,34	2000	0,85	51,48	0,21	34189	44664
0,55	2000	0,46	50,16	0,51	40174	41146
0,95	2200	0,11	49,03	0,38	37525	49504
0,76	2000	0,52	50,32	0,90	48026	44130
0,36	2000	0,08	48,91	0,83	46518	38033

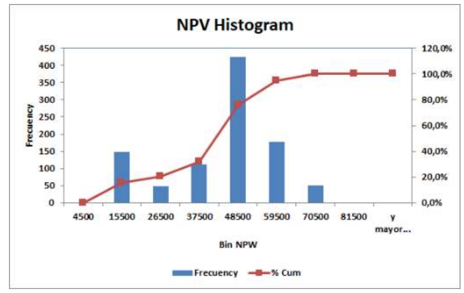


FIGURE 46 NPV VARIATION

With the assurance that 962 trials were sufficient, the relative frequencies can be interpreted as probabilities. The NPV values range between US$ 5138 and US$ 67582, indicating no loss in any situation. The NPV distribution has an expected value of US$ 39132 and a standard deviation of US$ 14932. Although, the expected NPV value (US$ 39132) is lower than the Base Case NPV (43443), the project may be approved.

Another option is analyzing a single project in a **deterministic way** and use the results to run sensitivity on some of the inputs. For example, using this cash flow of the Base Case and its results:

	Cash Flow Economic Analysis					Option 1			
Input						Use as equivalent rate:		15,00%	i = i'+f+i'*f
Base Case NPV			Clear Input			Option 2			
Project Life (N, yrs)	5		*Max 40 Yrs*			15,0%	is corrected to		
Initial Investment (P_{oINF}, $)	125000						To be added to i		
Interest (I)	15,0%		inflation (f)	0,0%	r corected by f =		15,00%		r = (i - f)/(1 + f)
Fill Rows 8 & 10		Yr	0	1	2	3	4	5	
Annty Outflow (C, $ per year)		Ci		0,0	0,0	0,0	0,0	0,0	
Annty Inflow (A, $ per year)		Ai		43500	48750	44925	42248	40373	
Sunk Cost @ N (S, $)	9796								
Salvage Value @ N (Sv, $)	50000								

Results	
Profit to Investment Ratio (PIR)	**0,348**
Discounted Payout (Payback yrs)	**3,84**
NPVI	**0,348**
Net Future Worth (FW_{PC}, $)	**87380**
Net Present Value (NPV, $)	**43443**
Benefit/Cost Ratio (PI)	**1,335**
Internal Rate of Return (IRR,%)	**27,76%**
	Calc IRR
Eqvlnt Total Annual Cost (A_{PC}, $)	**12960**

TABLE 144 Individual Project Sensitivity

Using the Annual Equivalent Value (A) and varying A, N and i from Table 144, applying maximum and minimum estimated parameters from Table 142; it is also possible to estimate the impact on NPV:

Sensitivity on NPV

Sensitivity on Input Values		A	N	i
	Max	27170	6	16,0%
	Min	-1660	4	14,0%

Known Values	P	A	N	i	NPV
	125000	12960	5	15,0%	43443
Results / Sensitivity		82	4	15,0%	234
		13018	5	15,0%	43638
		6226	4	15,0%	17775
		25239	4	16,0%	70623
		8834	6	16,0%	32551
		11672	6	15,0%	44172
		15537	6	16,0%	57250
		1698	5	15,0%	5692
		10805	4	15,0%	30848

Project	
Runs	100
Max NPV	99994
Average NPV	38383
Min NPV	-5522
Std Dev NPV	27444

TABLE 145 Sensitivity on NPV

For most analysis 100 simulations or experiments will usually achieve the necessary steady-state conditions in the results. The NPV of the Base Case US$ 43443 is compared to the sensitivity run NPV of US$ 38383.

Monte Carlo Methodology

Probabilistic distributions are as accurate as the data from which the distribution was derived. If estimates of the evaluation parameters are accurate, full simulation risk analysis provides detailed information of the magnitude of risk that the corporation accepts if the decision is to continue with a given project. This procedure is the basis of what is called generally the Monte Carlo methodology. Additional examples will be presented along this chapter. As a brief introduction, the standard general process for a probabilistic method such as the Monte Carlo is illustrated in Figure 47:

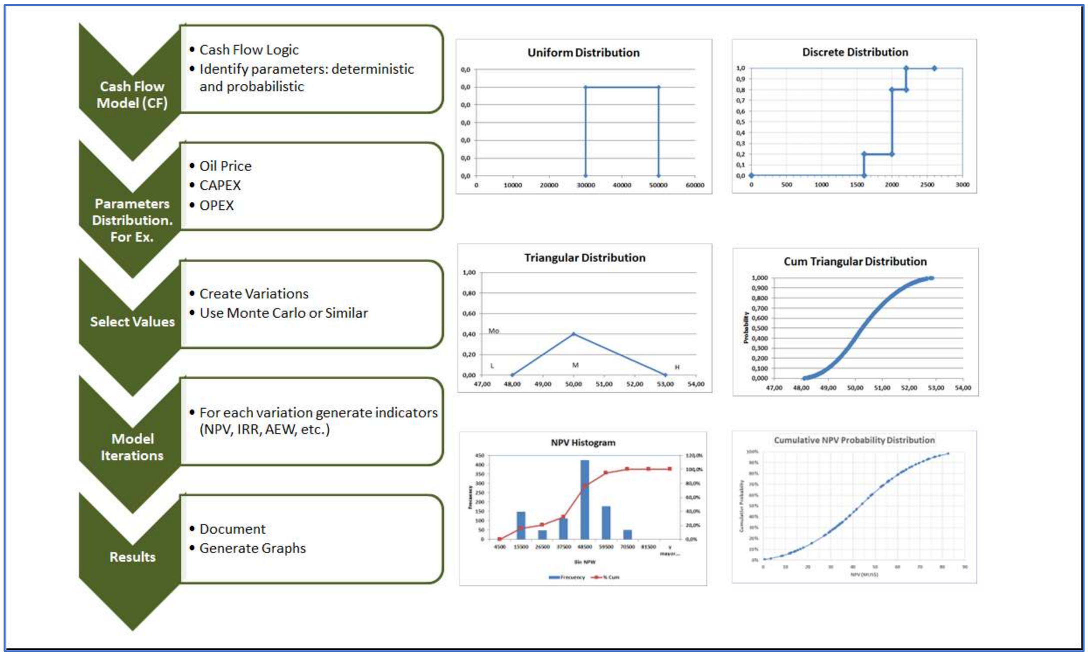


FIGURE 47 STANDARD GENERAL PROCESS FOR A PROBABILISTIC METHOD

Probability distributions are constructed for each of the uncertain parameters and used to create a series of possible "states of nature."

The selection process normally involves some form of random number generation. Each possible state of nature forms the basis of an iteration of the cash flow model, leading to the calculation of a value for an appropriate measure, such as NPV.

Results from several such iterations are collated and plotted to characterize the range of NPV values consistent with the data.

After enough repetitive simulations trials has been run, the analysis is essentially completed. The remaining tasks are to tabulate the computed NPV values to determine the expected value and to make various graphic displays useful to management.

Interpretation of Simulations Results

Once analysts obtain an NPV frequency distribution they need to make the assumption that the actual relative frequencies are representative of the probability of having an NPV in each range. That is, analysts assume that the relative frequencies they observe in the sampling are representative of the proportions they would have obtained if they had examined all the possible combinations.

After obtaining the probability distribution of the NPV, analysts face a question: how to use this distribution in decision making?

Consider that the probability distribution provides information regarding the probability that a random variable will attain some value $\underline{x}$. This information can be used in turn, to define the cumulative distribution, which expresses the probability that a random variable will attain a value smaller or equal to some ***x***.

If the NPV distribution is known, analysts can compute the probability that the NPV of a project will be negative. Analysts use this probabilistic information in judging the probability of the project meeting the desired targets.

Example of Stochastic Parameters and Probability Distribution

The conventional method of defining or describing uncertainty is to construct a probability density function or distribution. This plots likelihood or probability against parameter value. Some parameters are better understood than others. However, generally the plots and/or legends do not incorporate details beyond the most obvious. If the extreme range is all that is known, that is all that should be incorporated into the model. For example, these parameters could be considered stochastic:

- Project CAPEX
- Fixed OPEX
- Oil price
- Gas price
- Rate of inflation
- Exchange Rate
- Rate of Corporation Tax

In general, it could be considered a limited range of distribution types; but in practice, a variety of empirical and theoretical distributions are applied. Some of these distribution types were already described; however, considering the importance of the topic it is expanded in the following paragraphs.

UNIFORM DISTRIBUTION

A uniform distribution specifies a range of values, with no central tendency, the assumption being that probability of occurrence is uniform across the specified range.

Three of the following parameters are considered to have uniform distributions, namely:

- Rate of Inflation
- Exchange Rate
- Corporation Tax Rate

These are included in Table 146, which identifies distribution type and Minimum and Maximum values. For Inflation, these are given as 1.5% and 4%.

Parameter	Type	Units	Mín.	Value 1	Value 2	Value 3	Value 4	Value 5	Máx.
Inflation	Uniform	%	1.50	1.75	2.25	2.75	3.25	3.75	4.00
Exchange Rate	Uniform	Ratio	1.400	1.425	1.475	1.525	1.575	1.625	1.650
Tax Rate	Uniform	%	28.0	28.5	29.5	30.5	31.5	32.5	33.0
Fix OPEX	Triangular	MMUS$/year	15.00	17.20	18.85	19.95	21.10	22.71	25.00
CAPEX	Triangular Skewed	MMUS$	850	887	924	968	1012	1080	1200
Oil Price	Irregular	US$/bl	10.0	13.8	17.2	19.1	23.9	29.6	35.0
Gas Price	Dependent	US$/Temp	10.0	13.8	17.2	19.1	23.9	29.6	35.0

TABLE 146 Parameters and Distribution Type

Figure 48 is a plot of the probability distribution for Inflation. It shows the range of values, which have been chosen for this analysis. Values falling outside the given range are not considered. Within the range, all values are equally likely.

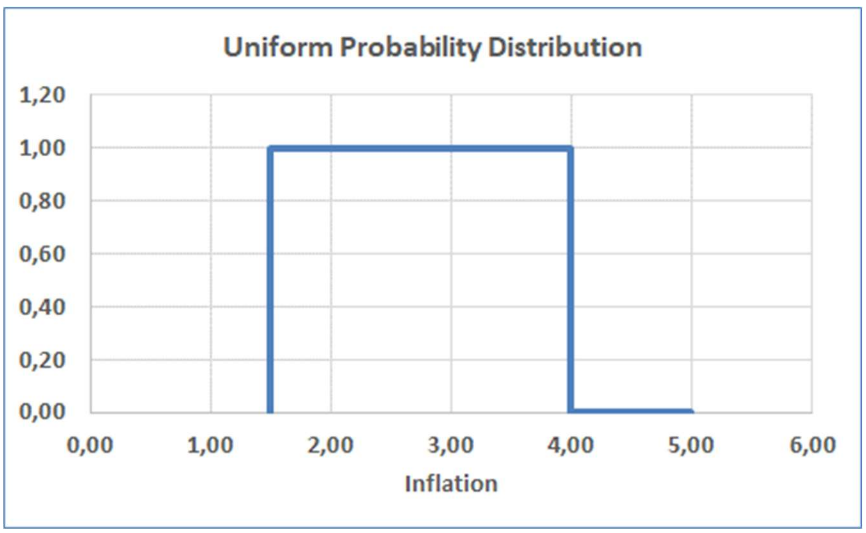


FIGURE 48 INFLATION UNIFORM PROBABILITY DISTRIBUTION

Triangular Distribution

A triangular distribution specifies a range of values and a most likely or modal value. If the mode is central to the range, the distribution is ***symmetrical***, if it is non-central, the distribution is ***skewed***. Two of the parameters are assumed to have triangular distributions:

- CAPEX
- Fixed OPEX

OPEX is considered in this example to have symmetrical distribution, with equal probability of being above or below the modal value.

Example of OPEX calculation distribution

OPEX	Mod	Prob	
15,00	0,12	0	
19,19	0,50	0,42	*(pMo-L)/(H-L)*
25,00	1,03	0	

CAPEX is skewed, having a greater probability of being higher.

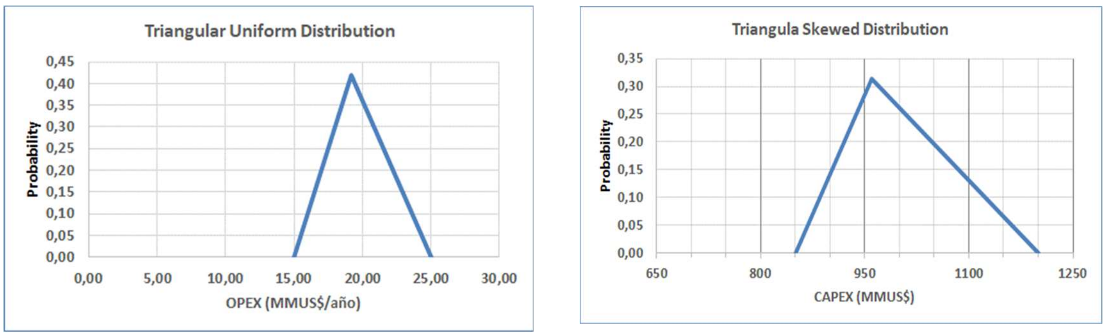


FIGURE 49 TRIANGULAR DISTRIBUTION UNIFORM AND SKEWED FOR OPEX AND CAPEX

Irregular Distribution

A non-geometrical and non-theoretical distribution may be described as "irregular." An irregular distribution may be derived from unadulterated, empirical or historical data. The distribution in Figure 50 is derived from historical prices.

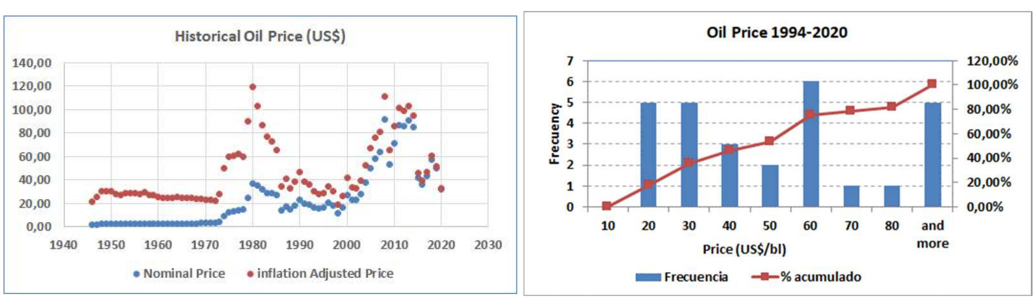


FIGURE 50 IRREGULAR DISTRIBUTION_ OIL PRICE

The "X-axis" label of the histogram in Figure 50 on the right, $10 means the range $10-20 In practice, such an analysis would reflect the purpose of the investigation and the level of knowledge and understanding contemporaneous with it. However, in this example the concern is with the mechanics of the process.

THEORETICAL DISTRIBUTION

Normal and lognormal distributions for example may be used in appropriate circumstances. None is used in this example. Normal distribution, also known as the Gaussian distribution, is a probability distribution that is symmetric about the mean, showing that data near the mean are more frequent in occurrence than data distant from the mean. In graphical form, normal distribution will appear as a bell curve. A lognormally-distributed random variable is a random variable whose logarithm is normally-distributed (right hand side Figure 51).

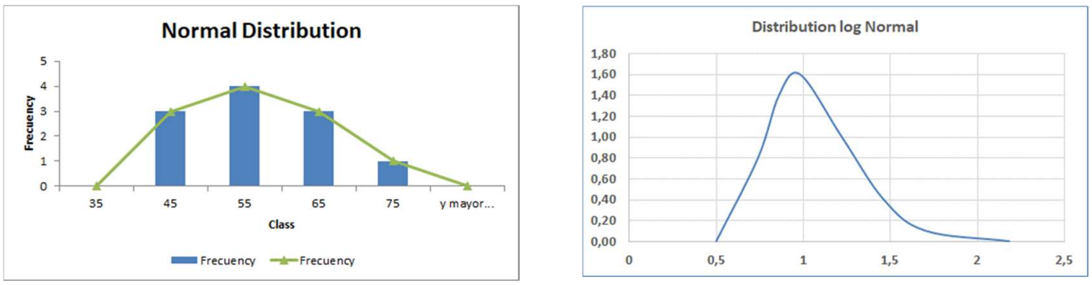


FIGURE 51 NORMAL DISTRIBUTION

RANDOM NUMBER SAMPLING AND THE MONTE CARLO SAMPLING (MCS)

Figure 52 illustrates the MCS method. It is derived from the CAPEX distribution in Table 146.

Parameter	Type	Units	Mín.	Value 1	Value 2	Value 3	Value 4	Value 5
Inflation	Uniform	%	1,50	1,75	2,25	2,75	3,25	3,75
Exchange Rate	Uniform	Ratio	1,400	1,425	1,475	1,525	1,575	1,625
Tax Rate	Uniform	%	28,0	28,5	29,5	30,5	31,5	32,5
Fix OPEX	Triangular	MMUS$/year	15,00	17,20	18,85	19,95	21,10	22,71
CAPEX	Triangular Skewed	MMUS$	850	887	924	968	1012	1080
Oil Price	Irregular	US$/bl	10,0	13,8	17,2	19,1	23,9	29,6
Gas Price	Dependent	US$/Temp	10,0	13,8	17,2	19,1	23,9	29,6

A random number (RN) was selected, by computer generation. Numbers are commonly present in pairs, therefore 100 pairs from 00 to 99.

Example of the probability distribution calculation

CAPEX	Mod	Prob	Cum Prob
850	0,2	0	0
960	0,5	0,31	0,31
1200	1,1	0	1

CAPEX (MMUS$)	p
869,793	0,011
875,469	0,018
878,717	0,023
886,641	0,037
890,935	0,046
903,733	0,080
911,988	0,107

For a random p = 0.011

$$p(x) = \frac{(x-L)^2}{(H-L)*(M-L)}$$

$$CAPEX = (p \ast (H-L) \ast (M-L))^{1/2} + L$$

$$\text{CAPEX} = (0.011 \ast (1200 - 850) \ast (960 - 850))^{1/2} + 850$$

$$\text{CAPEX} = 870$$

The total cumulative probability is presented in Figure 52. In the example, if a probability of 61.2 has been selected, entering the Figure 52 with p = 61.2, gives the parameter value selection MMUS$ 1017 (red dot).

Entering each distribution in turn, with a new random number, generates a feasible state of nature. Furthermore, if the numbers used are truly random, the values of each parameter should appear in proportion to their underlying probabilities.

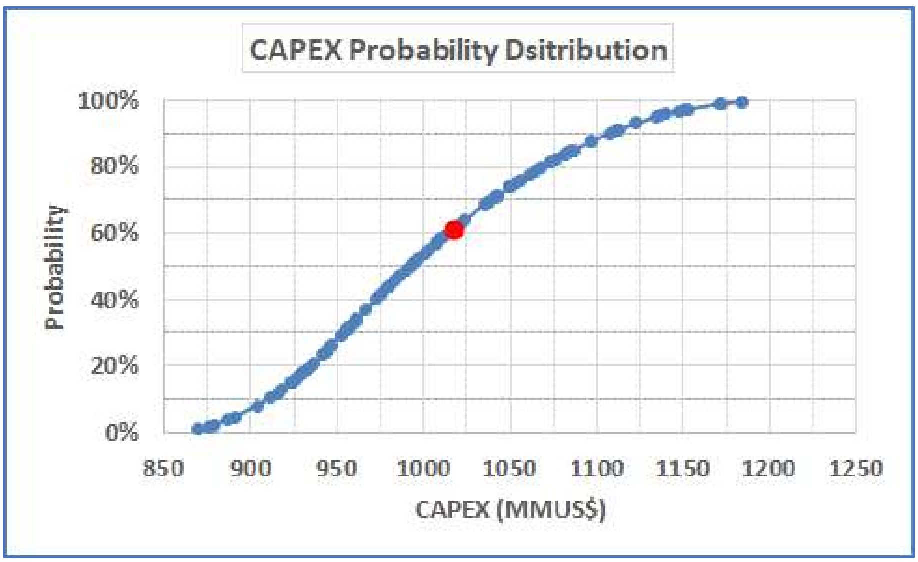


FIGURE 52 DERIVED PROBABILITY FOR CAPEX

PARAMETER DEPENDENCE

The process used to sample values from the various distributions is based on separate random numbers. This procedure is only valid if the parameters, which are sampled independently, are truly independent.

If there is a relationship, one parameter may be sampled independently and the other derived from an appropriate relationship.

For above example in Table 146, it is assumed that gas price is related to oil price. There is often a contractual link, often with a time-delay. For convenience, the complexity of the relationship is simplified and a perfect correlation is assumed. In the analysis, once a dollar price for oil is selected, an equivalent dollar price for gas is also selected. Any impact on this correlation, by varying exchange rate has been ignored.

The Monte Carlo Method for STOIIP Calculation

Sensitivity analysis is most appropriate when the effects of deviations in various forecasts are to be assessed one at a time. In a typical investment, however, many variables interact in a complex way to determine the overall risk of the project.

For example, a project NPV is determined by net cash flows for several periods. The net cash flows themselves are made up of several costs and revenues, and each of these items also depends on several operating variables, including production schedules and rates, market variables, and the like. With uncertainties inherent in each of these interrelated variables, the overall impact on the economic desirability of the investment is not straightforward.

In mathematical terms the problem can be expressed as follows: A measure such as IRR is a function of several variables, which can be labeled as $X_1, X_2\ X_3, \ldots\ldots X_n$.

This is: $IRR = f(X_1, X_2\ X_3, \ldots\ldots X_n)$

Each *X* is subject to uncertainty and has a probability distribution associated with it. These variables could be in some cases dependent.

Analysts may want to find the probability distribution for the IRR given that it is known the function *f* and analysts have received from operating personnel probability distributions for all the individual variables and their statistical interrelationships.

Direct mathematical solutions that derive such a probability distribution are possible only under very special circumstances. When such direct solutions are not available, as is typically the case, forecasters can resort to Monte Carlo simulation, which is a computer-based technique that offers solutions, based on sampling, to problems of this type.

This method uses the cumulative distribution functions of the variables. It enables the user to examine the effect of randomness upon the predicted outcome of numerical models.

The technique calls for:

- The identification of key variables that affect the investment's cash flow
- The assignment of probability distributions to each variable
- The specification of any statistical dependencies that may exist between different variables
- Then, giving due recognition to the likelihood of outcomes, the computer randomly selects values for each variable and combines them to generate cash flows and IRR (for example)
- Through enough trials, a distribution of IRR (in this case) is provided

The use of the NPV in this context can be problematic, as the discount rate on which NPV calculations are based should reflects an appropriate adjustment for the overall risk of the project.

As mentioned, the method requires a model that relates the input variables to the feature of interest. For example, in the case of hydrocarbon reserves the input variables are the reservoir properties, and the features of interest could be: original hydrocarbon in place, ultimate oil recovery, breakthrough time, or water cut. The variability of the output quantities is used to make decisions about: economic viability, additional data acquisition and exploitation strategy.

Similarly, the method can be used to estimate for example the Original Oil in Place (OOIP) or as it is also known the Stock Tank Oil Initially in Place (STOIIP), as follows:

$$STOIIP = \frac{\emptyset\,(1 - S_w)\,A\,h}{\beta_{oi}}$$

Where the input variables are:

A h = net reservoir bulk volume

B_{oi} = initial oil formation volume factor

S_w = interstitial or initial water saturation

Ø = porosity

In this case, the equation is the "model" used to calculate the continuous distribution function ($F_{(x)}$) of the STOIIP, and the input variables may all be considered as independent random variables, only if they represent average values over a given net reservoir volume (A h).

A random number creator generates random numbers for all the input variables in the STOIIP equation for a specified $F_{(x)}$. For each value of the input variables the STOIIP is computed. This process is repeated several hundred times. The output is a series of STOIIP values that, using the empirical Cumulative Distribution Function ($F_{(x)}$) procedure (triangular, discrete or uniform), will yield values of STOIIP $F_{(x)}$. In this case the distribution function is any value between the minimum and the maximum. From that Distribution Function, summary statistics such as the average or median will be calculated.

Sensitivity on STOIP

Sensitivity on Input Values		A h (ac-p)	Porosity	Sw	Boi (v/v)
	Max	1,2	0,24	0,24	1,36
	Min	0,8	0,16	0,16	1,12

	A h	Porosity	Sw	Boi	STOIP (stb)
Known Values	1,0	0,20	0,20	1,24	1001
Results / Sensitivity	0,8	0,24	0,22	1,28	908
	1,2	0,24	0,20	1,31	1364
	0,9	0,20	0,20	1,26	887
	1,1	0,17	0,23	1,22	916
	1,2	0,21	0,17	1,24	1309
	0,9	0,19	0,21	1,21	866
	1,2	0,20	0,19	1,16	1300
	0,9	0,18	0,17	1,34	778
	0,9	0,23	0,18	1,29	1021
	1,0	0,23	0,21	1,28	1101

Count	100
Max	1532
Average	1030
Min	645
Std Dev	188

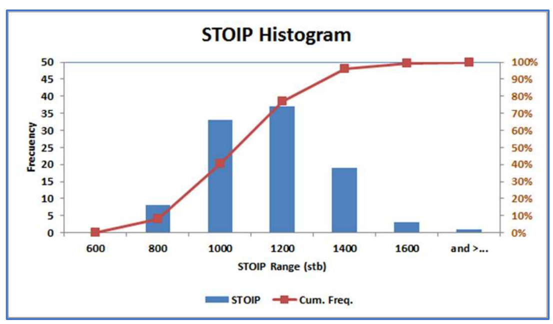


FIGURE 53 STOIIP CALCULATION USING MONTE CARLO METHOD

The distributions for all the input variables must be assigned to use the Monte Carlo Method. Experience from other fields, data from the field under study, and geological knowledge, all contribute to the selection of the function $F_{(x)}$ for each variable. The results are sensitive to the input $F_{(x)}$ and, consequently, the variability in the input data will affect the results. Interdependence between variables can be accommodated and represented if it is known how the variables are interrelated. For example, the handling of low porosity and high-water saturation can be reflected by known correlations between both parameters.

Implicit in the full "simulation of risk procedure" is that the random variables are independent. In other words, the value of any one parameter is not affected by the value of any other. If interdependence of variables exists it is advisable to treat the interdependent variables as constant values, rather than as random variables and use deterministic approaches.

The method is so popular that some governments require Monte Carlo Simulation results to be reported when a company wants to proceed to develop a "prospect." However, successful capital investment decision-making in hydrocarbon business projects requires diverse components such as: good data, engineering, and management.

Risk analysis is an analytical tool which helps to improve and conditions managerial judgement by allowing the development of a greater insight into the sources of risk and their impact on the project's financial evaluation results.

Latin Hypercube Sampling (LHS)

Latin hypercube sampling (LHS) is a method that can be used to sample random numbers in which samples are distributed evenly over a sample space.

It is widely used to generate samples that are known as "controlled random samples" and is often applied in Monte Carlo analysis because it can reduce the number of simulations needed to achieve accurate results.

Consider the following simple example: Suppose analysts would like to obtain a sample of 2 values from a dataset that is normally distributed with a mean of 0 and a standard deviation of 1. If they used a true random number generator to obtain this sample, it is possible that both values could be greater than 0 or that both values could be less than 0.

However, if analysts used Latin Hypercube sampling to obtain this sample, then it would be guaranteed that one value would be above 0 and one would be below 0 because they could specifically partition the sample space into one region with values above 0 and one region with values below 0, then select a random sample from each region.

Figure 54 illustrates the LHS method. It is derived from the Inflation distribution assuming the uniform type from Table 146.

Parameter	Type	Units	Mín.	Value 1	Value 2	Value 3	Value 4	Value 5
Inflation	Uniform	%	1,50	1,75	2,25	2,75	3,25	3,75
Exchange Rate	Uniform	Ratio	1,400	1,425	1,475	1,525	1,575	1,625
Tax Rate	Uniform	%	28,0	28,5	29,5	30,5	31,5	32,5
Fix OPEX	Triangular	MMUS$/year	15,00	17,20	18,85	19,95	21,10	22,71
CAPEX	Triangular Skewed	MMUS$	850	887	924	968	1012	1080
Oil Price	Irregular	US$/bl	10,0	13,8	17,2	19,1	23,9	29,6
Gas Price	Dependent	US$/Temp	10,0	13,8	17,2	19,1	23,9	29,6

The cumulative distribution is divided into several equal (equally likely) layers. In this example, five layers are used. The same number of layers is applied to each of the parameter distributions. The layer is selected in the random number process by 100 / 5 equals 20 random numbers and the layer is represented by a single parameter value, usually chosen to be the value at the mid-point of the probability range. More complex procedures can use a second sampling process to identify the position of the representative value in the layer. In this example, the first (bottom) layer is selected by random numbers in the range 00 - 19 and has a representative value of 1.75. A critical element of the LHS procedure is that sampling is forced to visit each of the layers. Repeat visits are not permitted until all layers have been visited once. Third visits are not permitted until all have received two visits, and so on.

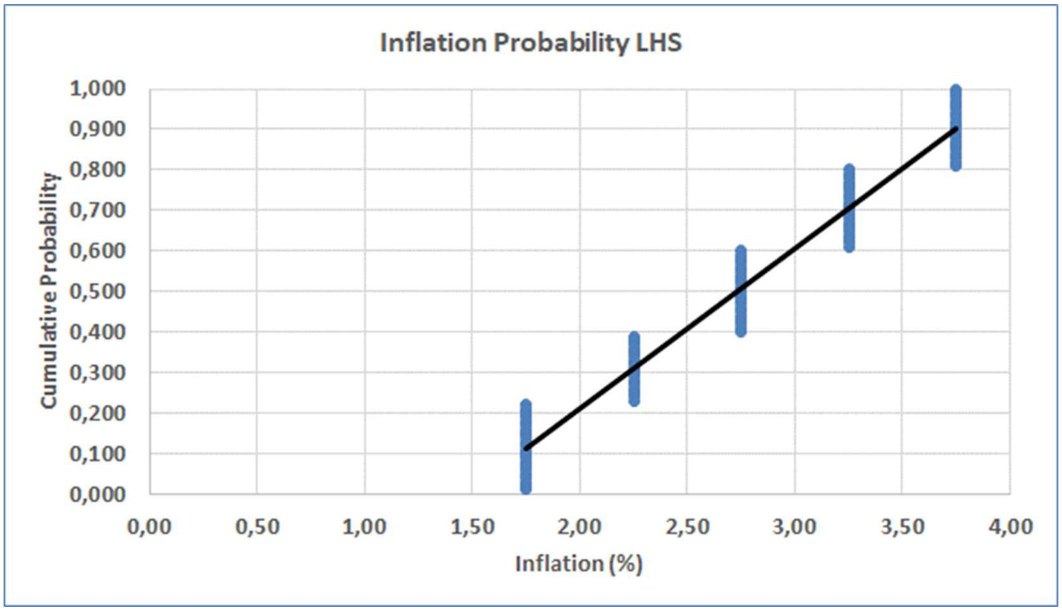


FIGURE 54 CUMULATIVE PROBABILITY LHS METHOD

LHS is a more efficient process as it produces a more evenly spread set of results. Latin Hypercube Sampling is included in specialized statistical packages for the creation of optimized LHS plans.

Monte Carlo Method for NPV Calculation Example

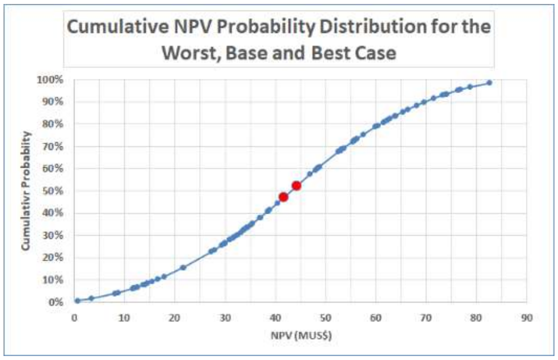
Using the data from the Chapter Scenario Analysis and Table 142 (repeated below) it is possible to generate the sampling process, which is summarized in Table 147 and based on triangular distribution for the NPV (@ i = 0.15). This has generated a cumulative probability distribution of the project and it is plotted in Figure 55.

Income Statement	**Worst Case**	**Base Case**	**Best Case**
Production (stbpd)	1600	2000	2200
Barrel Price (US$)	48	50	53
Variable Cost per Barrel (US$)	17	15	12
Income (US$)	76800	100000	116600
Total Variable Cost (US$)	27200	30000	26400
Fixed Costs (US$)	11000	10000	8000
Investment (US$)	125000	125000	125000
Depreciation	30%	30%	30%
Tax rate	40%	40%	40%
Net Cash Flow	**Worst Case**	**Base Case**	**Best Case**
Salvage Value (US$)	30000	50000	60000
Effect of the Abandonment Tax	-1796	-9796	-13796
Discount Rate	15.0%	15.0%	15.0%
NPV (US$)	-5565	43443	91077
AEW (US$)	-1660	12960	27170
IRR	13.2%	15.0%	40.7%

Assume that the “Base Case” has a probability close to 50%. The probability of a negative result is in the range below 10%. The outcome is likely to be around NPV of 43 MUS$.

NPV	**Probability**
-5565	0.10
43443	0.50
91077	1.00

TABLE 147 NPV Triangular Distribution Probability



if p =0	if p <= Mo	if p > Mo <= H	if p = 1	Calc Y	Random Prob	NPV (MUS$)	Probability
	21588,25			21588,25	0,15567	647	0,008
		49672,43		49672,43	0,62759	3311	0,017
	30482,29			30482,29	0,27435	8008	0,039
		51081,31		51081,31	0,65250	8676	0,043
	37100,61			37100,61	0,38434	11523	0,062
		46317,79		46317,79	0,56480	11788	0,064
	34361,89			34361,89	0,33658	11812	0,064
	18678,89			18678,89	0,12410	12028	0,065
	37677,42			37677,42	0,39480	12406	0,068
	39652,99			39652,99	0,43170	12537	0,069
		50053,45		50053,45	0,63441	13609	0,078
	7858,35			7858,35	0,03804	13953	0,080

FIGURE 55 CUMULATIVE NPV PROBABILITY DISTRIBUTION

Additional Notes on Probability

Probability is a specialized topic and is explained in several books. The content presented in this chapter is just a summary to relate the use of probability in the world of economic analysis.

Probabilities can be represented either by a probability distribution or a cumulative probability distribution. With a probability distribution, the proportion of the area under the curve that falls within any given range indicates the probability of the forecasted variable falling between the two specific values. The empirical rule, or the 68-95-99.7 rule, tells where most of the values lie in a normal distribution:

- Around 68% of values are within 1 standard deviation from the mean
- Around 95% of values are within 2 standard deviations from the mean
- Around 99.7% of values are within 3 standard deviations from the mean

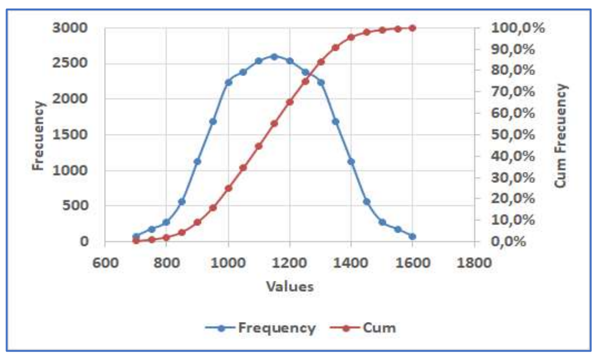


FIGURE 56 NORMAL DISTRIBUTION EXAMPLE

					Count
Mean	1150		**Total**		19
Std. Dev.	281		**Mean + Std. Dev**	1431	13
			Mean - Std. Dev	869	

TABLE 148 Results from Example

A cumulative probability distribution allows to read off on the vertical axis the probability that the forecasted value will be below the corresponding value on the horizontal axis.

Probability distributions are simply explicit presentations of forecaster's subjective views about a particular situation. Depending on the quantity of the analyst's information and judgement, a probability distribution may or may not be a good representation of reality.

Risk is typically associated with a variability of anticipated returns or with the dispersion of the underlying probability distribution. Therefore, a set of economic hurdles expectations and their associated probabilities can be generated.

Expected Value

Expectation has a sense of what is normal or average. Expected value is a popular method for combining estimates of both quantitative outcome and probability. ***Decision Trees*** provide a powerful mechanism for applying expected value to complex decision problems while ***Preference Theory*** explores situations where expected value may not be appropriate. Both topics will be described in a separate chapter.

Expected Monetary Value

The Expected Value (**EV**) of an event may be defined as the product of its numerical outcome and its probability of occurrence. If the outcome is financial, the product may be called Expected Monetary Value (**EMV**).

$EMV = c * p$

c = cash value or payoff of an event or outcome

p = its probability of occurrence

Thus, if an event has a cash value of MMUS$ 100 and a probability of occurrence of 0.5, its EMV equals 100 * 0.5 = $50 million.

The event has a:

Risk-adjusted value	of MMUS$ 50
Risk-weighted value	
Risked value	
Certainty money equivalent	

A positive value here signifies a benefit.

If a decision has more than one possible outcome, the EMV of the decision may be represented as:

$EMV = c_1*p_1 + c_2*p_2 + c_3*p_3 + ... + c_n*p_n$

Where:

c_1 - c_n represent payoffs associated with ***n*** possible outcomes

p_1 - p_n represent the probabilities of occurrence of these n outcomes and

$p_1 + p_2 + p_3 + ... + p_n = 1$

This condition ensures that all possible outcomes are included.

EMV Heads and Tails Example

Considering the example of tossing a coin where the outcome is “Heads” (H) or “Tails” (T).

The probability (p) of either appearing on a single toss is 0.5.

Player A wins $100, when the coin shows “Heads” and loses $100 when the coin shows “Tails”. Conversely, Player B wins $100 on Tails and loses on Heads.

Player	Heads	Tails	EMV Heads	EMV Tails
A	+ US$ 100	- US$ 100	+ US$ 50	- US$ 50
B	- US$ 100	+ US$ 100	- US$ 50	+ US$ 50

For Player A, the EMV of the event “Heads”:

EMV = +100 * 0.5

EMV = +$50

For Player B, the EMV of the event “Tails”:

EMV = -100 * 0.5

EMV = -$50

For Player A the EMV of this bet is:

EMV = ($P_{(H)}$ * 100) – ($P_{(T)}$ * 100)

EMV = (0.5 * 100) – (0.5 * 100)

EMV = 50 - 50

EMV = zero

If such a game is symmetrical, i. e. the payoff or value associated with a “Tail” is equal and opposite to that associated with a “Head”, the EMV of the decision to play is zero. This implies that there is no advantage to either player. If payment for a “Head” was greater than that for a “Tail”, the EMV for Player A would become positive and the player would have an advantage.

EMV Drilling Example

Consider a drilling situation with the following data:

Geological Prediction

Probability of commercial success: $P_{(S)} = 0.30$

Probability of dry hole / failure: $P_{(F)} = 0.70 = (1 - P_{(S)})$

Financial Implications

Cost of drilling the well = MMUS$ 10

Present value of ***production*** (including completion cost) = MMUS$ 100

Event “Success”

$EMV_{(s)} = (100 – 10) * 0.3$

$EMV_{(s)} = 90 * 0.3$

$EMV_{(s)}$ = MMUS$ 27

Event “Failure”

$EMV_{(f)} = (-10) * 0.7$

$EMV_{(f)}$ = - MMUS $7

Decision “Drill”

EMV = EMV “Success” + EMV “Failure”

EMV = 27 - 7

EMV = MMUS$ 20

Alternatively, Decision “Drill”

EMV = (Net Revenue of Success * $p_{(S)}$) – (Cost)

EMV = (100 * 0.3) - 10

EMV = 30 - 10

EMV = MMUS$ 20

EMV The Farmout Option Example

Two definitions are used in this example: farmout and carried interest.

As described in previous chapter a ***farmout*** is the assignment of part or all oil, natural gas, or mineral interest to a third party for development. The interest may be in any agreed-upon form, such as exploration blocks or drilling wells.

Carried interest is a share of the profits of a successful partnership that is paid to the manager of the partnership as a form of compensation.

In this example, the **Company** that owns the project can farmout the prospect to a local **Operator**, and is close to negotiating a carried interest of MMUS$ 20. Under the Farmout agreement in place, the Operator would be responsible for drilling the well and would benefit from any production, minus the carried interest payment to the Company.

The EMV of the Farmout decision to the **Company** would be:

*EMV = (Carried interest * p(S)) – (Cost)*

EMV = (20 * 0.3) – (zero)

EMV = MMUS$ 6

The EMV of this farmout agreement to the Operator would be:

*EMV = (Net Revenue of success * $p_{(S)}$) – (Cost)*

Remember that the Present value of production (including completion cost) = MMUS$ 100

EMV = ((100 - 20) * 0.3) – (10)

EMV = 24 - 10

EMV = MMUS$ 14

This Farmout agreement would enable the Operator to earn most of the EMV from this prospect. By varying the magnitude of the carried interest, the distribution of EMV would also vary. For example, to ensure equal EMV to both companies:

If Carried Interest = “z”

Company

Company EMV = (z * 0.3) – (zero)

Company EMV = 0.3 z

Operator

Operator EMV = ((100 - z) * 0.3) – (10)

Operator EMV = 30 - 0.3z -10

Operator EMV = 20 - 0.3 z

If EMV’s equal:

0.3 z = 20 - 0.3 z

z = MMUS$ 33.3

If the Company wanted to retain all the available EMV from the prospect:

Company EMV: 20 = 0.3 z

z = 20 /0.3

z = MMUS$ 66.7

Such a carried interest requirement would reduce the Operator EMV to zero:

Operator EMV = ((100 - 66.7) * 0.3) -10

Operator EMV = (33.3 * 0.3) -10

Operator EMV = zero

These numbers would be relevant to any negotiation between the Company and the Operator regarding the size of the carried interest. It is most unlikely that the Operator would agree to a value close to MMUS$ 66, since it would transfer to the Operator all the risk, while allowing the Company to retain the EMV.

These calculations depend significantly on the probability of success, which is inevitably a somewhat subjective value. It is always advisable in these circumstances to consider some form of sensitivity analysis. The problem may be analyzed with a spreadsheet and graphically over a range of values for $P_{(S)}$.

Geological Prediction		
Probability of commercial success:	$P_{(S)}$ =	0.3
Probability of dry hole / failure:	$P_{(F)} = (1 - P_{(S)})$	0.7
Financial Implications		
Cost of drilling the well	MMUS$	10
Present value of production (including completion cost)	MMUS$	100
Event “Success”		
EMV = (NPV – Cost of Drilling) * P(s)	MMUS$	27
Event “Failure”		
EMV = – Cost of Drilling * $P_{(F)}$	MMUS$	-7
Decision “Drill”		
EMV = EMV “Success” + EMV “Failure”	MMUS$	20
Alternatively, Decision “Drill”		
EMV = (Net Revenue of Success * $p_{(S)}$ – (Cost)	MMUS$	20
Carried interest (Farmout)	MMUS$	20
The EMV of the Farmout decision to the Company would be:		
EMV = (Net Revenue of success * P(S)) – (Cost)	MMUS$	6
The EMV of this farmout agreement to the Operator would be:		
EMV = (Net Revenue of success * $p_{(S)}$) – (Cost)	MMUS$	14
Varying the magnitude of the carried interest **(z)**	MMUS$	
Company EMV = (z * 0,3) – (zero)	MMUS$	
Operator EMV = ((NPV - z) * P(S)) – (Cost of Drilling)	MMUS$	
If EMV’s equal:		
P(S) * z = Original Carried Int - P(S) * z	MMUS$	33.3
If the Company wanted to retain all the available EMV from the prospect:		
Company EMV = Original Carried Int = (z' * P(S))	MMUS$	66.7
Such a carried interest requirement would reduce the Operator EMV to zero:		
Operator EMV = ((NPV - z') * P(S)) -Cost of Drilling		0

TABLE 149 EMV Drilling Decision Probability of Success = 0.3

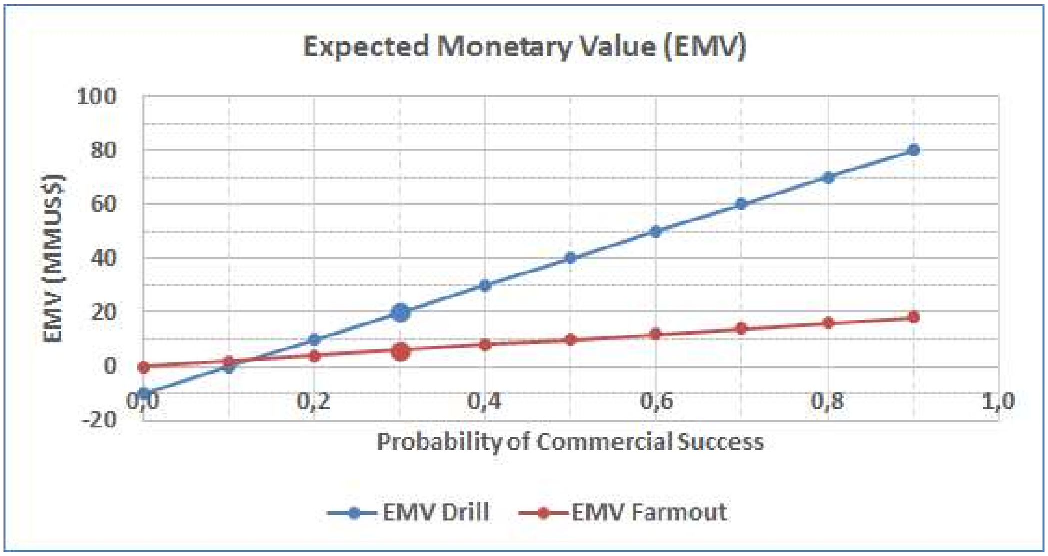


FIGURE 57 EMV Drilling Decision Various Probability of Success

Sens P(S)	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
EMV Drill	-10	0	10	20	30	40	50	60	70	80
EMV Farmout	0	2	4	6	8	10	12	14	16	18

TABLE 150 EMV Drilling Decision Various Probability of Success

Above $P_{(S)}$ = 0.1, EMV “Drill” > EMV “Farmout”

At $P_{(S)}$ = 0.3, EMV “Drill” equals MMUS$ 20 and EMV “Farmout” equals MMUS 6, as calculated above.

With a carried interest of MMUS$ 20, the optimum decision for the Company is to drill the well, or perhaps to try to negotiate a higher carried interest payment.

z @ $P_{(s)}$ = 0.3	20	30	40	50	60	70
Company EMV	6	9	12	15	18	21
Operator EMV	14	11	8	5	2	-1

Figure 58 illustrates how increasing the carried interest payment impact the EMV of the Farmout option. The higher the probability of success, the higher the carried interest that may be justified.

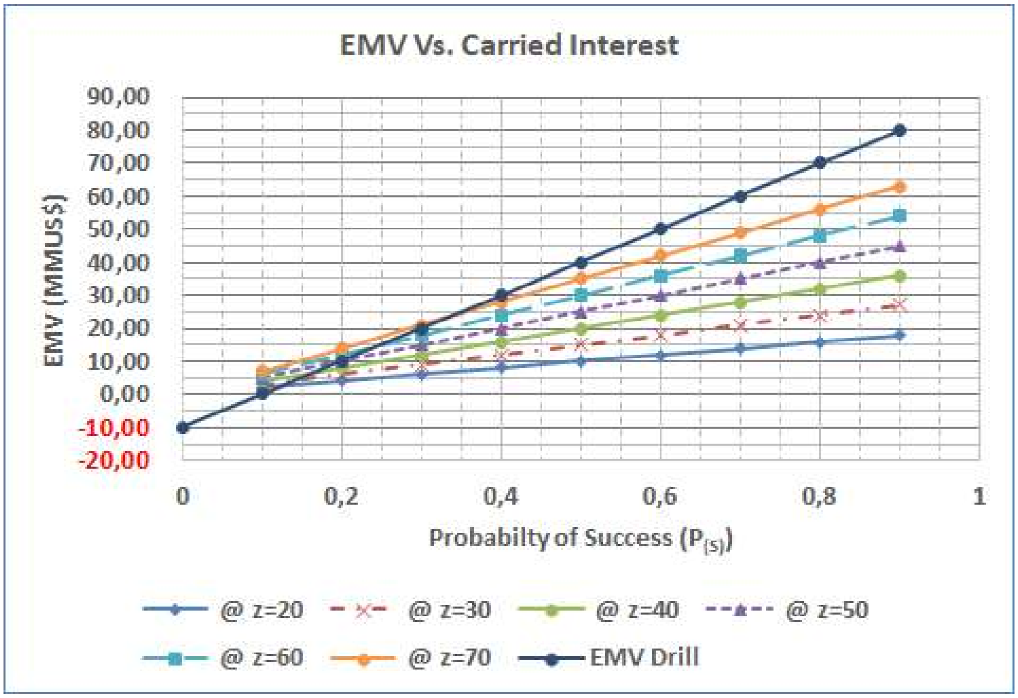


FIGURE 58 COMPANY EMV AT VARIOUS CARRIED INTEREST AND PROBABILITY OF SUCCESS

These simple graphical plots are a useful technique for analyzing this type of problem, where uncertainty is present and a range of investment options must be considered.

Significance of Expected Monetary Value (EMV)

An Expected Monetary Value (EMV) type parameter is a probabilistic or weighted average. It is an estimate of the likely average return over a series of similar investments.

In the previous example (Table 149), the EMV of drilling the well is MMUS$ 20, derived from a prospect, with a surplus of $90 million, if successful, or a loss of MMUS$ 10.

This means that if unsuccessful, the prospect will not return MMUS$ 20.

The model implies that for a series of ten similar prospects, three would be successful and seven unsuccessful ($P_{(S)}$ = 0.3).

- This would generate an income of 100 * 3 = MMUS$ 300
- An expenditure of 10 * 10 = MMUS$ 100
- The net income is therefore 300 – 100 = MMUS$ 200
- An average per well of 200/10 = MMUS$ 20

EMV Decision Rule for NPV

Once the expected value has been located from an NPV distribution, for example, it can be used to make an accept-reject decision because a single NPV is used when a single possible outcome is considered for an investment project.

- The decision rule is the expected value criterion and using it, managers may accept a single project if the expected NPV value is positive
- In the case of mutually exclusive alternatives, managers select the one with the highest expected NPV

The use of expected NPV has an advantage over the use of a point estimate, such as the likely value, because it includes all the possible cash flow events and their probabilities.

The justification for the use of expected value criterion is based on the ***law of large numbers***. It says that if many repetitions of an experiment are performed, the average outcome will tend toward the expected value.

This justification may seem to contradict the usefulness of the expected value criterion in economic analysis since most often in project evaluation analysts are concerned with a single nonrepeatable "experiment" or the investment alternative.

However, if a company adopts the expected value criterion as a standard decision rule for all its investment options over the long term the law of large numbers predict that accepted projects will tend to meet their expected values. Individual projects may succeed or fail, but for the average project result will tend to meet the company's standard for economic achievement.

The expected value criterion is simple and straightforward to use, but it fails to reflect the variability of investment outcome. Analysts can enrich the decision by incorporating the variability information along with the expected value. Since the variance represents the dispersion of the distribution, it is desirable to minimize it. In other words, the smaller the variance, the less the variability associated with the NPV.

Therefore, when analysts compare mutually exclusive projects, they may select the alternative with the smaller variance if its expected value is the same as or larger than those of other options. In cases where there are no clear-cut preferences, the ultimate choice will depend on the decision maker's trade-offs or how willing management is to accept variability to achieve a higher expected value.

Binomial Probability Function

The binomial distribution is a probability distribution that summarizes the likelihood that a value will take one of two independent values under a given set of parameters or assumptions.

The underlying assumptions of the binomial distribution are that:

- There is only one outcome for each trial
- Each trial has the same probability of success
- Each trial is mutually exclusive i.e., independent of each other

The binomial probability function is a mathematical representation of the probability of success over several trials, where there are two possible outcomes, e. g. Success or Failure.

$$Prob\ (R = s) = P_{(S)} = (\frac{I!}{s!\,(I-s)!}) * (p^S) * \ (1-p)^{I-s}$$

Where:

s = number of successes

I = number of trials

$I!$ = I factorial

$\frac{I!}{s!(I-s)!}$ it is the number of combinations of I things taken s at time = IC_s

p = probability of success for a single trial

Consider an experiment with only two possible outcomes:

E_1 is a "Success" with Prob(E_1) = p

E_2 is a "Failure" with Prob(E_2) = 1 - p

E_1 and E_2 are mutually exclusive in a single experiment, but several repetitions of the experiment would produce combinations of successes and failures. The experiments are taken to be mutually exclusive.

Consider the probability of getting ***s*** successes in ***I*** trials for a given p. This means calculate the Prob (R = ***s***)

Where:

R = $\sum X_i$ i =1, 2, 3…. *I*

and Prob (X_i = 1) = p

and Prob (X_i = 0) = 1-p

This problem might occur, for example, when estimating net-to-gross ratios for a sand-shale sequence:

X_i = 1 for sand and

X_i = 0 for shale

To answer this, it is considered all possible ways of getting ***s*** success in ***I*** trials with each combination multiplied by its appropriate probability ***p*** using the equation Prob (R=s):

$$Prob\ (R = s) = p_{(S)} = (\frac{I!}{s!\,(I-s)!}) * (p^S) * \ (1-p)^{I-s}$$

$$p_{(S)} = (A) * (B) * (C)$$

That is the binomial distribution equation. Each term in the equation has a physical significance:

- The first term (A) is the number of ways in which there can be exactly ***s*** successful outcomes
- The second term (B) is the aggregate probability of *s* successes (p^s)
- The third term (C) is the aggregate probability of (*I* - *s*) failures ($(1-p)^{I-s}$)

Using the Binomial Distribution _ Porosity Example

Using the Binomial Distribution for a Thin-Section-Porosity Estimation. Consider a point count on a vugular carbonate thin-section that has 50% porosity (ϕ).

The microscope cross hairs will fall either on:

- The matrix ($X_o = 0$)
- An opening ($X_1 = 1$)

The question is: what is the estimated porosity based on the number of samples to be taken?

If:

- ***I*** = the total number of samples taken and
- s = the number of times it is drawn ϕ = 1

Then the observed or *estimated* sample porosity is:

$$\phi = \frac{s\ * X_1 + (I-s) * \ X_0}{I} = \frac{s}{I}$$

Because the thin section has 50% porosity, Prob (ϕ = 1) = 1/2. Hence, the probability of *R* points out of *I* samples falling on vugs is:

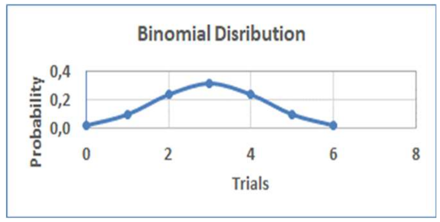
$$Prob\ (R = r) = p = \frac{I!}{r!\,(I-r)!} * (1/2)^I$$

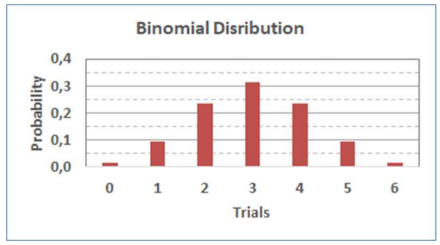
For I = 2 the outcome would be s = 0, 1, 2. For I = 6 the outcome would be s = 0, 1, 2, 3, 4, 5, 6

I =	6	p =	0.5
s	I!/s!(I-s)!	p(s)	1-p(s)
0	1	0.016	0.984
1	6	0.094	0.906
2	15	0.234	0.766
3	20	0.313	0.688
4	15	0.234	0.766
5	6	0.094	0.906
6	1	0.016	0.984

$$p_{(S)} = \left(\frac{I!}{s!\,(I-s)!}\right) * (p^S) * \ (1-p)^{I-s}$$

TABLE 151 Binomial Distribution N= 6 and P = 0.5





Apart from showing an application of the binomial distribution, this example illustrates the variability of estimates quantities, such as porosity in this case, that depend upon data.

People always estimate quantities based on some number of samples and those estimates are just that: estimated quantities that are themselves random variables.

Unless an extremely large number of points are included, average porosity would always be subject to some variability. It is possible to get some idea of the variability by looking at the probabilities of the extreme events Prob (ϕ = 0) and Prob (ϕ = 1).

For 2 trials Prob (ϕ = 0) = Prob (ϕ = 1) = 0.250

I =	2	p =	0,5
s	I!/s!(I-s)!	p(s)	1-p(s)
0	1	0.25	0.75
1	2	0.50	0.50
2	1	0.25	0.75

TABLE 152 Binomial Distribution N= 2 And P = 0.5

When *I* = 6 Prob (ϕ = 0) = Prob (ϕ = 1) = 0.016

By tripling the sample size, the probabilities of the extremes have dropped considerably.

USING THE BINOMIAL DISTRIBUTION _ EXPLORATION PROGRAM EXAMPLE

It is also possible using the binomial function to analyze an exploration program of ten wells.

Assume wells cost MMUS$ 10 and discounted net revenue for each successful well is MMUS$ 100. Furthermore, the probability of success "*p*" equals 0.30.

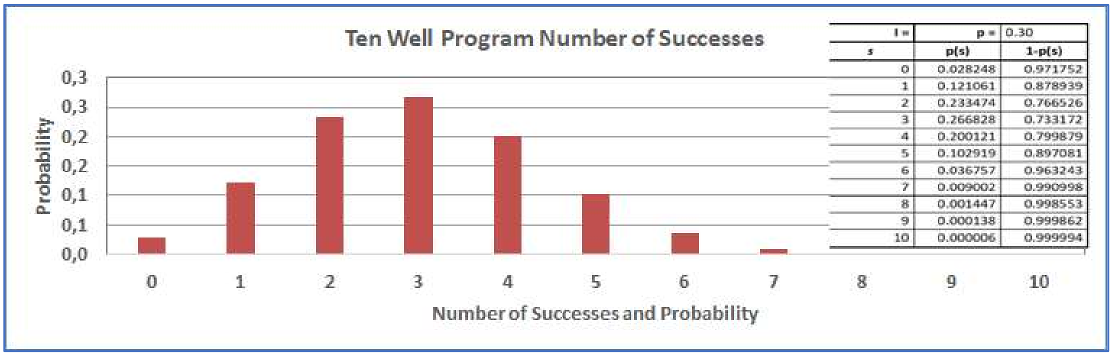


FIGURE 59 TEN WELL PROGRAM NUMBER OF SUCCESSES

Figure 59 is a discrete distribution of probabilities, which is derived from the binomial probability function, using 0.3 as value for "*p*" and using six decimal places.

Figure 59 plots the probability of each specific number of successes $p_{(S)}$ where $p_{(0)}$ is plotted as 0.028248. This is the probability of zero successes or 10 failures; therefore, $p_{(F)}$ is equal to $(1\text{-}0.3)^{10}$.

At the other end of the distribution, the probability of 10 successes is $(0.3)^{10}$ or 0.000006.

I =	10	p =	0.30
s	I!/s!(I-s)!	p(s)	1-p(s)
0	1	0.028248	0.971752
1	10	0.121061	0.878939
2	45	0.233474	0.766526
3	120	0.266828	0.733172
4	210	0.200121	0.799879
5	252	0.102919	0.897081
6	210	0.036757	0.963243
7	120	0.009002	0.990998
8	45	0.001447	0.998553
9	10	0.000138	0.999862
10	1	0.000006	0.999994

The weighted average or mean of this distribution is 3 successes as calculated and presented in Table 153. It is the summation of the numbers of successes multiplied by the corresponding probability in the drilling sequence.

I =	10	p =	0.30	Σ =	3.000		300.000
					Weighted Average	Revenue	Expected
s	I!/s!(I-s)!	p(s)	1-p(s)		S*P(s)	R (MMUS$)	R* P(s)
0	1	0.028248	0.971752		0.00000	0	0.000000
1	10	0.121061	0.878939		0.12106	100	12.106082
2	45	0.233474	0.766526		0.46695	200	46.694888
3	120	0.266828	0.733172		0.80048	300	80.048380
4	210	0.200121	0.799879		0.80048	400	80.048380
5	252	0.102919	0.897081		0.51460	500	51.459673
6	210	0.036757	0.963243		0.22054	600	22.054145
7	120	0.009002	0.990998		0.06301	700	6.301184
8	45	0.001447	0.998553		0.01157	800	1.157360
9	10	0.000138	0.999862		0.00124	900	0.124003
10	1	0.000006	0.999994		0.00006	1000	0.005905

TABLE 153 Ten Well Program Number of Successes

Thus, the mean of a binomial distribution is equal to *I* * *p* where "*I*" is the number of trials and "*p*" the probability of success at each trial. If ten wells are drilled, with a probability of success of 0.3 at each trial, the expectation is to have three successes.

It is possible to compute an expected value for this program. If ten wells are drilled, the outcome could be from zero to ten successes, with zero to a maximum of MMUS$ 100 * 10 of net revenue and probabilities derived from the binomial distribution. For the program, the expected net revenue is MMUS$ 300 as presented in Table 153.

Given a cost of MMUS$ 100, (10 wells * MMUS$ 10), the expected monetary value of this project is MMUS$ 200, (expected net revenue of 300 – cost of 100).

The EMV for ten wells (MMUS$ 200) equals ten times the EMV for one well (100*0.3-10 = MMUS$ 20).

Using the Binomial Distribution _ Tossed Coins Example

On the other extreme, the binomial distribution may be used to analyze simple questions as a sequence of tossed coins.

For example, to compute the probability of precisely two "Heads" from three trials, there are eight permutations of Heads and Tails in a sequence of three trials and three of these contain two Heads. The probability of this occurring is therefore 3/8 or 0.375. The probability of zero "Heads" is 0.125. The binomial calculation is presented in Table 154.

I =	3	p =	0.5
s	I!/s!(I-s)!	p(s)	1-p(s)
0	1	0.125	0.875
1	3	0.375	0.625
2	3	0.375	0.625
3	1	0.125	0.875

TABLE 154 Probability of Precisely Two "Heads" from Three Trials

Preference Theory

As indicated earlier the Preference Theory explores situations where expected value may not be appropriate

Consider the following bets:

a) "Heads" wins US$ 1 and "Tails" loses US$ 1

b) "Heads" wins US$ 1000"and Tails" loses US$ 1000

EMV(a) = (0.5 * (+1)) + (0.5 * (-1)) = zero

EMV(b) (0.5 * (+1000)) + (0.5 * (-1000)) = zero

According to the EMV criterion, any symmetrical bet of this form has zero EMV and all are therefore equally appealing (or not). However, in practice, "Option b" could be less interesting opportunity than "Option a", because of the possibility of a significant loss. Application of the EMV criterion implies a neutral attitude to risk across the range of monetary values incorporated in the calculation.

In general, managers are influenced by potential loss in relation to available or expendable resources, as well as to psychological attitude to gains and losses.

The Pessimist Rule handles the opposite case, where the decision taker avoids any potential loss.

The formal analysis of these issues is called ***Preference*** or ***Utility Theory***. Utility theory bases its beliefs upon individuals' preferences. It is a theory postulated in economics to explain behavior of individuals based on the premise that people can consistently rank and order their choices depending upon their preferences.

People will show different preferences, which appear to be linked with their inner willingness to take risks. It can be said that individuals' preferences are intrinsic. Utility theory is a positive theory that seeks to explain the individuals' observed behavior and choices. For example, consumers' preferences can be revealed by what they purchase under different circumstances, particularly under different income and price circumstances.

From one side the economic theories should be normative, which means they should be prescriptive and tell people what to do. On the other side, economic theories should be explanations of observed behavior of agents in the market, hence positive in that sense. This contrasts with a normative theory, one that dictates that people should behave in the manner prescribed by it. Instead, since the theory itself is positive, it is only after observing the choices that individuals make, that people can draw inferences about their preferences.

When people place certain restrictions on those preferences, they can be represented analytically using a utility function—a mathematical formulation that ranks the preferences of the individual in terms of satisfaction that different consumption packages provide. Thus, under the assumptions of utility theory, it can be assumed that people behave as if they had a utility function and acted according to it. Therefore, the fact that individuals do not know their utility function, or even deny its existence, does not contradict the theory. Economists have used experiments to decipher individuals' utility functions and the behavior that underlies the resulting individuals' utility.

For example, when managers formally record a non-neutral attitude to risk. Buying an insurance policy, could reveal a preference for making regular, small premium payments to avoid a larger loss. Insurance companies employ highly trained actuaries to ensure that such premiums have a negative expected value for the policy-holder. Similarly, gambling on the turn of a card, the spin of a roulette wheel or the result of a race all reveal an affinity for a large gain, but at the expense of a negative expected value. The casino is not a charitable organization and formal games of chance are biased against the customer.

Negative sums of money impact on individuals and organizations in different ways, depending on available resources and attitude to risk. A company with limited funds and/or a strong aversion to risk will be reluctant to make an investment, where there is a possibility of a significant loss.

Certainty Equivalents to Evaluate Risk

This represents a relatively complex approach to evaluating risk from as individual decision maker's perspective. For example, it overcomes some of the limitations of risk-adjustment discount rates. Assume that a decision maker is faced with the choice of:

1. Receiving at some point in time an uncertain cash flow characterized by a particular probability distribution
2. Receiving $\underline{\boldsymbol{\alpha}}$ dollars with certainty

If the amount $\underline{\boldsymbol{\alpha}}$, is such that the decision maker is indifferent between the two options, then $\underline{\boldsymbol{\alpha}}$ is termed the certainty equivalent of the uncertain cash flow.

Certainty Equivalent Example 1

Assume that net cash flows for the coming period are characterized by the following probability distribution:

Cash flow ((c), US$)	Probability (p)	EMV = (c) * (p)
17000	30%	5100
10000	40%	4000
3000	30%	900
Expected Monetary Value (US$)		**10000**

If a decision maker is indifferent between either the prospect of such cash flows or a certain receipt of US$ 8000 in the same period, then, US$ 8000 becomes the ***Certainty Equivalent*** of the uncertainty cash flows.

The discrepancy between the cash flow's expected value of US$ 10000 and the certain value of US$ 8000 reflects the decision maker´s risk-aversion. The certainty equivalent can also be expressed as:

*Certainty equivalent = (certainty-equivalent coefficient) * (expected value of uncertain cash flows)*

In this case the certainty-equivalent coefficient is 0.8 (8000/10000). It is like the use of conservative estimates to scale down an uncertain forecast. In using this method to evaluate investments, analysts first assign certain equivalent to the cash-flow distribution specified for each period.

These certainty equivalents are then discounted to derive the project's NPV. Because risk has been fully accounted for, the discount rate applied should reflect only the time value of money without including any premium for risk.

Certainty Equivalent Example 2

Consider an investment of US$ 900 that is to yield uncertain cash flows with expected values of US$ 500 at the end of each of the next 3 years.
If ***α*** denotes the certainty-equivalent coefficient for period *t*, and ***α*** varies between 0 and 1. The lower the ***α*** the higher the risk.
Assume that risk increases over time and that management views the certain-equivalent coefficients as indicated in Table 155. The time value of money for riskless investment is given as i_k = 10%.

	i_k =	10.0%

Year (n)	Uncertain Cash flows Expected Values (US$)	Certainty-equivalent coefficients	Certainty equivalents (US$)	NPV (US$) $A_n/(1+i_k)^n$
0	-900	1.0	-900	-900.00
1	500	0.9	450	409.09
2	500	0.8	400	330.58
3	500	0.7	350	262.96

Total NPV (US$)	102.63

TABLE 155 Certain-Equivalent Coefficients for the Project

Because the project yields a positive NPV, after full considerations of the risks involved it should be accepted.

- The drawback of this method is the need to specify a certainty equivalent for every distribution of uncertain cash flows.

Certainty Equivalent Example 3

An Oil Company considers a small investment project with the following characteristics

Year	A (US$)
0	-500000
1	481750
2	90286
3	88333

Salvage value at the end of year 3 is equal to US$ 34550

The risk-free discount rate is 6%. Assume the risk increases over time, so the management views the certainty-equivalent coefficients for the cash flows anticipated as follows:

Year	Certainty-Coefficient
1	0,8
2	0,7
3	0,6

a) Using certainty-equivalents coefficients calculate the IRR of the investment
   - Would the company accept the project?

b) If, instead risk-adjusted discount rates were used to evaluate this project
   - How high could the risk premium on the discount rate be (over and above the 6% risk-free rate) for the project to break-even?

Solution

a) Given the certainty-equivalent coefficients the cash flows are as follows:

Year	A (US$)	Certainty-Coefficient	A_ adjusted (US$)
0	-500000	1,0	-500000
1	481750	0,8	385400
2	90286	0,7	63200
3	88333	0,6	53000

The IRR calculated using a spreadsheet is:

	Cash Flow Economic Analysis					Option 1		
Input						Use as equivalent rate:	6,00%	i = i'+f+i'*f
IRR adjusted			Clear Input			Option 2		
Project Life (N, yrs)	3		*Max 40 Yrs*			6,0%	is corrected to	
Initial Investment (P_{oINF}, $)	500000					To be added to i		
Interest (I)	6,0%		inflation (f)	0,0%	r corected by f =		6,00%	r = (i - f)/(1 + f)
Fill Rows 8 & 10		*Yr*	*0*	*1*	*2*	*3*		
Annty Outflow (C, $ per year)		Ci		0,0	0,0	0,0		
Annty Inflow (A, $ per year)		Ai		385400	63200	53000		
Sunk Cost @ N (S, $)	0							
Salvage Value @ N (Sv, $)	34550							
Results								
Profit to Investment Ratio (PIR)	-0,013		NPW/Po					
Discounted Payout (Payback yrs)								
NPVI	-0,013		NPV (@i) / MCO (@i)					
Net Present Value (NPV, $)	-6659		$P = F / (1 + i)^n$		$P = A * \{[(1 + i)^n - 1]/(i (1 + i)^n)\}$			
Benefit/Cost Ratio (PI)	0,987		ΣPV of net cash inflows/ΣPV of net cash outflows				Net Operating Income to Investment Ratio	
Internal Rate of Return (IRR,%)	5,00%		Click Calc IRR Setting D26=0 Varying B24					
	Calc IRR		0,00	NPV Target		1	IRR Possible solutions	

TABLE 156 IRR For the Adjusted Cash Flow

Because the IRR of 5% is lower than 6%, the project should be ***rejected***

b) Computing the project for the original cash flow without the risk-equivalent the result is:

	Cash Flow Economic Analysis					Option 1			
Input						Use as equivalent rate:		25,40%	i = i'+f+i'*f
IRR Original Cash Flow			Clear Input			Option 2			
Project Life (N, yrs)	3		*Max 40 Yrs*			14,0%	is corrected to		
Initial Investment (P_{oINF}, $)	500000						To be added to i		
Interest (I)	6,0%		inflation (f)	10,0%	r corected by f =		3,64%	r = (i - f)/(1 + f)	
Fill Rows 8 & 10		*Yr*	*0*	*1*	*2*	*3*			
Annty Outflow (C, $ per year)		Ci		0,0	0,0	0,0			
Annty Inflow (A, $ per year)		Ai		481750	90286	88333			
Sunk Cost @ N (S, $)	0								
Salvage Value @ N (Sv, $)	34550								
Results									
Profit to Investment Ratio (PIR)	0,276		NPW/Po						
Discounted Payout (Payback yrs)	1,55								
NPVI	0,276		NPV (@i) / MCO (@i)						
Net Present Value (NPV, $)	138010		$P = F / (1 + i)^n$		$P = A * \{[(1 + i)^n - 1]/(i (1 + i)^n)\}$				
Benefit/Cost Ratio (PI)	1,276		ΣPV of net cash inflows/ΣPV of net cash outflows					Net Operating Income to Investment Ratio	
Internal Rate of Return (IRR,%)	26,12%		Click Calc IRR Setting D26=0 Varying B24						

TABLE 157 IRR for the Original Cash Flow

The resulting IRR is 26.12%; therefore, only if the risk-adjusted discount rate is raised above 26.12% would the project be rejected.

Hence, any risk premium of less than 26.12% - 6% = 20.12% would imply that the project remains acceptable.

Decision-Making Tools

As indicated in previous chapters, Expected Value is a common method used for combining estimates of quantitative outcome and probability. ***Decision Trees*** provide a mechanism for applying expected value to decision problems.

A decision tree is a graphical representation of a sequential decision problem, incorporating logic, value and probability. It is a methodology with several applications. Construction of the tree requires the compilation of relevant data and organization of the problem into a logical sequence of information gathering and decision-making.

The solution of the tree provides an optimum sequence of decisions and a value for the initial decision. This may be in terms of NPV or some other appropriate parameter. Analysis of the tree may provide insight into the value of information.

In constructing a decision tree, it is critical to order the information and the decisions correctly, because an illogical tree will not generate an optimum decision.

In decision trees, time increases from left to right without a scale of time. Therefore, activities above and below do not relate to the same moment. For each branch it is also possible to incorporate a probability and a cost to improve the use of the decision tree.

In general, decisions are concerned with the acquisition of information. The cost of the acquisition may be justified in terms of reduced risk and higher expected value for subsequent decisions. For example, in the decision tree (Figure 60) two methods of exploring a prospect are indicated:

A. Based on existing information
B. Based on acquiring additional information

In option B, the additional information is a seismic survey to help the decision maker to optimize expenditures on drilling.

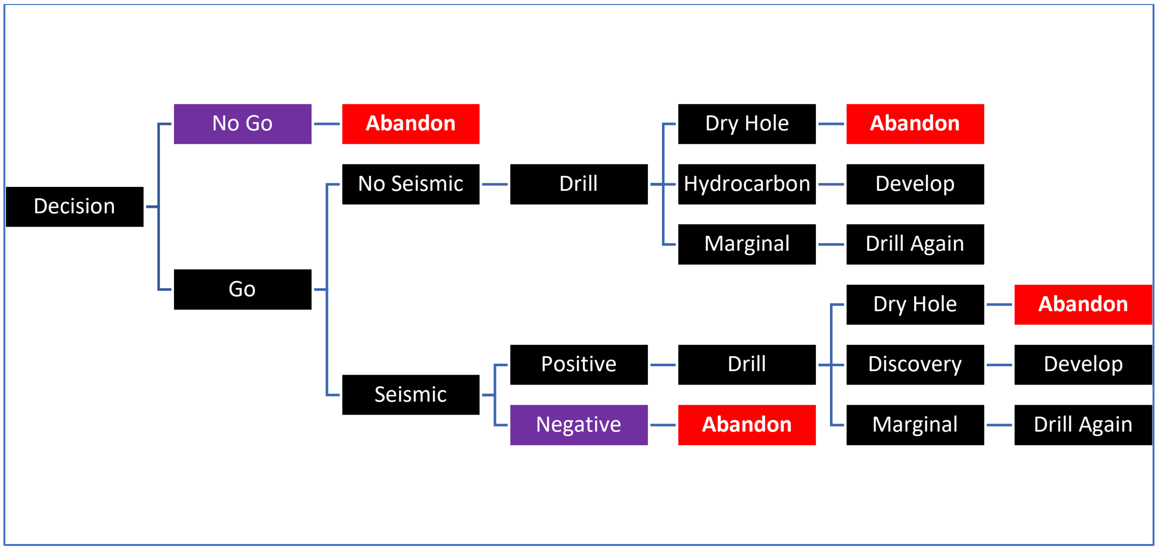


FIGURE 60 DECISION TREE _ TWO METHODS OF EXPLORING A PROSPECT

The decision tree may be used to determine the “value” of the information. The meaning of “value” is whether the acquisition of the data is beneficial to the project. If NPV is the decision hurdle, the benefit is quantified in terms of NPV.

In this example for the seismic information to have any value, it must be demonstrated that the expenditure on seismic has increased the expected value of the prospect.

- The benefit coming from the seismic information is greater than the cost of its acquisition

Some considerations on the value are for instance:

- Cost of Data Acquisition
    - This includes the value of the information
        - It is defined as the difference in NPV between the value of the decision option containing the cost of acquisition and the next best option
    - Value, therefore implies the increase in NPV for the initial investment decision because of acquiring the data
- Expenditure Limit
    - The analysis may determine the limit on expenditure for data acquisition, consistent with being no worse off if the data is not acquired
    - This value indicates the maximum that may be spent
        - Any reduction would represent a benefit
    - This measure is equivalent to the first plus cost of data acquisition
- Zero NPV

- If the project NPV is negative, the information has zero value, even if it improves knowledge
- It is suggested that the maximum value of information should be the positive NPV for the decision to acquire the data

Figure 61 is an example of a simple decision tree, derived from part of the logic contained in Figure 60. It includes standard tree elements and data and is styled for convenience of understanding and of analysis.

There are two types of branching point or node

- "Decision" □
- "Chance" ○

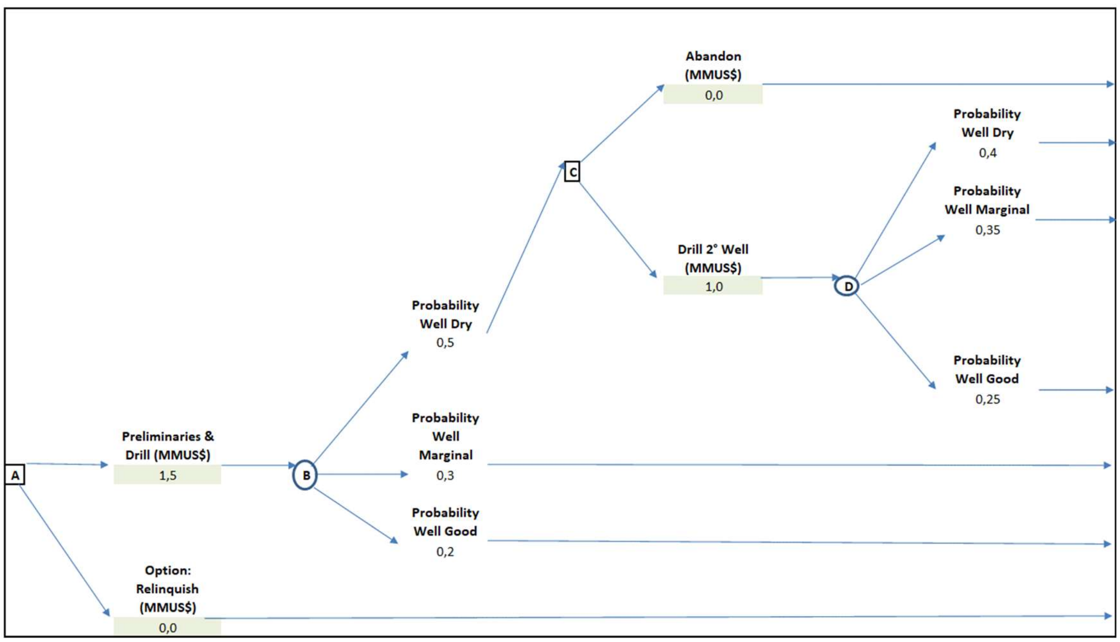
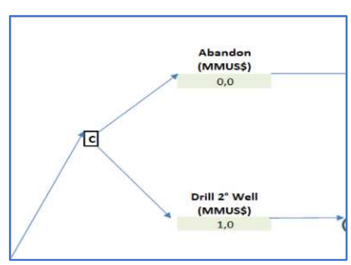


FIGURE 61 EXAMPLE OF A SIMPLE DECISION TREE

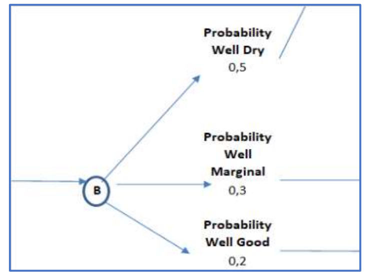
A Decision Node is a point at which the decision maker may choose the route.



- At Node "**C**" for example the choice is between
  - Abandoning the project
  - Drilling a second well
- Decision Nodes are commonly represented by a **square symbol**
- There are two Nodes in this example: **"A"** and **"C'**
- A decision tree always begins with a Decision Node

A Chance Node is a point at which there is an uncertain outcome.

- The relevant information may or may not exist in the present, but it is unknown to the decision maker until this stage in the project
- At Node "**B**" data from the first well indicates whether the prospect will be a:
    - Dry hole
    - Marginal producer
    - Good producer
- Chance Nodes are commonly represented by **circular symbol**
- There are two Nodes: **"B"** and **"D"**



A "branch" as a line joining two nodes and a "composite branch" as a sequence of nodes and branches, forming a route through the tree.

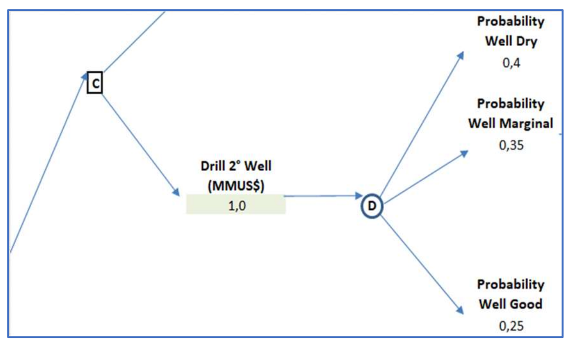
Tree branches normally represent activities or information.

In Figure 61 activities may have cost implications and these are included as monetary units.

The cost of drilling the second well is given as MMUS$ 1.

Information may have probability values estimated or measured by the manager.

The probability of drilling a dry hole at the second attempt is given as 0.40. This is estimated to be less than for the first well (p = 0.5), presumably based on geological factors.



At any chance node, the sum of probabilities necessarily sums to unity.

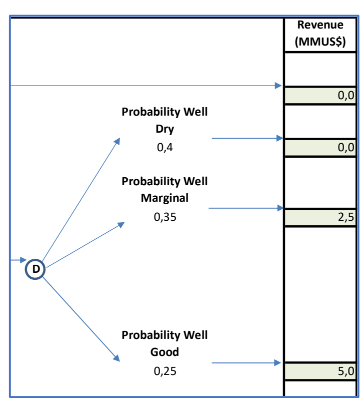
At Node "D", the individual probabilities for a well are:

- Dry = 0.4
- Marginal = 0.35
- Good = 0.25

Each composite branch termination has an associated revenue, positive for a successful project (producing well):

- Zero for a failed investment (dry hole)
- A marginal producer MMUS$ 2.5
- A good producer is assumed to generate MM US$ 5

Under normal circumstances, these numbers represent NPV's as presented in Figure 62.



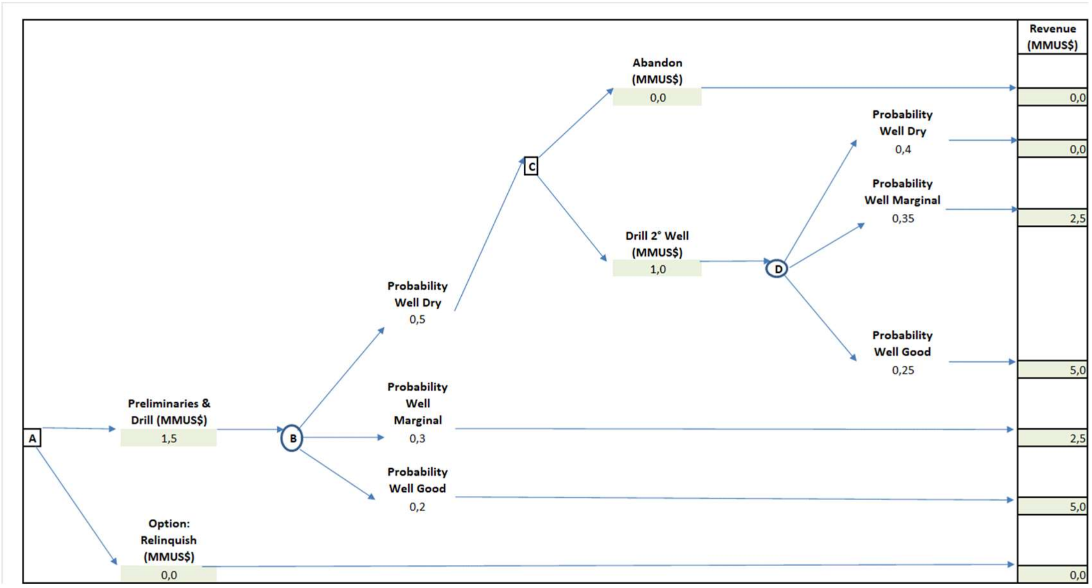


FIGURE 62 EXAMPLE OF A SIMPLE DECISION TREE WITH ASSOCIATED REVENUE

Sometimes a tree is used to analyze costs rather than net revenues, in which case there are no revenues.

Decision Tree Solution

Solution of this tree is presented in Figure 63, the first step being to compute payoff values for each composite branch termination. Payoff for one composite branch is the algebraic sum of all costs and revenues associated with that composite branch from beginning (Node **A**) to end.

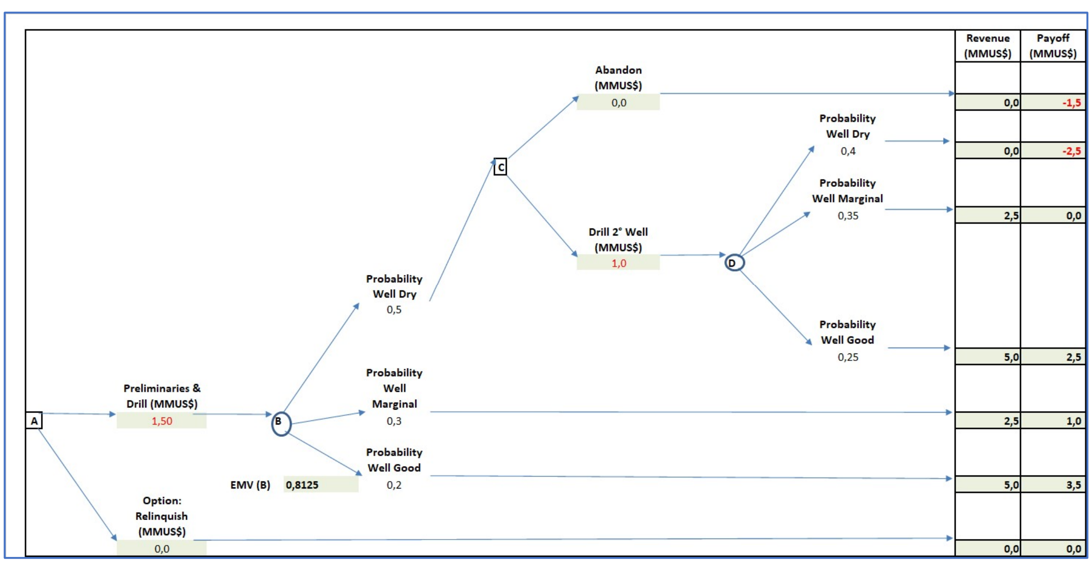
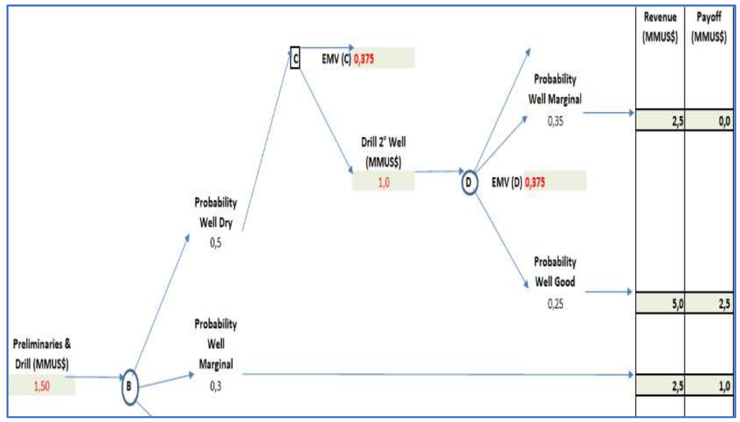


FIGURE 63 PAYOFF VALUES FOR EACH COMPOSITE BRANCH TERMINATION

For example, the zero value calculated for the "Marginal" termination, which is attached to Node "D" is derived from:

- - MMUS$ 1.5 (Preliminaries & Drill)
- - MMUS$ 1 (Drill 2nd Well) +MMUS$ 2.5 (Marginal Producer).



These payoff values are the only monetary amounts to be used during the solution process. Revenue and Cost data are no longer required.

Solution of a tree involves solving each Node in turn, starting at the right and working towards the principal decision node on the left (**A**).

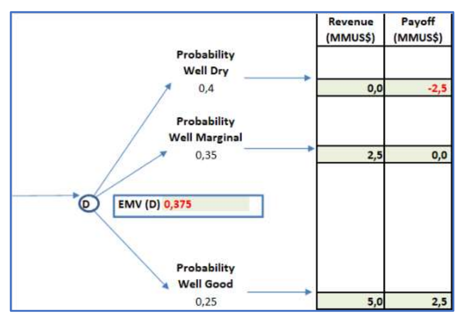
It is not possible to solve a node if there exists an unsolved node, which is connected to its right.

In this example, therefore, Node "**D**" must be solved first. "**D**" is a chance node and therefore is commonly solved as an EMV. Other criteria, such as expected utility may be used as an alternative.

$$EMV(D) = (0.40 * -2.5) + (0.35 * 0) + (0.25 * 2.5)$$

$$EMV(D) = -1 +0 + 0.625$$

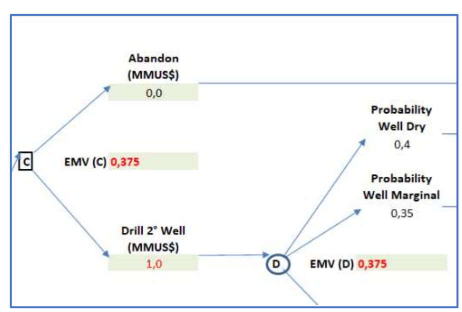
$$EMV(D) = -0.375$$



Having solved Node "**D**", Node "**C**" may now be solved. "**C**" is a decision node and its solution conventionally involves choosing the branch with the highest value.

In this case the choice is between minus MMUS$ 0.375 (Drill 2nd Well) and minus MMUS$ 1.5 (Abandon).

The decision is to choose "Drill 2nd Well" and the value of Node "C" becomes minus MMUS $0.375.



A risk averse decision maker may wish to avoid specific branches where significant financial loss is possible.

Having solved Node "**C**", Node "**B**" may now be solved.

Node “B” is a chance node and EMV is used to calculate its value, as follows:

*EMV(B) = p * payoff or EMV*

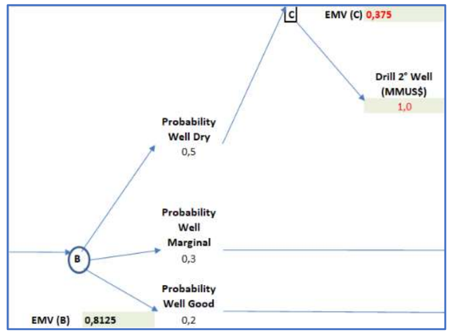
EMV(B) = (0.50 * -0.375)

+ (0.30 * 1.0)

+ (0.20 * 3.5)

EMV(B) = -0.1875 + 0.30 + 0.70

EMV(B) = 0.8125



Node “**B**” becomes MMUS $0.8125

The complete solution of the tree is presented in Figure 64

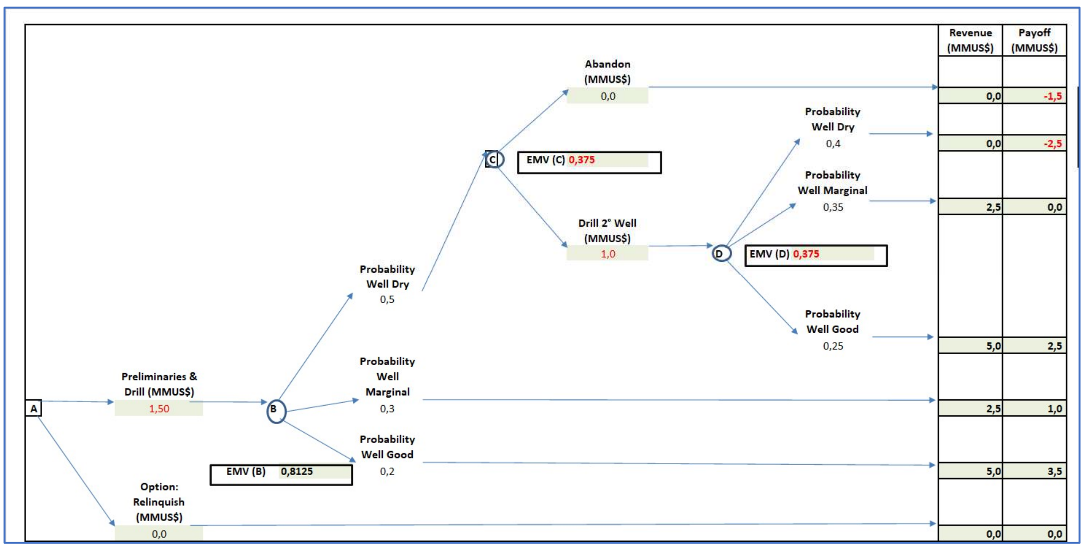


FIGURE 64 EMV FOR EACH COMPOSITE BRANCH TERMINATION

Decision trees can be very large, with many nodes and branches.

Therefore, analysts should develop a style of layout, which enhances comprehension and eliminates ambiguity.

Some characteristics of a decision tree are:

- Branches include text
- Nodal symbols have associated numbers or letters representing identification or value
- All composite branch terminations should locate at the right of the diagram
- At the right the Revenue and Payoff values may be stacked neatly and unambiguously

Decision Tree and Conditional Probability

The logic and data included in a decision tree relate to the ***timing*** of the principal or first decision node. "Timing" here relates the real time and the sequence of events represented by the tree.

For example, the cost and probability data included in Figure 64 were based on estimates made before the first well was drilled. Each node and each branch of a tree is unique, even if there seems to be repetition.

Figure 65 is a section of a decision tree, with two main composite branches, comparing the relative merits of drilling based on seismic information or only on geological information.

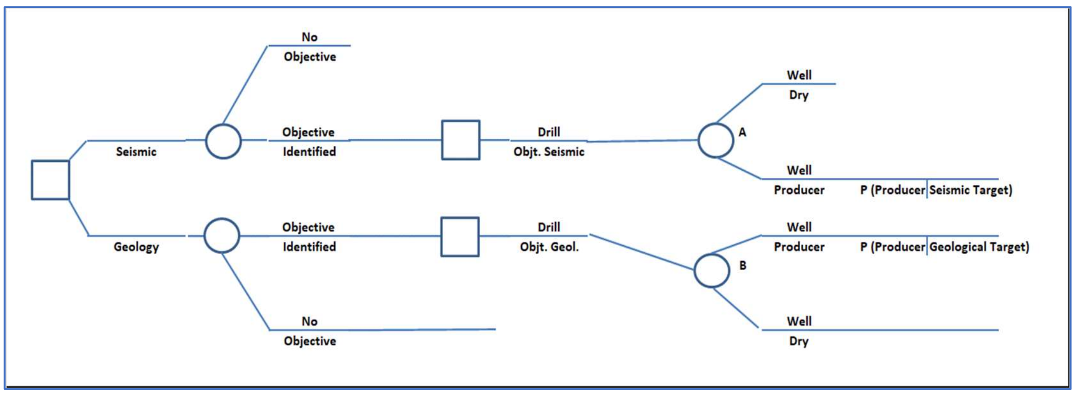


FIGURE 65 FRAGMENT OF A TREE, WITH TWO MAIN COMPOSITE BRANCHES

Chance nodes "**A**" and "**B**" look similar. Each follows the drilling of a well and each has two possible outcomes: "Producer" and "Dry Hole." However, they are different:

- Node "**A**" follows the drilling of a seismic target
- Node "**B**" follows the drilling of a geological target

Seismic information frequently provides a more precise definition of a drilling target with a higher probability of success.

The basic principle here is that each branch is part of a unique sequence of activity and outcome, and probability must reflect project history along its composite branch.

Conditional Probability

Tree probabilities are known as ***conditional probabilities***, the condition is that a unique sequence path to that position in the tree has been followed.

At Node "**A**", the probability of drilling a producer may be better defined as:

p (Producer | Seismic Target)

This means the probability of drilling a producer, given the condition that a seismic target was drilled.

The convention is to draw a vertical line between the event and the specific condition.

At Node "**B**". the probability of drilling a producer may be written as:

p (Producer | Geological Target)

Probability conditions are normally assumed when preparing decision trees.

If E_1 and E_2 are any two events, then the conditional probability of E_1 given E_2, is denoted $p(E_1 \mid E_2)$, and if $p(E_2) > 0$, the conditional probability is:

$$p(E_2 \mid E_1) = \frac{p(E_1\ E_2)}{p(E_1)}$$

Conditional probability plays a role in many practical applications of probability. In these applications, conditional probabilities are often affected by apparently small changes in the basic information from which the probabilities are derived.

Venn Diagrams

Conditional probability requires further consideration. ***Venn Diagrams*** are used to illustrate important probability relationships. A Venn Diagram uses area to represent probability, with a rectangular boundary enclosing unit area, equivalent to a unit probability.

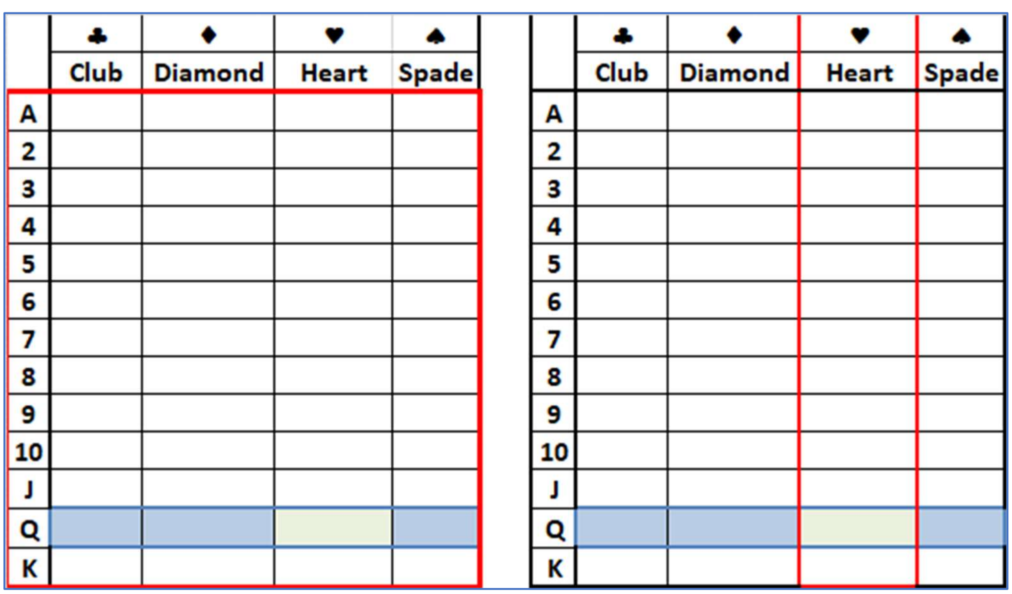


FIGURE 66 VENN DIAGRAM

The Venn Diagrams in Figure 66 to the left represent a pack of 52 playing cards, ordered into four suits, Clubs, Diamonds, Hearts and Spades, each suit having 13 cards.

In the left diagram the thick red line forms a rectangle of unit area, containing 52 equal sub-areas representing these cards. A single card, the Queen of Hearts, is highlighted. As one out of fifty-two, the probability of drawing that card is 1/52.

It is represented as: p(Queen of Hearts) = 1/52.

Strictly, every probability has a condition, whether specified or implied. In this case, for completeness, it should be written as p(Queen of Hearts | Pack of cards) = 1/52.

In the right diagram (Figure 66) the thick red line forms a rectangle of unit area. the Venn Diagram boundary has been altered to enclose only the Heart Suit of 13 cards. The probability of drawing the Queen of Hearts has now increased to 1/13.

It is represented as: p (Queen of Hearts | Hearts) = 1/13.

Placing a condition on probability changes the boundary of the appropriate Venn Diagram and subsequently changes the value of probability.

If the question now is calculating the probability of selecting two Hearts in two draws from the same 52-card deck, when each card is returned to the deck and the deck is shuffled before the next draw.

Since each card draw could produce any one of 52 cards, the total number of outcomes is

N = 52*52 = 2704.

Therefore, each card could be any one of 52 and that 2704 is just the number of ways to take 52 things, two at a time, with replacement.

The event E is the drawing of two Hearts and the number of outcomes favorable to the event is: 13*13 = 169 since there are 13 Hearts in the deck and the second draw is with replacement. Thus, the probability of E is:

$p(E) = 169/2704 = 0.0625$

The experiment is more complicated when the card is not replaced after each draw. This is because withholding the first card alters the probability in the second draw. Such changes confirm the notion of conditional probability.

If E_1 and E_2 are mutually exclusive, then if E_1 has occurred, E_2 cannot occur, so $P(E_2 \mid E_1) = 0$.

If in the previous experiment the first card is held out before the second draw; then, the sample space now consists of fewer pairs of cards because pairs with identical cards are no longer possible.

The events are now defined differently. For example, if E_1 is drawing a Heart on the first card, and $E_2 \mid E_1$ is drawing a Heart on the second card given that the first card is Heart. The question now is looking for the probability that have a set of outcomes that belongs to both E_1 and E_2.

p(E_1) = 13/52 = 0.2500 because there are 13 Hearts in 52 cards
and
p(E_2 | E_1) = 12/51 = 0.2353 because there remain now 12 Hearts in 51 cards

This is because the lack of replacement changes both the number of cards in the deck and the Hearts available to be chosen.

Rearranging the definition of conditional probability results in a decrease in probability caused by the lack of replacement:

$$p(E_1\ E_2) = p(E_2 \mid E_1) * \ p(E_1)$$

$$p(E_1\ E_2) = 0.2353 * 0.2500 = 0.0588$$

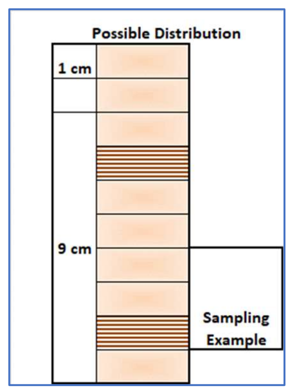
Multiplication Rule

The definition of ***conditional probability*** can be generalized to provide a multiplication rule for the intersection of several dependent events.

For example: a probe permeameter is set to take a measurement every 3 cm along a 10 cm section of core.

The formation consists of 1-cm shale and sand layers interspersed at random, with 80% sand in total or a net-to-gross ratio of 0.8.

The question is: What is the probability that only sand will be measured? Assuming the probe falls either on a sand or shale, not a boundary.



Because of the 3-cm spacing, the measurements are only made over the interval. The possibilities are:

C = Probe measures a layer of sand, p(C) = 8/10

B = Probe measures a layer of sand given first is sand, $p(B \mid C)$ = 7/9

A = Probe measures a layer of sand given first wo layers are sand, $p(A \mid BC)$ = 6/8

The probability of measuring only sand is:

p(ABC) = (8/10) * (7/9) * (6/8)

p(ABC) = 0.4667 = 46.67%

With three measurements, there is 53.3%, (100-46.67), chance of shale layers going undetected over a 10-cm sample.

Conditional Probability for Exploration Targets Example

Figure 67 represents a Venn Diagram for exploration targets in a certain sedimentary basin.

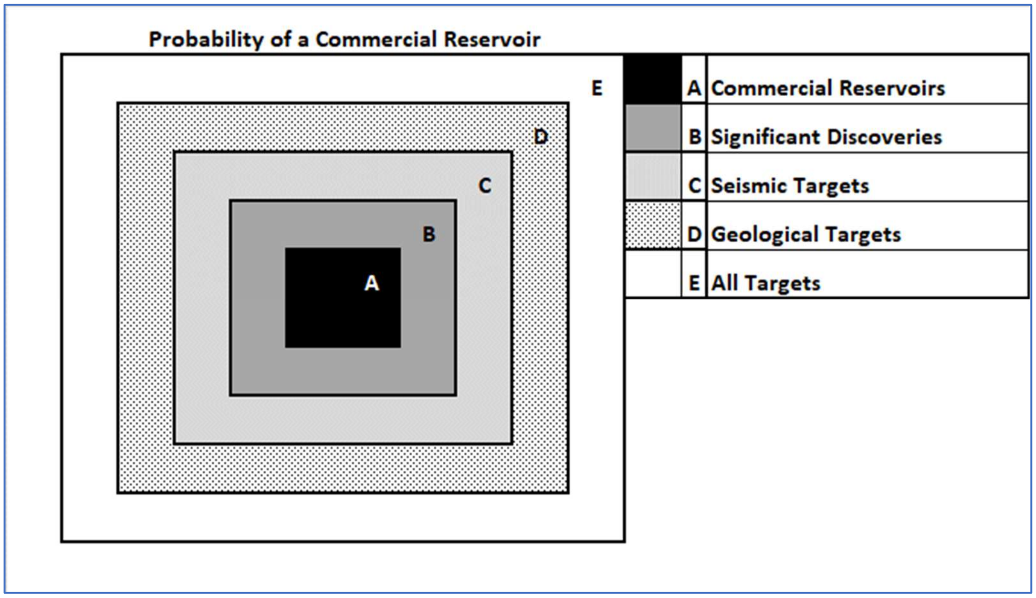
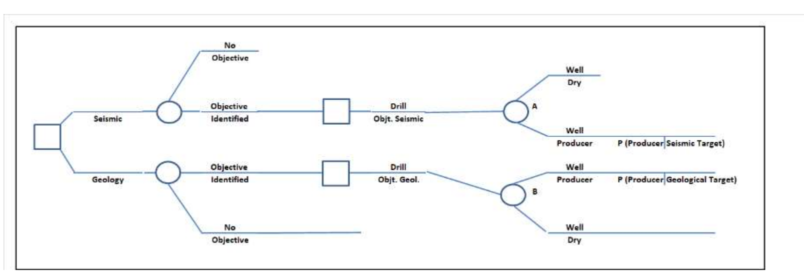


FIGURE 67 EXPLORATION TARGETS

- The outer (white) population, labelled "**E**" and described as "All targets" includes all possible drilling sites in the designated area
- "Geological targets" (**D**) are those identified because of basin analysis and extrapolation of known structural and sedimentological features into the subsurface
- "Seismic targets" (**C**) are geometrical features identified from seismic investigation
- "Significant discoveries" (**B**) are petroleum-bearing formations intersected by wells
- "Commercial reservoirs" (**A**) are accumulations of petroleum, sufficiently large and productive to justify commercial development

The diagram indicates that the probability of identifying a "Commercial reservoir (**A**)" depends on the condition applied. p(A | C) is greater than p (A | D), since the underlying population is smaller. It is like drawing cards from a smaller deck.

These two conditional probabilities are equivalent to those discussed previously in relation to the decision tree (Figure 65) previously discussed.



These examples are straightforward in the sense that the populations used fit wholly within one another.

- The suit of “Hearts” is wholly contained within the “Pack of cards”
- The “Seismic targets” are wholly contained within “Geological targets”

	♣ Club	♦ Diamond	♥ Heart	♠ Spade
A				
2				
3				
4				
5				
6				
7				
8				
9				
10				
J				
Q				
K				

There exist, however sets of populations, whose boundaries overlap. For example, using the pack of cards, the population “Hearts” overlaps the population “Queens”, with one common element, the Queen of Hearts.

From this figure, it can be seen that:

P (Heart | Queens) = 1/4 and that

P (Queen | Hearts) = 1/13

The specified condition determines the size of the sampling population and the boundary of the Venn Diagram.

Overlapping populations may also occur in the previous subsurface example, which is reformatted in Figure 68 where a new population termed “***<u>Structures</u>***” was added.

Structure is represented by a red rectangle boundary. The term “***<u>Structures</u>***” here is intended to include all trapping geometries.

Seismic investigation identifies trapping geometries in the subsurface, but this process is imperfect. Some tectonic structures are easily identified. However, other trapping geometries, such as diagenetic and sedimentological boundaries may have no seismic signature.

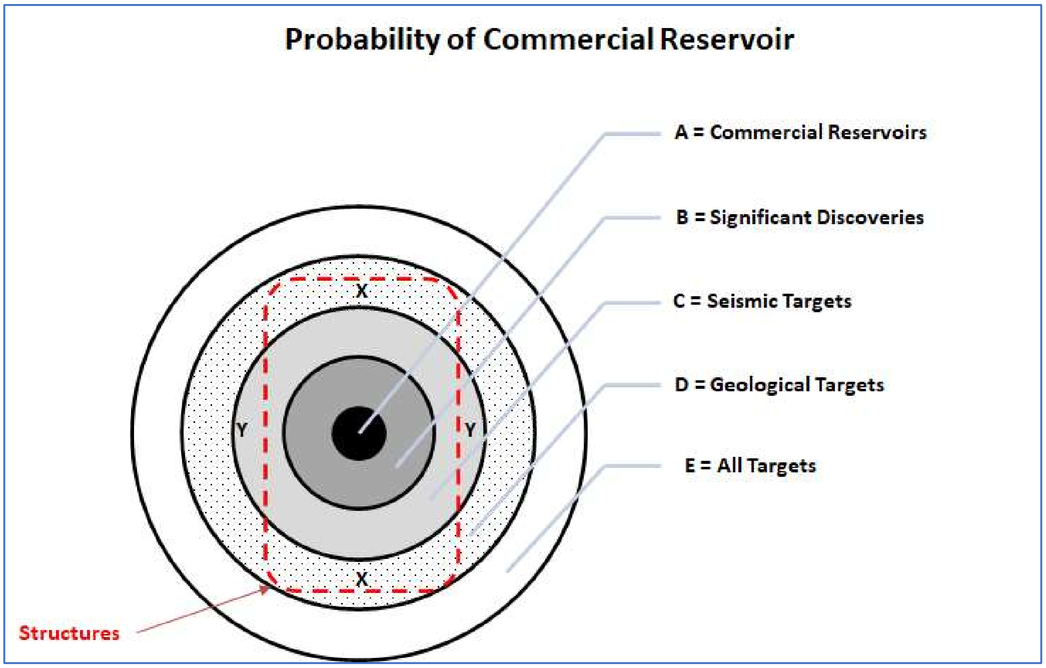


FIGURE 68 OVERLAPPING POPULATIONS

Structures, which are invisible to seismic investigation fall within the "***Structures***" population, but outside the "***Seismic target***" population. These areas are labelled "**x**" in the Venn Diagram.

It is possible to include in this "***Structures***" category those structures, which may be overlooked for other reasons, such as complex geology, or imperfect procedure or interpretation.

Imperfect interpretation, may also generate phantom or non-existent targets. These fall within the "***Seismic target***" population, but outside the "***Structures***" population. They are represented in the diagram by areas labelled "**y**".

Overlapping predicament is handled by the Bayes Theorem and it is the subject of the following section.

Bayes Rule or Bayes Theorem

Formal analysis of overlapping populations may be solved using Bayes Theorem. This is based on the notion of a partition of a sample space.

From a definition of conditional probability:

$$P(E_1\ E_2) = P(E_1 \mid E_2) * P(E_2)$$

$$P(E_2\ E_1) = P(E_2 \mid E_1) * P(E_1)$$

$$P(E_1\ E_2) = P(E_2\ E_1)$$

$$P(E_1 \mid E_2) * P(E_2) = P(E_2 \mid E_1) * P(E_1)$$

Solving for one conditional probability in terms of the other:

$$P(E_1 \mid E_2) = \frac{P(E_2 \mid E_1) * P(E_1)}{P(E_2)}$$

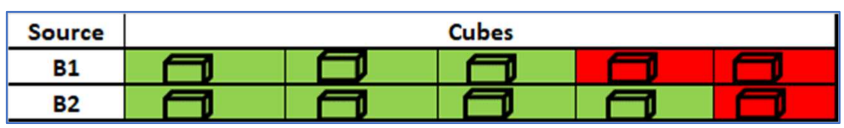
This relationship is called ***Bayes' Theorem***. It can be extended to ***I*** events to give:

$$P(E_i \mid E) = \frac{P(E \mid E_i) * P(E_i)}{\sum_{i=1}^{I} P(E \mid E_i) * P(E_i)}$$

Where ***E*** is any event associated with the (mutually exclusive and exhaustive) events $E_1, E_2, E_3, \ldots E_I$. It provides a way of incorporating previous experience into probability assessments for a current situation.

The theorem is demonstrated by the following discussion using a simplistic example as it may be applied to a sampling problem, where "**E**" represents a sample, which could have originated from population B1 or B2. The probability of the sampled population being B1 (or B2) can be updated because of the sampling process.

This figure illustrates a sampling problem, based on two identical boxes, each containing 5 colored cubes.



- Box B1 contains 3 green and 2 red cubes
- Box B2 contains 4 green and 1 red cube

Sample **E** is 1 red and 1 green cube, taken without replacement. Bayes Theorem allows to compute the probability that the sampled box is B1 (or B2).

P (E | B1) = 2/5 * 3/4 = 0.3 2 red cubes out of 5, 3 green out of 4 remaining after sampling

P (E | B2) = 1/5 * 4/4 = 0.2 1 red cubes out of 5, 4 green out of 4 remaining after sampling

Before sampling, *P(B1)* = *P(B2)* = 0.5

After sampling, what is the probability that the sampled box is B1?

Bayes Formula

$$p(E_1 \mid E) = \frac{p(E \mid E_1) * p(E_1)}{p(E \mid E_1) * p(E_1) + p(E \mid E_2) * p(E_2)}$$

From Bayes, P(B1 | E)

p(B1 | E) = (p(E | B1) * p(B1)) / (p(E | B1) * p(B1) + p(E | B2) * p(B2))

p(B1 | E) = (0.3 * 0.5) /((0.3 * 0.5) + (0.2 * 0.5))

p(B1 | E) = 0.6

				Sampling		Probability	
Source	**Objects**	**Type 1**	**Type 2**	**Type 1**	**Type 2**	**Source**	**After Sampling**
B1	5	3	2	1	1	0.3	0.6
B2	5	4	1			0.2	0.4

As a result of the sampling, the probability of the sampled box being B1 has increased from 0.50 to 0.60.

Bayes calculations may be presented in an alternative format, which may be easily remembered. It breaks the theorem into a series of logical steps.

a	Nature of the State (NE)		B1	B2	Total
b	Initial Probability		0.50	0.50	1.00
c	Conditional probability:	Probability of sample E. given the nature of the state p(E I NE)	0.30	0.20	?
d	Combined Probability:	Product of Probabilities and Conditional	0.15	0.10	0.25
e	Revised probabilities:	Probability of the nature of the state given sampling E	0.15/ 0.25 = 0.60	0.10/ 0.25 = 0.40	1.00

TABLE 158 Bayes Theorem Alternative Format

a) States of Nature

These are all possible specified outcomes.

In this case, there are two possibilities, defined as:

- Box B1
- Box B2

Figure 69 presents the Venn Diagram of this example.

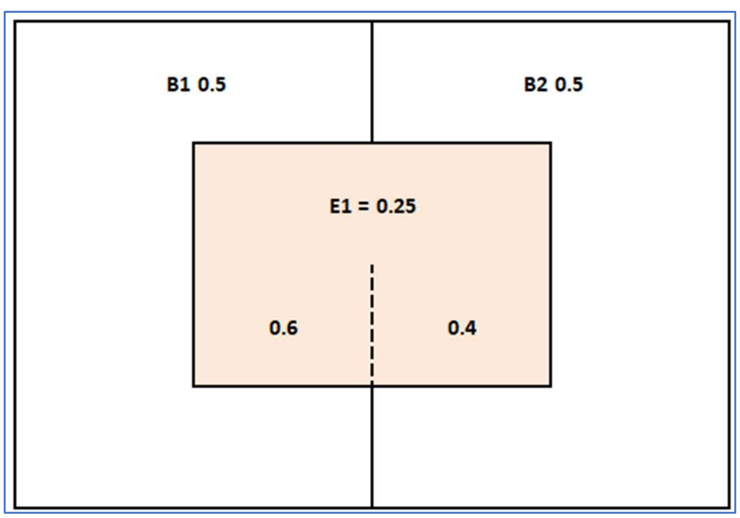


FIGURE 69 VENN DIAGRAM FOR EXAMPLE OF BOXES AND CUBES

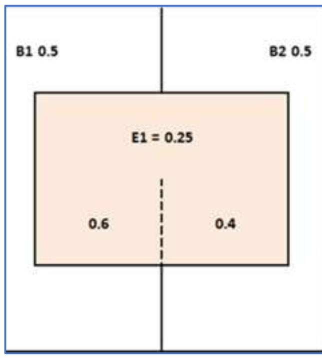
b) Initial Probabilities

These are estimates or calculations of the probabilities of the various states of nature, available before the sampling occurs. In this case, since the boxes are externally identical, their "***a priori***" probabilities are equal. *P(B1)* = *P(B2)* = 0.5.

c) Conditional probabilities

These are the probabilities of the specified samples or events arising, from each identified state of nature. For example, if the box were B2, what would be the probability of the specified sample appearing *P (E| B2)*. This is represented in the figure as the proportion of B2 overlapped by E, this is equal to
0.20 = ((1/5)*(5-1))/(5-1)

d) Combined probabilities



a	Nature of the State (NE)		B1	B2	Total
b	Initial Probability		0,50	0,50	1,00
c	Conditional probability:	Probability of sample E, given the nature of the state p(E I NE)	0,30	0,20	?
d	Combined Probability:	Product of Probabilities and Conditional	0,15	0,10	0,25
e	Revised probabilities:	Probability of the nature of the state given sampling E	0,60	0,40	1,00

For each state of nature, multiplying its initial probability by the conditional probability of the sample arising from that state of nature, gives the probability of both occurring together. For example, for B1, it gives the probability that the box is B1 and that it produces the sample E.

In the figure, this is represented by E1. Adding these products together for all states of nature, gives the probability of the specified sample appearing, in other words *p(E)*.

e) Revised Probabilities

Once the sample **E** has appeared, there is the opportunity to revise the estimates for *p(B1)* and *p(B2)*. There are the probabilities of **E** coming from each of the states of nature, in this case E1 (0.15) and E2 (0.10). Accepting the appearance of E, effectively sets p(E) equal to unity and redefines the boundary of the Venn Diagram as the perimeter of E.

If E = 1

P(B1 | E) = 0.15 / 0.25

P(B1 | E) = 0.60

P(B2 | E) = 0.10 / 0.25

P(B2 | E)= 0.40

0.60 and 0.40 are two numbers, in the ratio of E1 to E2, which add to unity. These are the "***a posteriori***" probabilities, based on additional information. From this analysis, the following probability values are available:

Probability that the Sample **E** will appear, *P(E)* = 0.25

Probability of sampled box being B1, given sample **E**, *P(B1 | A)* = 0.60

Probability of sampled box being B2, given Sample **E**, *P(B2 | A)* = 0.40

Bayes Theorem Geological Information Example

Assume there is a geological information to be used on a current prospect. Suppose there is a well into a previously unexplored sandstones horizon.

From regional studies and wireline logs, geologists think they have hit a channel in a distal fluvial system, but they do not have enough information to suggest whether the well is in a channel fill (low sinuosity) or meander-loop (high sinuosity) sand body.

Outcrop studies of a similar settings; however, have produced a table of thickness probabilities that could help them make an assessment.

Thickness (X) < (in m)	Low-Sinuosity Probability	High-Sinuosity Probability
1	0.26	0.03
2	0.38	0.22
3	0.56	0.60
4	0.68	0.70

TABLE 159 Thickness Probabilities

If a well is drilled and it is observed "***X***" meters of sand thickness, what are the probabilities that the reservoir is a low (or high) sinuosity channel?

The difference could have implications for reserves, and one of the critical calculations in reservoir engineering is to quantify the original volumes of Original Hydrocarbons in Place (**OHIP**) (Original Oil in Place (OOIP), Stock Tank Oil Initially in Place (STOIIP), or Original Gas in Place (OGIP)) and reserves.

$$STOIIP = \frac{\emptyset\,(1 - S_w)\,A\,\boldsymbol{X}}{\beta_{oi}}$$

In any reservoir targeted for exploitation an inadequate estimate of OHIP and the subsequent estimation of producible reserves, may lead to unreliable development plans and inadequate or simply wrong decisions by operators and investors. This is because the evaluation of a hydrocarbon producing property is typically divided into three estimates: the remaining reserves, the production schedule, and the cash flow schedule.

Going back to the example, in outcrop, high-sinuosity channels were observed to be generally wider and have higher connectivity than the low-sinuosity variety.

If E_1 is the low-sinuosity and E_2 is high-sinuosity before any well is drilled there is no way of preferring one over the other. Therefore, $p(E_1) = p(E_2) = 0.5$.

After one well is drilled, the probabilities will change according to whether the observed sand thickness is less than 1, 2, 3, or 4 m.

If for example, ***"X"*** = 2.5 m. then from Table 159 ***"X"*** falls between two rows of thickness-less than entries, $X \geq 2$ and $X < 3$. Therefore, this event E falls withing an interval $[2 \leq X < 3]$. Now it is possible estimate the occurrence of ***E*** by calculating the interval probability of ***E*** as the difference in interval bound probabilities. In this example, the conditional interval probabilities are calculated as follows:

$p(E \mid E_1) = p(E_1$ at upper bound$) - p(E_2$ at lower bound$)$. From Table 159:

$p(E \mid E_1) = 0.56 - 0.38$

$p(E \mid E_1) =$ **0.18**

While $p(E \mid E_2) = 0.60 - 0.22$

$p(E \mid E_2) =$ **0.38**

Applying Bayes Theorem with E associated to two mutually exclusive events E_1 and E_2 the revised probabilities are:

$$p(E_1 \mid E) = \frac{p(E \mid E_1) * p(E_1)}{p(E \mid E_1) * p(E_1) + p(E \mid E_2) * p(E_2)}$$

$$p(E_1 \mid E) = \frac{0.18 * 0.50}{0.18 * 0.50 + 0.38 * 0.50} = \mathbf{0.32}$$

And $p(E_2 \mid E) = 1 - p(E_1 \mid E) = 1 - 0.32 =$

$p(E_2 \mid E) =$ **0.68**

Therefore, the outcrop information has tipped the balance in favor of the high-sinuosity type channel.

The maximum information from the thickness measured in the previous solution has been used. That is: it was known that the thickness was less than 3 m and more than 2 m. If the information were only that the thickness was less than 3 m., the probabilities would be:

$$P(E_1 \mid E) = \frac{0.56 * 0.50}{0.56 * 0.50 + 0.60 * 0.50} = \mathbf{0.48}$$

And $P(E_2 \mid E)$ = 1 - 0.48 = **0.52**

Thus, the change in probabilities is small because less prior information has been added to the assessment.

Bayes Theorem Example			Init Prob (IP) =	0,5	All the information		Partial Information	
Thickness < (m)	Low-Sinuosity Probability	High-Sinuosity Probability	*P(E | E1) Low Sinuosity*	*P(E | E2) High Sinuosity*	*$P(E_1 \mid E)$*	*$P(E_2 \mid E)$*	*$P(E_1 \mid E)$*	*$P(E_2 \mid E)$*
(i)	(II)	(III)	(IV) = $(II)_n$-$(II)_{(n-1)}$	(V) = $(III)_n$-$(III)_{(n-1)}$	(VI) = ((IV)*(IP))/((IV)*(IP)+(V)*(IP))	(VII) =1-(VI)	(VIII) = (II)*(IP)/((II)*(IP)+(III)*(IP))	(IX) = 1-(VIII)

Bayes Theorem Example					All the information		Partial Information	
Thickness < (m)	**Low-Sinuosity Probability**	**High-Sinuosity Probability**	***P(E | E1) Low Sinuosity***	***P(E | E2) High Sinuosity***	***$P(E_1 \mid E)$***	***$P(E_2 \mid E)$***	***$P(E_1 \mid E)$***	***$P(E_2 \mid E)$***
1	0.26	0.03						
2	0.38	0.22	0.12	0.19	0.39	0.61	0.63	0.37
3	0.56	0.60	**0.18**	**0.38**	**0.32**	**0.68**	**0.48**	**0.52**
4	0.68	0.70	0.12	0.10	0.55	0.45	0.49	0.51

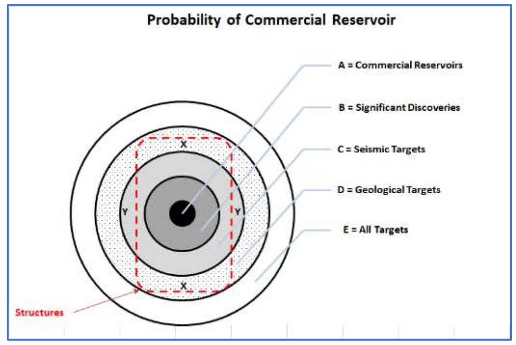
TABLE 160 Bayes Theorem Geological Example

In this example the probability values for E_1 and E_2, as stated earlier before measuring the formation thickness are called ***a priori probabilities***.

$P(E_1 \mid E)$ and $P(E_2 \mid E)$ are called ***posteriori probabilities***.

If there were further information with associated probabilities available (e. g. transient test data showing minimum sand body width), the posteriori probabilities could be amended still further.

Bayesian Decision Tree



Bayesian analysis is commonly required in decision situations involving imperfect information. Information may be imperfect, because the underlying data was defective, or perhaps because the interpretation is ambiguous.

Recalling Figure 68, the population C "Seismic targets" was an imperfect model for "Structures." It is possible to use the seismic data as a subsurface sample, to investigate the two populations "Structures" and "No structures", which are mutually exclusive.

Bayesian Decision Tree Example

Figure 70 is an example of decision tree logic, including imperfect seismic information. The tree has been simplified by reducing some of the options, which should be included in a full analysis. These decision options not included at Nodes C, D and E are:

Node C = Option to Drill, followed by (small) probability of Oil

Node D = Option to Abandon, with no further cost or revenue implications

Node E = Option to Abandon, with no further cost or revenue implications

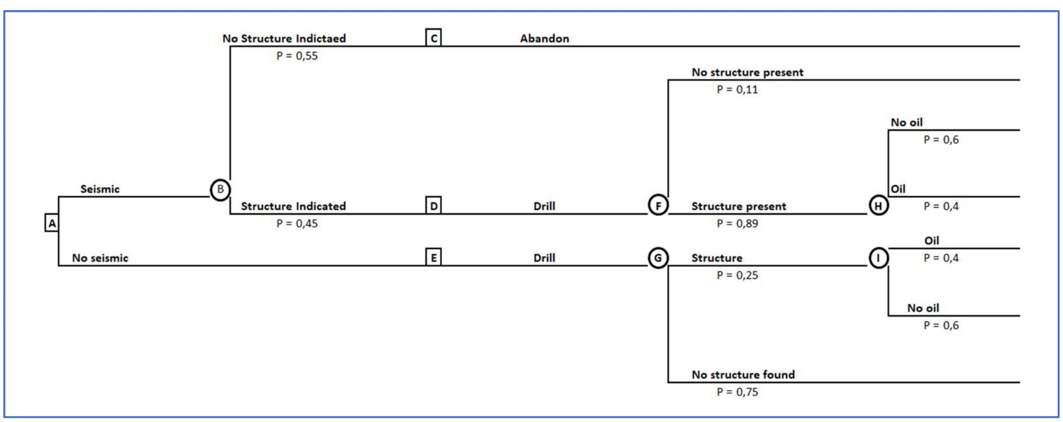


FIGURE 70 BAYESIAN EXAMPLE OF DECISION TREE LOGIC

The appropriate probabilities for the example are listed in Table 161:

Description of probability	Symbol	Probability
Structure exists on the property It is based on knowledge of regional trends and wells on adjacent properties	P (E)	0.50
No structure exists on the property (1-P(S))	P (NE)	0.50
Seismic indicates a structure, if it exists on the property. Some structures, may be overlooked because of discontinuous reflectors or because of faulty interpretations	P (Ind | E)	0.80
Seismic indicates a structure, even though none exists on the property. This is because of complex geology, deep targets or incorrect procedures.	P (Ind | NE)	0.10
Drillers will intersect a structure on the property, based on existing geological information. This is based on knowledge of regional trends, wells on adjacent properties and size distribution of interesting structures	P (Hit)	0.25
Oil is present, if structure exists, based of knowledge of regional trends and adjacent wells	P (Oil | E)	0.40

TABLE 161 Relevant Probabilities for the Example

Seismic information is imperfect and Bayesian analysis is required on the “Structure indicated” at composite branch from Node “B.” The calculation is illustrated in Table 162

a	States of Nature (SN)		Structure exists	No structure	Total
b	Initial probabilities		0.50	0.50	1.00
c	Conditional probabilities:	Probability of a positive seismic indication. given the specific state of nature P (Ind | SN)	0.80	0.10	?
d	Combined probabilities:	Product of initial and conditional probabilities	0.40	0.05	0.45
e	Revised probabilities:	Probability of specific state of nature. given sample A	0.40/0.45 = 0.89	0.05/0.45 = 0.11	1.00

TABLE 162 “Structure Indicated” At Composite Branch from Node “B”

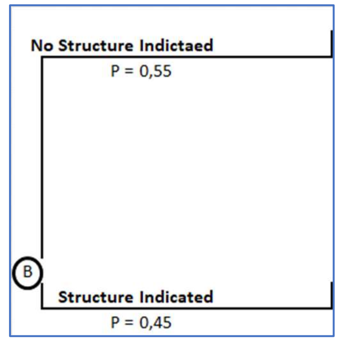
A summary of the results is:

P (Structure indicated) *P(Ind)* = 0.45

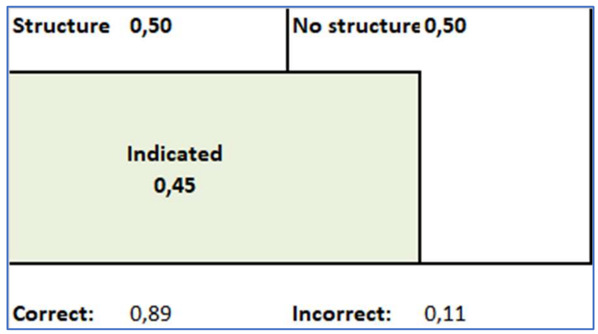
P (No structure indicated) = 1 - *P(Ind)* = 0.55

P (Structure exists given that it had been previously indicated); $P(S \mid Ind) = 0.89$

P (No structure, despite having been previously indicated); $P(NS \mid Ind) = 0.11$



These probability data were included on above decision tree figure and on this figure:



Value of Information in Bayesian Decision Tree

Figure 71 includes cost and revenue information for a general case, where:

R = Revenue (NPV) associated with finding oil

S = Cost of seismic survey

D = Cost of drilling a well

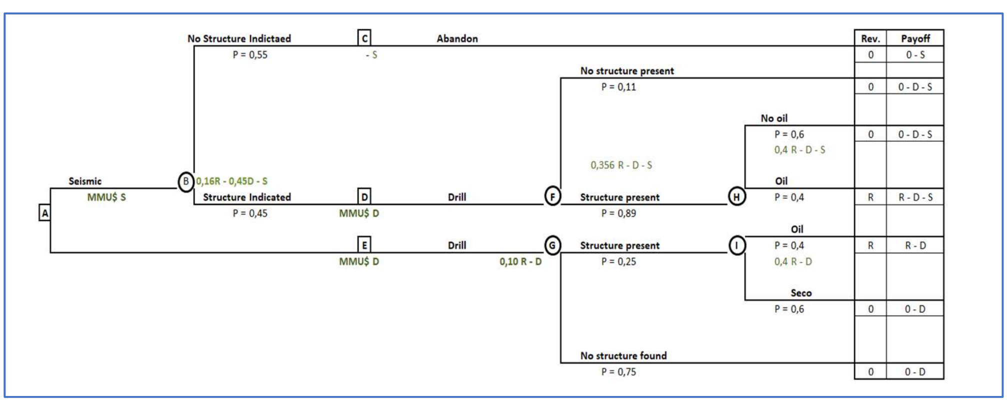
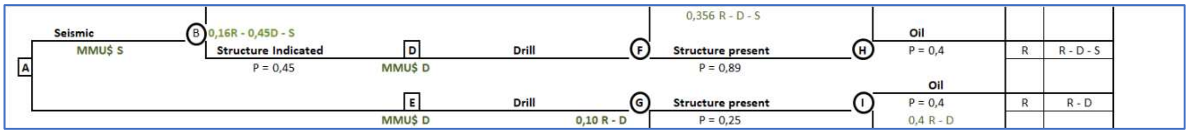


FIGURE 71 COST AND REVENUE INFORMATION

At Node "A" the choice is between:



Solving from right to left

Seismic option

(0.40 * 0.89 * 0.45) R – 0.45 D - S

0.16 R - 0.45 D - S

and geology option

(0.40 * 0.25) R - D

0.10 R - D

The "Seismic" option has a positive EMV if:

0.16 R - 0.45 D - S > 0, or

S < 0.16 R - 0.45 D

The "Seismic" option adds value to the decision if:

(0.16 R - 0.45 D – S) > (0.10 R – D)

The value of the information is the difference between "Seismic" and "Geology", i. e.

(0.16 R - 0.45 D – S) – (0.10 R – D)

Which equals:

0.06 R + 0.55 D - S

When this has a positive value:

(0.06 R + 0.55 D – S) > 0 or

(0.06R + 0.55 D) > S

With the data in this general form, the decision tree may be analyzed to investigate the various parameters.

Figure 72 is a plot of EMV vs. Drilling Cost "D" for two reservoir sizes MMUS$ 50 and MMUS$ 250.

In Figure 72, “Spend Limit” represents the difference between EMV “Seismic” and EMV “Geology” for each Revenue size. The “R50” case, for Drilling Cost beyond MMUS$ 5, “Spend Limit” is constrained since EMV: “Geology” is negative.

R50 Case

		0.16 R - 0.45 D	0.10 R - D	Seis- Geol
R	**D**	**Option Seismic 50**	**Option Geology 50**	**Spend Lím 50**
50	0	8.00	5.00	3.00
	1	7.55	4.00	3.55
	2	7.10	3.00	4.10
	3	6.65	2.00	4.65
	4	6.20	1.00	5.20
	5	5.75	0.00	5.75
	6	5.30	-1.00	4.30
	7	4.85	-2.00	2.85
	8	4.40	-3.00	1.40
	9	3.95	-4.00	-0.05
	10	3.50	-5.00	-1.50

R250 Case

R	D	Option Seismic 250	Option Geology 250	Spend Limit 250
250	0	40.00	25.00	15.00
	1	39.55	24.00	15.55
	2	39.10	23.00	16.10
	3	38.65	22.00	16.65
	4	38.20	21.00	17.20
	5	37.75	20.00	17.75
	6	37.30	19.00	18.30
	7	36.85	18.00	18.85
	8	36.40	17.00	19.40
	9	35.95	16.00	19.95
	10	35.50	15.00	20.50

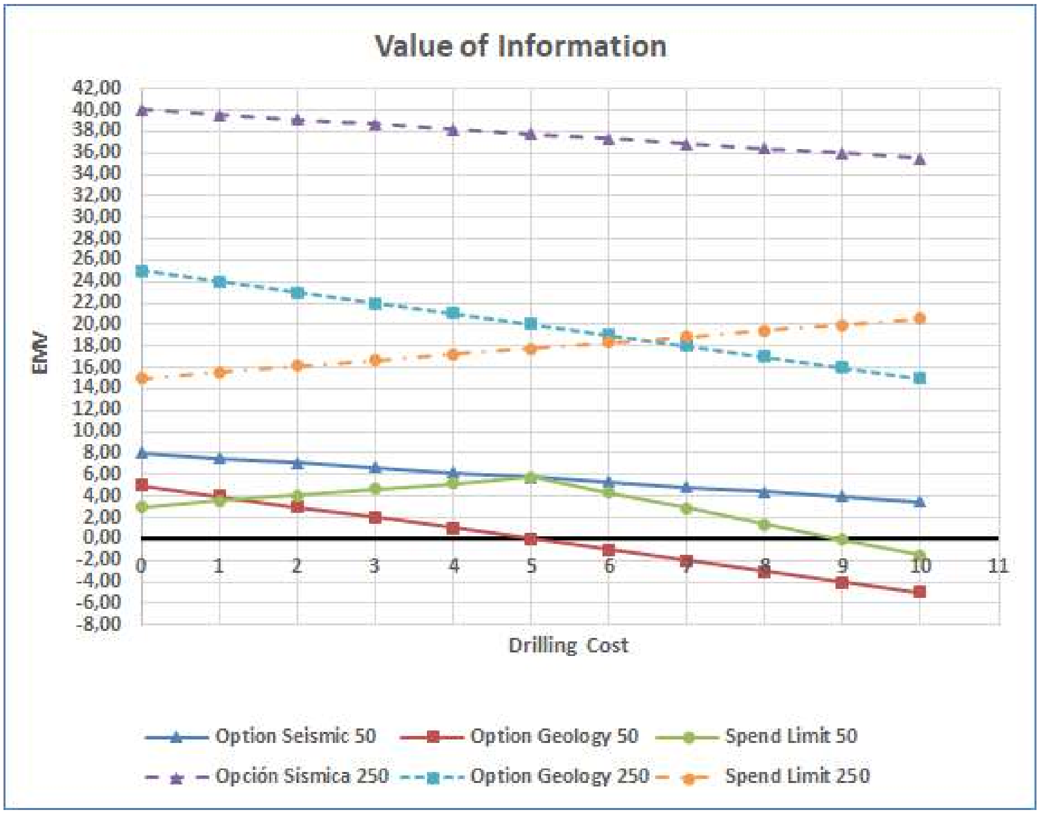


FIGURE 72 EMV VS. DRILLING COST

In summary, the value of the seismic information increases with increasing Revenue and increasing Drilling Cost.

The Expected Value of The Random Variable

Although knowledge of the probability distribution of a random variable allows to make a specific probability statement, a single value that may characterize the random variable and its probability distribution is often desirable. Such a quantity is the expected value of the random variable.

Analysts also want to know something about how the values of the random variable are dispersed about the mean. In investment analysis, this dispersion information is interpreted as the degree of a project risk. Mean indicates the weighted average of the random variable, and the variance captures the variability of the random variable.

Measure of Expectation

The expected value (the **mean**) is a weighted average value of the random variable where the weighting factors are the probabilities of occurrence. All the distributions (discrete and continuous) have an expected value.

It is used *E/[X]* (or *μ*) to denote the expected value of random variable *X.* For a random variable *X* that has either discrete or continuous value analysts compute an expected value with:

$$EMV(X) = \mu = p_j \;\; x_j \;\; discrete\ case$$

$$EMV(X) = \int x f_{(x)} d_x \;\; continuous\ case$$

The expected value of a distribution tells information about the “average” or expected value of a random variable such as the NPV, but it does not tell anything about the variability on either side of the expected value. The next section deals with the range of possible values of the random variable and the nearness to the expected value.

Measure of Variation

Another measure that analysts need in evaluating probabilistic situations is a measure of the risk due to the variability of the outcomes. There are several measures of variation of a set of numbers that are used in statistical analysis:

- The range
- The variance
- The standard deviation
- Others

The variance and the standard deviation are applied most frequently in the analysis of risk situations. It will be called *Var[X]* or σ^2 to denote the variance of a random variable X, and as indicated in previous chapters *S* denotes the standard deviation of a random variable X.
The variance indicates the degree of spread, or dispersion, of the distribution on either side of the mean value. As the variance increases the spread of the distribution increases.
The smaller the variance, the narrower the spread about the expected value.
To determine the variance:

- Calculate the deviation of each possible outcome x, from the expected value ($x_j = \mu$)
- Raise the result to the second power, and multiply it by the probability of x_j (this is p_j)
- The summation of all these products serves as a measure of the distribution’s variability

For a random variable that has only discrete values, the equation to compute variance is:

$$Var(X) \ = \ \sigma_x^2 \ = \sum (x_j - \ \mu)^2 \ * \ p_j$$

Where p_j is the probability of occurrence of value x_j of the random variable, and the expected value μ is defined by:

$$E(X) = \mu = \sum p_j * x_j \; discrete \; case$$

For a continuous random variable, the equation to compute variance is:

$$Var[X] \;=\; \int (x \;-\; \mu\,)^2 \; \mathrm{f}_{(\mathrm{x})}\, d_x$$

To be most useful, any measure of risk should have a definitive value. This measure is the standard deviation. To calculate the standard deviation, take the positive root of Var[X] which is measured in the same units as is X.

$$S_x \;=\; \sqrt{Var[X]}$$

In this case the ***standard deviation*** is now a probability-weighted deviation from the expected value or the square root of the sum of squared deviation from the expected value. Thus, it gives an idea of how far above or below the expected value compares to the actual value.

Example of Computation of Measure of Variance

Using oil rate (X) and oil price (Y) as estimated in the following table:

Oil Rate (X)	
Bls/d (x)	**P(X=x)**
1600	0.20
2000	0.60
2400	0.20

Oil Price (Y)	
$/Bl (y)	**P(Y=y)**
48	0.30
50	0.50
53	0.20

Compute the means, variance and standard deviation for the random variables X (oil rate) and Y (oil price)

Solution

For the oil rate variable (X)

	Oil Rate (X)				
	stbpd (x)	**P(X=x)**	**stbpd (X) * P(x)**		
	(I)	**(II)**	**(I)*(II)**		**(III) = (II)*((I)-E(X))^2**
	1600	0.20	320		32000
	2000	0.60	1200		0
	2400	0.20	480		32000
E(X)	**2000**			**Var{X] =**	**64000**
				S_x =	**252.98**

For the variable oil price (Y)

	Oil Price (Y)				
	$/bl (y)	**P(Y=y)**	**Unit(y) * P(y)**		
	(I)	**(II)**	**(I)*(II)**		**(III) = (II)*((I)-E(X))^2**
	48	0.30	14.4		1.63
	50	0.50	25.0		0.06
	53	0.20	10.6		1.42
E(Y)	**50**			**Var[Y] =**	**3.11**
				S_y =	**1.76**

TABLE 163 Means, Variance and Standard Deviation for The Random Variable X (Oil Rate) and Oil Price (Y)

Therefore, considering the standard deviation the weighted oil rate of 2000 stbpd and oil price of 50 $/bl vary as follows:

Oil Rate (stbpd)

X = 2000 - 252.98 and 2000 + 252.98

X = 1747.02 and 2252.98

Oil Price ($/bl)

Y = 50 - 1.76 and 50 + 1.76

Y = 48.24 and 51.76

Joint Probabilities Example

As a refresher, this topic is related to the values of variables influencing the values of others. This means that the value of some parameters will be dependent on the value of others and it is also called ***conditional probabilities***. For example, the oil demand can be influenced by the price of an oil barrel.

The joint probability is defined as:

$$p_{(x,y)} = p_{(X = x|Y = y)} p_{(y)}$$

Where $p_{(X=x|Y=y)}$ is the ***conditional probability*** of observing x given Y = y, and $P_{(y)}$ is the ***marginal probability*** of observing all events Y. If X and Y are independent, the joint probability is:

$$p_{(x,y)} = p_{(x)} * p_{(y)}$$

From the previous example the oil rate and oil price, the estimates are:

Oil Rate (X)	
Bls/d (x)	**P(X=x)**
1600	0.20
2000	0.60
2400	0.20

Oil Price (Y)	
$/Bl (y)	**P(Y=y)**
48	0.30
50	0.50
53	0.20

The marketing staff estimates that for a given oil price of US$ 48 the conditional probability that the company can produce 1600 bls/d is 0.10. Then, the probability of this joint event (oil rate = 1600 bls/d and oil price US$ 48 is:

$P_{(x,y)} = P_{(x = 1600,\ y = 48)}$

$P_{(x,y)} = P_{(x = 1600\ |\ y = 48)} * P_{(y=48)}$

$P_{(x,y)} = 0.10*0.30$

$P_{(x,y)} = 0.03$

The ***given*** unconditional probabilities and the resulting joint probabilities are:

Oil Price (Y)	P(Y=y)	Oil Rate (X)	P(X=x)	Unconditional Probability	Joint Probability
(I)	(II)	(III)	(IV)	(V)	(VI) = (V) * (II)
48	0.30	1600	0.20	0.10	0.030
		2000	0.60	0.64	0.192
		2400	0.20	0.26	0.078
50	0.50	1600	0.20	0.17	0.085
		2000	0.60	0.66	0.330
		2400	0.20	0.17	0.085
53	0.20	1600	0.20	0.50	0.100
		2000	0.60	0.40	0.080
		2400	0.20	0.10	0.020

TABLE 164 Unconditional Probabilities and the Resulting Joint Probabilities

The oil rate (X) ranges from 1600 to 2400 bls/d, the price (Y) ranges from US$ 48 to US$ 53 and there are nine possible joint events. The sum of these joint probabilities must equal 1. The marginal distribution for ***x*** can be developed from the joint ***y*** fixing ***x*** and summing over ***y*** or:

Oil Rate (X)	Oil Price (Y)	Joint Probability
(III)	(I)	(VI) = (V) * (II)
1600	48	0,030
1600	50	0,085
1600	53	0,100
	Sub Total 1600	**0.215**
2000	48	0,192
2000	50	0,330
2000	53	0,080
	Sub Total 2000	**0.602**
2400	48	0,078
2400	50	0,085
2400	53	0,020
	Sub Total 2400	**0.183**
	Total	**1.000**

This distribution indicates that 60.2% of the time it can be expected to have the oil rate of 2000 Bls/d, and that 21.5% and 18.3 % of the time it can be expected to have the rate of 1600 and 2400 Bls/d respectively.

If it is known that the oil rate was 1600 and analysts want to develop a conditional distribution for the oil price (y) that accounts for this fact, then, the conditional distribution is:

$$P(y \mid x) = 1600 = \frac{P(x,y)}{P(x)}$$

Oil Rate (X)	Oil Price (Y)	Joint Probability	Conditional Distribution
(I)	(II)	(III)	(IV) = (III)/Sub Total
1600	48	0.030	0.140
1600	50	0.085	0.395
1600	53	0.100	0.465
	Sub Total 1600	**0.215**	**1.000**
2000	48	0.192	0.319
2000	50	0.330	0.548
2000	53	0.080	0.133
	Sub Total 2000	**0.602**	**1.000**
2400	48	0.078	0.426
2400	50	0.085	0.464
2400	53	0.020	0.109
	Sub Total 2400	**0.183**	**1.000**
Total		**1.000**	

TABLE 165 Conditional Distribution for the Oil Price (Y) Given the Oil Rate (X)

This distribution differs from the original unconditional distribution for Y. For example, here, for a given oil rate of 1600 Bls/d, there is 0.140 probability that the oil price will be US$ 48.

Probability Distribution of NPV

After identifying the random variables in a project and assessing the probabilities of the possible events the next step is to develop the probability distribution of the project's NPV.

Procedures for Developing an NPV Distribution

Consider he situation where all random variables used in calculating NPV are independent. To develop the NPV distribution, follow these steps:

1. Express the NPV as a function of unknown random variables
2. Determine the profitability distribution for each random variable
3. Determine the joint event and their probabilities
4. Evaluate the NPV equation at these joint events
5. Order the NPV values in increasing order of value

Example of Probability Distribution of NPV

Consider the same case as before but now the oil rate (**X**) and oil price (**Y**), and their distribution are given as follows:

		Project Years					
	Items	**0**	**1**	**2**	**3**	**4**	**5**
Cash Inflow							
	Net Salvage value	0	0	0	0	0	40204
	Revenue	0	0.6XY	0.6XY	0.6XY	0.6XY	0.6XY
	Credits	0	7500	12750	8925	6248	4373
Cash Outflow							
	Investment	125000	0	0	0	0	0
	Variable Cost	0	9X	9X	9X	9X	9X
	Fixed Cost	0	6000	6000	6000	6000	6000
	Net Cash flow	-125000	0.6X(Y-15) + 1500	0.6X(Y-15) + 6750	0.6X(Y-15) + 2925	0.6X(Y-15) + 248	0.6X(Y-15) + 38577

TABLE 166 Distribution of The Oil Rate (X) And Oil Price (Y)

Net Cash Flow Year 1 = 0.6XY + 7500 - 9X – 6000 = 0.6X *(Y – 9/0.6) + 7500 – 6000 = 0.6X(Y-15) + 1500

Probability distribution for oil rate (X) and oil price (Y) where X and Y are independent random variables.

Oil Rate (X)	
Bls/d (x)	**P(X=x)**
1600	0.20
2000	0.60
2400	0.20

Oil Price (Y)	
$/Bl (x)	**P(Y=y)**
48	0.30
50	0.50
53	0.20

Develop the NPV distribution and calculate the mean and variance of the NPV distribution for the 15% opportunity cost of capital (MARR).

Solution

Based on the cash flow the NPV can be calculated as follows:

	Cash Flow Economic Analysis					Option 1				
Input						Use as equivalent rate:	15,00%	i = i'+f+i'*f		
Base Cash Flow			Clear Input			Option 2				
Project Life (N, yrs)	5		Max 40 Yrs			15,0%	is corrected to			
Initial Investment (P_{oINF}, $)	125000					To be added to i				
Interest (I)	15,00%		inflation (f)	0,0%	r corected by f =		15,00%	r = (i - f)/(1 + f)		
Fill Rows 8 & 10		Yr	0	1	2	3	4	5		
Annty Outflow (C, $ per year)		Ci		6000	6000	6000	6000	6000		
Annty Inflow (A, $ per year)		Ai		7500	12750	8925	6248	4373		
Sunk Cost @ N (S, $)	0									
Salvage Value @ N (Sv, $)	40204									
Results										
Profit to Investment Ratio (PIR)	-0,779		NPW/Po							
Discounted Payout (Payback yrs)										
NPVI	-0,779		NPV (@i) / MCO (@i)							
Net Present Value (NPV, $)	-97347		$P = F / (1 + i)^n$		$P = A * \{[(1 + i)^n - 1]/(i (1 + i)^n)\}$					
Benefit/Cost Ratio (PI)	0,329		ΣPV of net cash inflows/ΣPV of net cash outflows					Net Operating Income to Investment		
Internal Rate of Return (IRR,%)	-18,46%		Click Calc IRR Setting D26=0 Varying B24							

Results	
Profit to Investment Ratio (PIR)	-0,779
Discounted Payout (Payback yrs)	
NPVI	-0,779
Net Present Value (NPV, $)	-97347
Benefit/Cost Ratio (PI)	0,329
Internal Rate of Return (IRR,%)	-18,46%
	Calc IRR
Eqvlnt Total Annual Cost (A_{PC}, $)	-29040,14
Future Value of P (F_1, $)	251419,65
Capital Recovery (CR_{PC}, $)	37289,44

TABLE 167 Base Cash Flow Credits and Fixed Cost

From Table 167 NPV of the credits and fixed cost is US$ -97347. Therefore, in addition to the corresponding annuity outflows (Variable Cost) and inflows (Revenues) and using the annuity formula:

$$P = (A/i)\ (1 - 1/(1+i)^n)$$

or

$$NPV = (A/i) * (1 - 1/(1+i)^n)$$

$$NPV\ inflow = (0.6XY/0.15) * (1 - 1/(1+0.15)^5)$$

$$NPV\ outflow = (-9X/0.15) * (1 - 1/(1+0.15)^5)$$

Therefore, the total NPV = -97347 + (0.6XY/0.15) (1 - 1/(1+0.15)^5) + (-9X/0.15) (1 - 1/(1+0.15)^5)

If the oil rate X and the oil price Y are random variables, the NPV at 15% will also be a random variable. To determine the NPV distribution, it is necessary to consider all the eight combinations of possible outcomes. They are summarized with their joint probabilities in Table 168.

Oil Price (Y)	P(Y=y)	Oil Rate (X)	P(X=x)	Joint Probability p(x,y)	Cumulative Joint Probability	NPV (US$)
(I)	(II)	(III)	(IV)	(V) = (II)*(IV)	(VI) = ∑(V)	(VII)
48	0.30	1600	0.20	0.06	0.060	8849.27
50	0.50	1600	0.20	0.10	0.160	15285.41
53	0.20	1600	0.20	0.04	0.200	24939.62
48	0.30	2000	0.60	0.18	0.380	35398.34
50	0.50	2000	0.60	0.30	0.680	43443.51
53	0.20	2000	0.60	0.12	0.800	55511.27
48	0.30	2400	0.20	0.06	0.860	61947.41
50	0.50	2400	0.20	0.10	0.960	71601.62
53	0.20	2400	0.20	0.04	1.000	86082.93

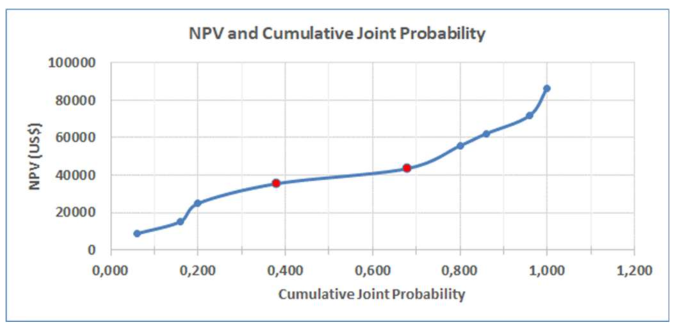
Where (VII) = -97347 + (0.6*(I)*(III)/0.15) $(1 - 1/(1+0.15)^5)$ + (-9*(III)/0.15) $(1 - 1/(1+0.15)^5)$

TABLE 168 NPV Distribution with Oil Rate (X) And Oil Price (Y) as Random Variables

The NPV probability distribution in Table 168 indicates that the project's NPV varies between US$ 8849 and US$ 86083

There is no loss under any of the cases examined.

From the cumulative distribution there is 38% probability that the project would realize an NPV less than the forecast for the base-case situation (US$ 43444) or mid case presented in Table 169 (this is the same case already presented in Table 142).



On the other hand, there is a 32% (100-68) probability that the NPV will be greater than US$ 43444. The probability distribution provides more information on the likelihood of each possible event than does the scenario analysis for example:

Item	Scenario		
	Worst Case	Most Likely	Best Case
Oil Rate	1600	2000	2400
Oil Price (US$)	48	50	53
Variable Cost (US$/Bl)	17	15	12
Fixed Cost (US$)	11000	10000	8000
Investment (US$)	125000	125000	125000
Salvage Value (US$)	30000	50000	60000
NPV (US$)	-5565	43443	91077
AEW (US$)	-1660	12960	27170
IRR	13.2%	15.0%	40.7%

TABLE 169 Optimistic (Best), Pessimistic (Worst) And Base Case Estimates for The Key Variables

It has been developed a probability distribution for the NPV considering the random cash flows. As it has been observed the probability distribution helps to conclude what the data imply in term of risk of the project.

The next step is to summarize the probabilistic information (the mean and the variance). The computed expected value of the NPV distribution is shown in Table 170:

Oil Price (Y)	P(Y=y)	Oil Rate (X)	P(X=x)	Joint Probability p(x,y)	Cumulative Joint Probability	NPV (US$)	Weighted NPV
(I)	(II)	(III)	(IV)	(V) = (II)*(IV)	(VI) = ∑(V)	(VII)	(VIII) = (V)*(VII)
48	0.30	1600	0.20	0.06	0.060	8849.27	530.96
50	0.50	1600	0.20	0.10	0.160	15285.41	1528.54
53	0.20	1600	0.20	0.04	0.200	24939.62	997.58
48	0.30	2000	0.60	0.18	0.380	35398.34	6371.70
50	0.50	2000	0.60	0.30	0.680	43443.51	13033.05
53	0.20	2000	0.60	0.12	0.800	55511.27	6661.35
48	0.30	2400	0.20	0.06	0.860	61947.41	3716.84
50	0.50	2400	0.20	0.10	0.960	71601.62	7160.16
53	0.20	2400	0.20	0.04	1.000	86082.93	3443.32
						Summa	43443.51

TABLE 170 Computed Expected Value of the NPV Distribution

This expected value (US$ 43443.51) is the same as the most-likely value of the NPV distribution This equality was expected because X and Y have symmetrical probability distribution, respectively.

The variance of the NPV distribution is obtained, assuming independence between X and Y and using the equation

$$Var(X) = \sigma_x^2 = \sum (x_j - \mu)^2 * p_j$$

		MARR =	15%						
		n =	5	(VII) = -97347 + (0.6(I)(III)/0.15) (1 - 1/(1+0.15)5) + (-9(III)/0.15) (1 - 1/(1+0.15)5)					
Oil Price (Y)	**P (Y=y)**	**Product Demand (X)**	**P (X=x)**	**Joint Probability p(x,y)**	**Cumulative Joint Probability**	**NPV (US$)**	**Weighted NPV**	**Weighted ((NPV-µ)^2)*p**	**Variance**
(I)	**(II)**	**(III)**	**(IV)**	**(V) = (II)*(IV)**	**(VI) = ∑(V)**	**(VII)**	**(VIII) = (V)*(VII)**	**(IX) = ((VII)-Summa)^2)*(V)**	**(X) = ∑(IX)**
48	0.30	1600	0.20	0.06	0.060	8849.27	530.96	71805689.01	71805689.01
50	0.50	1600	0.20	0.10	0.160	15285.41	1528.54	79287875.46	151093564.47
53	0.20	1600	0.20	0.04	0.200	24939.62	997.58	13695766.90	164789331.37
48	0.30	2000	0.60	0.18	0.380	35398.34	6371.70	11650463.33	176439794.70
50	0.50	2000	0.60	0.30	0.680	43443.51	13033.05	0.00	176439794.70
53	0.20	2000	0.60	0.12	0.800	55511.27	6661.35	17475695.00	193915489.70
48	0.30	2400	0.20	0.06	0.860	61947.41	3716.84	20543650.34	214459140.04
50	0.50	2400	0.20	0.10	0.960	71601.62	7160.16	79287875.46	293747015.50
53	0.20	2400	0.20	0.04	1.000	86082.93	3443.32	72724781.12	366471796.62
						Sum	43443.51	**Std Deviation**	19143.45

TABLE 171 Variance of the NPV Distribution

The probability distribution offers more information on the probability of each possible event than does the scenario analysis and according to the normal distribution around 68% of values are within 1 standard deviation from the mean. Therefore, using the standard deviation (19143.45) and the weighted NPV of US$ 43443.51, it is assumed that 68% of the NPV values vary between US$ 24300.06 and US$ 62586.97.

Dependent Random Variable

All the previous examples have considered independent random variables. However, some of the random variables affecting NPV may be related to one another. If they are, it is required to sample from distributions of the random variable in a manner that accounts for any dependency. For example, the oil rate and the oil price are not known with certainty. Both parameters would be on the list of variables for which analysts need to describe distributions, but they could be related inversely. When describing distribution for these parameters, analysts must account for dependency.

Summary of Chapter 7

Often, cash flow elements and other aspects of investment projects analysis are uncertain. Whenever such uncertainty exists, analysts are faced with project risks or the possibility that an investment project will not meet the minimum requirements for acceptability and success.

The most basic tools for assessing project risk are:

- Sensitivity Analysis
  - Identifying the project variables which, when varied have the greatest effect on project acceptability
- Breakeven Analysis
  - Identifying the value of a particular project variable that causes the project to exactly breakeven
- Scenario Analysis
  - Comparing a "base case" or expected project measurement (such as NPV) to one or more additional scenarios, such as best and worst case, to identify the extreme and most likely project outcomes

Sensitivity, breakeven, and scenario analyses are reasonably simple to apply, but also simplistic and imprecise in cases where analysts are dealing with multifaceted project uncertainty. Therefore, ***probability concepts*** allow to further refine the analysis of project risk by assigning numerical values to the likelihood that project variables will have certain values.

The end goal of probabilistic analysis of project variables is to produce an NPV distribution. From the distribution the analysts can extract useful information as the ***expected value***, the extent to which other NPV values vary from or are clustered around the expected value (***variance***) and the best and worst case NPV's.

Risk simulation, in general, is the process of modeling reality to observe and weigh the likelihood of the possible outcomes of a risky undertaking. ***Monte Carlo sampling*** is a specific type of randomized sampling method in which a random sample of outcomes is generated for specific probability distribution. Because Monte Carlo sampling and other simulation techniques often rely on generating a significant number of outcomes they can be more conveniently performed on a computer.

In dealing with risk, companies distinguish between:

- Risk assessment
  - The effect of risk on project valuation
- Risk management

Risk Assessment is a widely used technique that includes:

- Sensitivity and breakeven analysis
- Payback period determination (simple or discounted)
- Computer based simulation for sensitivities on NPV, IRR, PIP, etc.

To be consistent with the objective of maximizing shareholders´ wealth, the view of risk of investors rather than the view of risk of managers should be relevant in evaluating projects.

Managers should accept only those projects that earn an expected return higher than the return investors anticipate when making funds available to the company.

There are ***evaluation techniques*** based on management's subjective assessment of risk. For example:

- Risk-adjusted discount rates
  - This means that the rates applied to individual projects are adjusted to reflect perceived risk
  - Only projects with individual risk characteristics that closely match those of the overall company are discounted at the company's average cost of raising funds
  - Conceptual difficulties of risk-adjusted discounting include the fact that time and risk become linked through the discounting process, implying that risk increases with time in a specified manner
  - Nevertheless, risk adjusted discounting often represents a reasonable compromise between operational simplicity and theoretical validity
- Conservative estimates
  - These estimates that potentially yield reasonable results are often applied in an informal and intuitive way. However, in large corporations where several divisions are involved in the decision-making process, this can lead to severe distortions
- The payback-period
  - Despite its weakness as a primary evaluation criterion, it can be useful in the evaluation of risk, particularly if several refinements are added. These include:
    - Specific considerations of the time value of money
    - Portraying the NPV as a function of time under the assumption of premature abandonment

Risk Management: risk can be managed, at least to some extent, within the company. Different techniques for doing this include:

- Improving forecasting through better information implementing various operating measures that enhance flexibility (such as subcontracting and renting rather than owning)
- Shifting risk through insurance-type arrangements
- Financial engineering
- Diversification