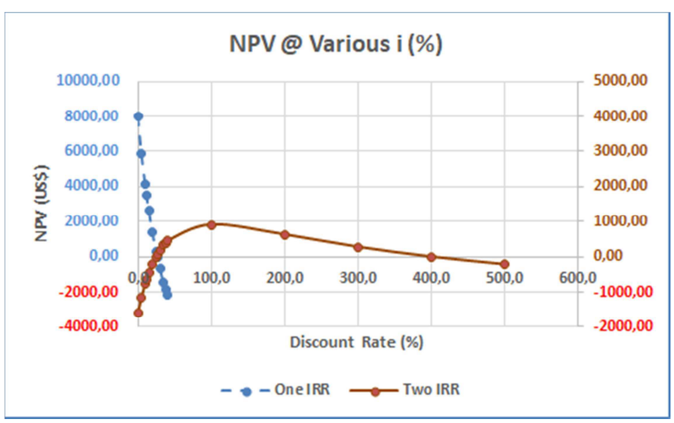
CHAPTER 4 MEASURES OF VALUE _ ECONOMIC HURDLES

Engineering Economics Topics in Hydrocarbon Subsurface_ a Bibliographical Review Handbook



1/11/2022 ROLANDO GARCÍA LUGO CARACAS VENEZUELA

Engineering Economics Topics in Hydrocarbon Subsurface _a Bibliographical Review Handbook

Chapter 4 Measures of Value _ Economic Hurdles

Learning Objectives Chapter 4 Project Parameters

Having worked through this chapter the reader will be able to:

- Cash Flow Modelling
    - Define project screening and ranking
    - Explain the important advantages and disadvantages of working with cash flows in real terms and in money of the day
    - List the stages in preparation of cash flow models
- Cumulative Cash flow and Simple Measures of Value
    - Define: Payback period. Profit to investment ratio
    - Explain the uses and limitations of Payback period
    - Explain the uses and limitations of profit to investment ratio
    - Define: CAPEX per barrel. CAPEX per daily barrel. Interest during construction (IDC)
    - Explain the uses and limitations of using CAPEX per barrel and CAPEX per daily barrel as measures of investment
- Project interdependencies
    - Describe the characteristics of mutually exclusive projects
- Discounted Measures of Value
    - Calculate Net Present Value (NPV) from project cash flows
    - Describe the characteristics of NPV
    - Explain the significance of NPV
    - Calculate NPV index (NPVI)
    - Explain the significance of NPVI
    - Define Annual Capital Charge (ACC)
    - Derive the ACC Factor
    - Explain the significance of the ACC method
    - Apply the ACC method in appropriate investment situations
    - Define the Internal Rate of Return (IRR)
    - Derive IRR using both numerical and graphical methods
    - Explain the significance of the IRR
    - Explain the inaccuracy from late-stage negative cash flows
    - Normalize cash flow using the extended yield method
    - Explain the origin of multiple roots in the context of IRR calculation
    - Explain the characteristics of cash flow associated with acceleration-type projects
    - Calculate and interpret NPV and IRR for acceleration projects
    - Explain the application of NPV as a transaction criterion
    - Define project screening
    - Explain the use of NPV, NPVI and IRR as screening criteria
    - List possible reasons for project ranking
    - Explain the use of NPV, NPVI and IRR as ranking criteria

  - Explain the decision process related to the replacement problem when comparison between defender and challenger and the determination replacement analysis when the required service period is long
- Replacement analysis
  - Explain the basis for selecting a new project or equipment (Challenger) or continuing with current one (Defender)
  - Describe the opportunity cost concept
- Multiple-Options Comparison
  - Select the best option based on the NPV method

Refreshing Previous Concepts

Cash Flow Modelling

A cash flow model in the context of this handbook is a detailed picture of investments, income and expenditure which is projected yearly, using assumed rates of growth, income, inflation, and interest rates. Individual items of expenditure are grouped and aggregated over time to generate a cash flow model. This allows to consider the implications of time value on these models.

Currency Units

Cash flow models may be compiled in "money of the day" or "Constant" terms.

Money of the Day

- A cash flow model, in money of the day ("MoD") terms, incorporates the expenditures and revenues for each project year, using currency units (€, £, $ etc.) appropriate to that year
- With inflation, the purchasing power of the currency units in the model varies from year to year
- Derivation of a cash flow, from the physical project model, requires an explicit assumption about future rates of price inflation
- The further into the future, the less confidence there will be in such estimates
- Project cost normally increases with inflation and this may be modelled, using RPI or some other price index (sector index or specialized industry-specific index)
- Revenues may or may not change with inflation. Commodity markets, such as oil, are dominated by supply / demand interaction, and price may be insensitive to inflation
- The advantages of having a cash flow model in "MoD" terms:
  - The model can interface directly with the world outside the project. For example, in the model, the negative cash flow in project year 5 is the management's estimate of actual expenditure in that year and may therefore be used as input to a corporate budget and accounts
  - Model data may also be used directly to calculate tax liability, since official tax calculation is based on "MoD" revenues and costs
- The disadvantages of having a cash flow in "MoD" terms:

  - The purchasing power of the data varies from year to year. This is true, for the model itself and for the NCF derived from it

Constant Terms

- The advantages of having a cash flow model in "Constant" terms:
  - The NCF derived from it is also in "Constant" terms
  - A cash flow model, in "Constant" terms, incorporates the expenditures and revenues for each project year, using currency units (€, £, $ etc.) of constant purchasing power
  - The base year is normally, but not necessarily, the current year
  - Costs, which are usually estimated in constant or real terms, may be entered directly into the model
  - If "MoD" revenue can be estimated, inflation must be removed to generate data in "Constant" terms
- The disadvantage of having a cash flow model in "Constant terms
  - The model does not relate to the world outside
  - The data may not be used directly as input to corporate budgets, as a basis for financial decisions or for accurate calculation of tax liability

Cumulative Cash Flow in Constant Terms

The starting point for this type of analysis is the net cash flow in Constant terms. Because it is inappropriate to add together cash flows in "MoD" terms, particularly over a long period of time, since purchasing power changes with inflation and totals are meaningless.

Price inflation implies declining purchasing power. Negative inflation caused by falling prices have the opposite result.

The following example presents an annual NCF and cumulative equivalent where there is an early negative (investment) phase, followed by positive cash flows and apparent profitability.

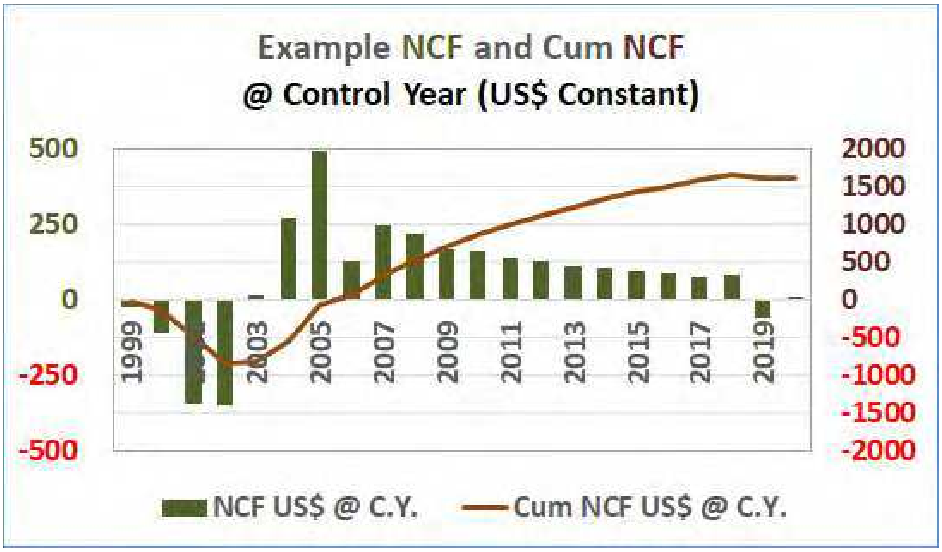


FIGURE 6 NCF AND CUMULATIVE NCF IN CONSTANT TERMS

Managing "MoD" and "Constant" Terms

The use of "MoD" or "Constant" data depends on the nature of the business and of the investment.

- Managers who concentrate on short term investments, have regular dealings with the public or other organizations, focus on "MoD" data
- Managers involved with longer term and more self-contained investments as in the hydrocarbon industry may prefer to use "Constant" data

Despite which form of data is used it can be converted from "MoD" to "Constant" or vice versa if the analyst indicates which data are used and on which terms.

Accounting systems are usually based on "MoD" data and this relates to conventional budgeting and annual reports, banking and tax calculation. Measures of value or profitability should be based on increasing purchasing power.

If the model is in "Constant or real" terms, the derived NCF will also be in "Constant" terms with the same currency units.

Any measure of profit derived from the NCF will be in the same units and will relate directly to purchasing power. Any interest or growth rate derived from such data will be "Constant" and of the type designated "r" in Chapter 3:

f = rate of inflation

r = real growth rate

$r = (i - f) / (1 + f)$

This “r” will reflect a change in purchasing power. If the model is in “MoD” terms, the NCF derived from it will also be in “MoD” terms. The NCF may be converted to “real” terms using simple conversion factors based on inflation data. Derived measures of profitability are then based on purchasing power as before.

If analysis proceeds with the NCF in “MoD” terms, any derived measure of value or profitability becomes distorted by inflation.

Any measure of profit, based on the difference between revenue (later) and expenditure (earlier), will be increased by inflation. Any rate of growth derived from this data will be of the type “i” designated in Chapter 3. Such interest rates include a “Constant” part and inflation, and these may be separated using the relationship developed in Chapter 3, if they are based on a constant rate of inflation. Most of the interest rates which are offered by financial institutions are of the “i” types where “i” is fixed, “f`” is variable and “r” is the residual.

As noted previously the constant-dollar technique is more used when comparing projects to select or rank them

Comparing Cash Flows Across Time Periods

One of the main motivations for compounding and discounting is to allow comparisons between cash flows that occur at different points in time.

- Through compounding is possible to shift any given cash flow forward in time
- Discounting moves future cash flow back toward the present
- Once cash flows are based in the same period they can be compared through simple addition or subtraction

It is possible to compare cash flows that occur at different points in times by:

1. Choosing a particular point in time as the basis for economic comparison
2. Shifting all cash flows into economically equivalent amounts at the chosen point in time through compounding or discounting
3. Adding or subtracting all these equivalent cash flows to obtain a net total

The point in time selected as the basis of economic comparison Is arbitrary. The convention is to choose the present and bring all future cash flows back to the origin through discounting to derive an overall present value.

Present value is easy to interpret economically as it represents the current dollar amount that is equivalent to the set of cash flows under consideration.

Another option is to choose the end of some planning period and compute the terminal wealth as given by the future value of all cash flows as of that date.

Cash Flow Model Creation

Usually, a cash flow model is used to calculate a range of economic parameters for making an investment decision. It is important to:

- Make the best use of available data
- Ensure that the analysis is appropriate to the problem or application

Cash Flow Model Steps

- Physical Model
    - The starting point for any cash flow is a model of the physical project
    - The model incorporates best estimates for development and production phases and all the relevant timing
    - The quantity and quality of data available will reflect the nature of the project and the current stage of investigation or development
    - A pre-decision analysis is based on estimates of many of the project details
- Relevant Data
    - Any decision to invest changes a pre-existing condition
    - For the cash flow method, it must be distinguished between
        - Those cash flows which result from the decision or investment
        - Those which would have occurred regardless
    - Only those cash flows that depend on the decision are relevant
- Cash Flow Considerations
    - The cash flow transfers funds from one account to another. Therefore, depreciation is not a cash flow
    - Costs may be estimates, derived from previous experience or quotes from potential suppliers
    - Revenues are further into the future and subject to greater uncertainty
    - Units of currency are important. In an international business, hydrocarbon companies operate in many countries and must make choices appropriate to their situation. At some point, cash flows must be converted into local currency for tax calculation. For every conversion process, there is a risk that the rate will vary
    - As discussed previously, a decision must be taken between "MoD" and "Constant" terms
        - "MoD" is slightly more involved, but provides greater flexibility with respect to integrating the model to the outside world. To do this, there must be an explicit assumption made about inflation over the life of the project and with all derived estimates
        - It is preferable to remain simple and keep the rate "***<u>Constant</u>***", unless there is clear justification to do otherwise
- Cumulative Cash Flow
    - Any project may be represented by a series of annual, net cash flows (NCF's), which characterizes the financial impact of the investment

  - The cumulative of NCF over time, provides a measure of performance or value and gives rise to several useful parameters.
- Net Cash Flow (NCF)
  - It is the aggregate cash flow for a specified time
  - The currency units for NCF are the same as for the cash flows from which it is derived
  - If cash flow is in "MoD", NCF will also be in "MoD"
  - To simplify further analysis, usually it is preferable for NCF to be converted to "***Constant***" terms
    - This way all derived parameters are related to constant purchasing power
- Timing
  - It is normal to sub-divide project time into calendar years, unless there is a particular reason for doing otherwise
    - Certain forms of tax, for example, are based on six-month periods.
    - If precision is required, then six-month periods become necessary
    - Short projects
      - Less than five years might be better modelled monthly
      - Less than a few months, daily
  - Within each period, aggregation of cash flows takes place, to produce a single net cash flow. It is normal to model the aggregation time as the mid-point of the period since, for randomly distributed cash flows, this produces the most accurate representation. If cash flows are not randomly distributed, it may be appropriate to choose a different timing for the aggregation point.
  - Time origin
    - This usually relates to the present day or year, or to a significant decision point in the life of the project
    - The origin is quantitatively important, because it is the date to which any derived economic parameter is calculated
    - If the origin is-mid year, it fits in with mid-year aggregation and an annual process of discounting.
    - This is the simplest structure. If it is set at some other point in the period, it implies a part-year step in the discounting process

Derivation of discount Factors for Two Forms of Cash Flow Model

The following examples explain the derivation of discount factors for two common forms of cash flow model structure:

- The mid-point of project year one
- Mid-year aggregation, with the origin at the beginning of project year one

Example (a) is the simplest form, with mid-year aggregation and discounting back to the origin, which is the mid-point of project year one. This implies that discounting takes place from the mid-point of one project year, back to the mid-point of the previous year. Values of "n" are integers.

(a) Simple Structure	
Aggregation Point	
n = 0	n =1
Year 1	Year 2

Example (b) also has mid-year aggregation, but the origin is now at the beginning of project year one. There is a half-year step between project year one aggregation and project origin. Each aggregation point is separated by a one-year step. A way to manage the discounting of these cash flows is using a discount exponent of 0.5 to year one cash flows. The exponent for year two becomes 1.5, and so on.

	(b) Alternate Structure	
	Aggregation Point	
n = 0	n = 0,5	n =1,5
	Year 1	Year 2

Year	n	$1/(1 + i)^n$
1	0	1.000
2	1	0.909
3	2	0.826
4	3	0.751
5	4	0.683
6	5	0.621
7	6	0.564
8	7	0.513
9	8	0.467
10	9	0.424
11	10	0.386

Discount Rate = 10%

Year	n	$1/(1 + i)^n$
1	0.5	0.953
2	1.5	0.867
3	2.5	0.788
4	3.5	0.716
5	4.5	0.651
6	5.5	0.592
7	6.5	0.538
8	7.5	0.489
9	8.5	0.445
10	9.5	0.404
11	10.5	0.368

Discount Rate = 10%

TABLE 36 Aggregation Point Structure

The usual option is using the end of the year aggregation and the result is like the simple form. In this case there is a year zero. In this example the last year is 10 and there are 11 years in total as indicated above.

	(c) End of Year Structure			
	Aggregation Point			
n = 0		**n = 1**		**n = 2**
	Year 1		**Year 2**	

Discount rate = 10%

Year	n	$1/(1+i)^n$
0	0	1.000
1	1	0.909
2	2	0.826
3	3	0.751
4	4	0.683
5	5	0.621
6	6	0.564
7	7	0.513
8	8	0.467
9	9	0.424
10	10	0.386

Project interdependencies

Before applying economic hurdles for selecting or ranking investments, the projects must be identified as:

- Independent projects of each other
    - Analysis is performed assuming isolation between projects
- Interdependences between projects
    - It exists where the cash flows generated by one project depend on the investments that the company may consider
    - Subclassification:
        - Mutually Exclusive Projects
        - Contingent Projects

Mutually Exclusive Projects Characteristics

Location problems are a typical example of mutually exclusive projects. If a company considers any of three different locations for a new refinery, the options are mutually exclusive as the choice of one site will preclude a refinery in any other location, even if a separate analysis of each option reveals that a positive NPV have been attainable in more than one location.

Another example would be the purchase of a new pump system where a choice of different pumps may be available for one operation. In this situation, all mutually exclusive options are analyzed and compared to each other, and only the best one is chosen. This topic will be expanded, considering its importance, in Chapter 6

Contingency Investments

These investments imply that the viability of one project is conditional on the previous selection of another. Contingent projects must be considered as a group and evaluated together. For example, acquisition of a gas lift systems may depend on the results of a drilling campaign, and vice versa. There is a possibility that the root project that is the base for the contingent projects may be accepted even if it, by itself, is not economically viable. This may be the case where an offsetting gain is provided by returns received from other projects in the contingent group.

Complementary or negative interdependence

The interdependency of a project may be either complementary or negative. For example, fuel oil and diesel may have some market overlap creating a negative interdependence. The complementary option is for example, lubricants and gasoline that may complement each other, thus increasing total sales.

Under conditions of dependence between projects, ideally all possible subgroups of investments that make up a capital budget should be evaluated to find the best possible combination, at least at the level of the more important interdependences.

Economic Hurdles or Yardsticks Derived from Cash Flows

Refreshing the Cash Flow characteristics:

- It is the most appropriate methodology for evaluating long term investments
- Time is an important factor in determining the value of individual cash flows
- An investment or project may include many cash flows
- When the cash flows are related to time in a proper manner, the total can be represented by a series of Net Cash Flows (**NCF**s)

The objective of this chapter is to derive from project NCF a range of numerical parameters, which may be used to characterize economic potential or profitability. These measures of value or profitability fall into two groups:

- Undiscounted
    - The simpler, undiscounted group includes
        - Payback
        - Profit to Investment Ratio
- Discounted
    - When time value of money is included, it may be calculated parameters such as:
        - Net Present Value
        - Internal Rate of Return

- Discounted Payback Period

Once derived, these yardsticks may be applied to screening and ranking, which are the basic stages in the investment decision process.

- **Screening** involves management in the testing of each available project against a set of appropriate criteria or standards, to determine whether these opportunities are suitable or profitable
- **Ranking** follows and requires comparison between suitable projects to determine the best candidates for investment

Undiscounted Payback Period or Payout Time

An investment's payback period is the time required for its expected after-tax cash inflows to equal (and thereby to recover) the original expenditure.

The payback period is related to break-even analysis and it is used as quick way to assess exposure to risk.

The uncertainty of cash flows typically increases the further one moves into the future; therefore, managers usually set a payback constraint.

In cases where there is a chance that an investment can be abandoned prematurely, knowing a project's Payback Period is useful in assessing exposure.

Additional definitions for the Payback Period or Payout Time are:

- It is the time taken by a project to break-even
- Payback Period is the length of time or period required to return the original investment to the investor or lender of the funds used
- It is the time of recovering the investment
- The Payback Period is the time taken, from the start of the project to the point at which the cumulative NCF returns to zero

As a stand-alone criterion to determine investment decision the Payback Period is basic and often misleading.

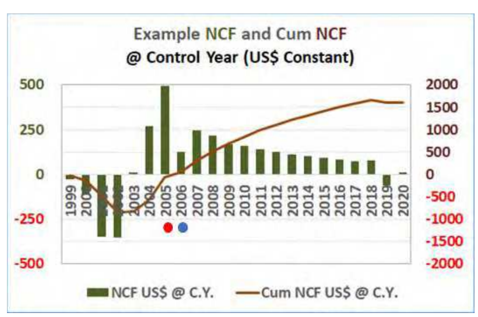
Payback Period Example:

Control Year (C. Y.) = 2005						
End of Year	RPI %	RPI Indices	Converted @ C. Y.	NCF US$ MoD	NCF US$ @ C. Y.	Cum. NCF US$ @ C. Y.
(I)	(II)	(III)	(IV) = $(iii)_{C.Y.}/(III)_n$	(V)	(VI) = (V)*(IV)	(VII) = Σ(VI)
1999	2.49	84.90	1.178	-22.00	-25.91	-25.91
2000	3.47	87.80	1.139	-99.00	-112.76	-138.67
2001	2.41	90.00	1.111	-312.51	-347.23	-485.90
2002	3.14	92.80	1.078	-327.39	-352.79	-838.70
2003	3.43	96.00	1.042	10.92	11.38	-827.32
2004	1.66	97.60	1.025	262.83	269.30	-558.02
2005	2.50	100.00	**1.000**	492.53	492.53	-65.50
2006	2.50	102.50	0.976	127.46	124.35	58.85
2007	2,50	105,10	0,951	257,15	244,67	303.52
2008	2,50	107,70	0,929	233,74	217,03	520.55
2009	2,50	110,40	0,906	184,11	166,76	687.31
2010	2,50	113,10	0,884	181,91	160,84	848.15
2011	2,50	116,00	0,862	161,27	139,03	987.18
2012	2,50	118,90	0,841	148,94	125,26	1112.44
2013	2,50	121,80	0,821	135,96	111,63	1224.07

TABLE 37 Data for Payback Calculation

This indicates the cumulative NCF curve crossing the axis between 2005 and 2006, so the Payback is near mid-2006.

Date	Cum NCF US$ @ C. Y.
Dec-2005	-65.50
Jul-2006	0.00
Dec-2006	58.85



As the project started at the end of 1999, this indicates a Payback Period of about 6 years.

- It is to be expected that larger, more complex projects have longer Payback Periods

- Projects with shorter Payback Periods may be better (than projects with longer Payback Periods)
- The interpretation of the Payback Period is subjective because Payout analysis is incomplete and reveals nothing about the future of a project beyond that point in time
- Projects with similar Payback periods may be different in later performance and value
- In general Payback Period is a useful indicator of risk
- The longer the Payback Period, the longer the investor must endure a cumulative loss on the project
- This may well be relevant to a situation where risk is time related, as for example political discontinuity

Comments on Payback Period

When a new project initiates, its useful life may be difficult to forecast as this will depend on uncertain events as: reservoir reaction, technological incompatibility, environmental issues, etc.

If the project has an expected Payback Period of three years, management knows that at least the original investment should have been recovered by that time. The estimated useful life of a project can be viewed as this: if the useful life is expected to exceed the payback period significantly, this may be perceived as providing adequate protection against risk.

The payback period can be made more useful by adding several refinements. The time value of money should be incorporated, and the Payback Period should be redefined as Discounted Payback Period.

Discounted Payback Period (DPP) as a Project Screening Method

To offset some weakness of the simple Payback Period a discounted version is used. The discounted version allows to screen projects based on the “number of years” required to recover the investment made in a project from ***discounted*** cash flows.

This is the time required for the present value of cash inflows to equal the original investment. The company must be concentrated with recovering the original investment and with the cost of funding the investment over time.

DISCOUNTED PAYBACK PERIOD EXAMPLE 1

Using the same data as in Table 37

	Cash Flow Economic Analysis					Option 1												
Input						Use as equivalent rate:		5,00%	i = i'+f+i'*f									
Payout Case			Clear Input			Option 2												
Project Life (N, yrs)	15		Max 40 Yrs			5,0% is corrected to												
Initial Investment (P_{oPYO}, $)	0						To be added to i											
Interest (I)	5,0%		inflation (f)	0,0%	r corected by f =		5,00%	r = (i - f)/(1 + f)										
		Yr	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Annty Outflow (C, $ per year)	Variable	Ci		25,91	138,67	485,90	838,70	827,32	558,02	65,50	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Annty Inflow (A, $ per year)	Variable	Ai		0,00	0,00	0,00	0,00	0,00	0,00	0,00	58,85	303,52	520,55	687,31	848,15	987,18	1112,44	1224,07
Sunk Cost @ N (S, $)	0																	
Salvage Valuet at N (Sv, $)	0																	
Results																		
Discounted Payout (Payback yrs)	13,74		Only for the variable case															

TABLE 38 Discounted Payback Period Calculation

The Discount Payback Period (DPP) is the number of years required for the discounted cash income from a project to return the initial cash investment in such project, using the single payment, present value equation.

In above example, the initial investment is zero, and costing 5% per year, will be paid-up in 13.7 years. This is the Discounted Payback-Period of this loan or investment and is larger than the 6 years calculated using the Simple Payout Period.

DISCOUNTED PAYBACK PERIOD EXAMPLE 2

A company plans to invest funds to purchase a new equipment. The company requires a rate of return of 15%. Determine the period necessary to recover both the capital investment and the cost of funds required to support the investment. The projected after-tax cash flow are as follows:

Year	Cash Flow (US$)
0	-1000
1	-500
2	500
3	700
4	1000
5	1500
6	500

Solution

	i=	15.0%	
	US$		
(I)	(II)	(III) = (II)/(1+i)$^{(I)}$	(IV) = Sum (III)
Year	Cash Flow	NPV	Cum NPV
0	-1000	-1000.00	-1000.00
1	-500	-434.78	-1434.78
2	500	378.07	-1056.71
3	700	460.26	-596.45
4	**1000**	**571.75**	**-24.70**
5	**1500**	**745.77**	**721.07**
6	500	216.16	937.23

Year	Cum NPV (US$)
4.00	-24.70
4.03	0.00
5.00	721.07

TABLE 39 DPP Detailed Calculation

	Cash Flow Economic Analysis					Option 1			
Input						Use as equivalent rate:	15,00%	i = i'+f+i'*f	
DPP			Clear Input			Option 2			
Project Life (N, yrs)	6		*Max 40 Yrs*			15,0%	is corrected to		
Initial Investment (P_{oINF}, $)	1000						To be added to i		
Interest (I)	15,00%		inflation (f)	0,0%	r corected by f =		15,00%	r = (i - f)/(1 + f)	
Fill Rows 8 & 10		Yr	0	1	2	3	4	5	6
Annty Outflow (C, $ per year)		Ci		500,0	0,0	0,0	0,0	0,0	0,0
Annty Inflow (A, $ per year)		Ai		0	500	700	1000,0	1500,0	500,0
Sunk Cost @ N (S, $)	0								
Salvage Value @ N (Sv, $)	0								
Results									
Profit to Investment Ratio (PIR)	0,937		NPW/Po						
Discounted Payout (Payback yrs)	4,03								
NPVI	0,653		NPV (@i) / MCO (@i)						
Net Present Value (NPV, $)	937,233		$P = F / (1 + i)^n$		$P = A * \{[(1 + i)^n - 1]/(i (1 + i)^n)\}$				
Benefit/Cost Ratio (PI)	1,653		ΣPV of net cash inflows/ΣPV of net cash outflows				Net Operating Income to Investment Ratio		
Internal Rate of Return (IRR,%)	32,29%		Click Calc IRR Setting D26=0 Varying B24						

TABLE 40 DPP Direct Calculation with Spreadsheet

Discussion

The total investment is recovered in 4.03 years

If the company's stated maximum payback period of 3 years; then, the project should be rejected.

Assuming that all cash flows are year-end flows, the project must remain in use at least 5 years for the company to recover its cost of capital and recover the funds invested in the project.

Another way to analyze the problem is considering the cost of funds during the first year. With US$ 1000 invested at the beginning of the year, the interest would be US$ 150 (1000 * 0,15 = 150). With no receipt from the project during the first year, the total committed grows to US$ 1150.

During year 2 with US$ 500 additional investment at the end of year 1 or at the beginning of year 2, the total commitment in the projects reaches to US$ 1650. Then, the cost of funds during the second year would be US$ 248 (1650 * 0,15). But with the receipt from the project the net commitment reduces to US$ 1397.50.

When the process repeats for the remaining project years, it is found that the net commitment to the project is over during year 5.

		i=	15.0%
	US$		
(I)	(II)	(III) = (II)$_{(n-1)}$ * i	(IV) = Sum (III) + (II)
Year	Cash Flow	Cost of Funds	Cum NCF
0	-1000	0.00	-1000.00
1	-500	-150.00	-1650.00
2	500	-247.50	-1397.50
3	700	-209.63	-907.13
4	**1000**	**-136.07**	**-43.19**
5	**1500**	**-6.48**	**1450.33**
6	500	217.55	2167.88

Year	Cum NPV (US$)
4	-43.19
4.03	0.00
5	1450.33

Cost of funds = Unrecovered beginning balance * interest rate

TABLE 41 DPP Considering Beginning of The Year Investment

Discounted Payback Period Example 3_Comparing Projects

Consider two mutually exclusive projects (A and B), a 3 years payback requirement and the following data and results:

	Cash Flow Economic Analysis					Option 1				
Input						Use as equivalent rate:		10,00%	i = i'+f+i'*f	
Project A			Clear Input			Option 2				
Project Life (N, yrs)	4		Max 40 Yrs			10,0%	is corrected to			
Initial Investment (P_{oA}, $)	400						To be added to i			
Interest (I)	10,0%		inflation (f)	0,0%	r corected by f =		10,00%	r = (i - f)/(1 + f)		
		Yr	0	1	2	3	4			
Annty Outflow (C, $ per year)		Ci		0	0	0	0			
Annty Inflow (A, $ per year)		Ai		100	100	200	200			
Sunk Cost at N (S, $)	0									
Salvage Value @ N (Sv, $)	0									
Results										
Profit to Investment Ratio (PIR)	0,151		NPW/Po							
Discounted Payout (Payback yrs)	3,53		Only for the variable case							
NPVI	0,151									
Net Future Worth (FW_A, $)	88,46		$F_1 = P\ (1 + i)^n$		$F_2 = \Sigma(A_x\ (1 + i)^{(n-x)})$		$F_3 = \Sigma(C_x\ (1 + i)^{(n-x)})$	$F = - F_1 + F_2 - F_3 - S + Sv$		
Net Present Value (NPV, $)	60,42		$P = F / (1 + i)^n$		$P = A * \{[(1 + i)^n - 1]/(i\ (1 + i)^n)\}$					
Benefit/Cost Ratio (PI)	1,151		ΣPV of net cash inflows/ΣPV of net cash outflows					Net Operating Income to Investment Ratio		
Internal Rate of Return (IRR,%)	15,90%		Click Calc IRR Setting D26=0 Varying B24							

TABLE 42 Project "A" DPP Data and Results

	Cash Flow Economic Analysis					Option 1					
Input						Use as equivalent rate:		10,00%	i = i'+f+i'*f		
Project B			Clear Input			Option 2					
Project Life (N, yrs)	4		Max 40 Yrs			10,0%	is corrected to				
Initial Investment (P_{oB}, $)	400						To be added to i				
Interest (I)	10,0%		inflation (f)	0,0%	r corected by f =		10,00%	r = (i - f)/(1 + f)			
		Yr	0	1	2	3	4				
Annty Outflow (C, $ per year)		Ci		0	0	0	0				
Annty Inflow (A, $ per year)		Ai		200	100	100	500				
Sunk Cost @ N (S, $)	0										
Salvage Value @ N (Sv, $)	0										
Results											
Profit to Investment Ratio (PIR)	0,703		NPW/Po								
Discounted Payout (Payback yrs)	3,16		Only for the variable case								
NPVI	0,703										
Net Future Worth (FW_B, $)	411,56		$F_1 = P\ (1 + i)^n$		$F_2 = \Sigma(A_x\ (1 + i)^{(n-x)})$		$F_3 = \Sigma(C_x\ (1 + i)^{(n-x)})$		$F = - F_1 + F_2 - F_3 - S + Sv$		
Net Present Value (NPV, $)	281,10		$P = F / (1 + i)^n$		$P = A * \{[(1 + i)^n - 1]/(i\ (1 + i)^n)\}$						
Benefit/Cost Ratio (PI)	1,703		ΣPV of net cash inflows/ΣPV of net cash outflows					Net Operating Income to Investment Ratio			
Internal Rate of Return (IRR,%)	34,21%		Click Calc IRR Setting D26=0 Varying B24								

TABLE 43 Project "B" DPP Data and Results

Both projects provide around a 3-year payback (A= 3.5, B= 3.2) and hence appear equivalent. However, Project B is superior to Project A because:

- Project B generates larger inflows in the year following the payback limit (US$ 500 vs. US$ 200 in year 4),
- During the payback period the larger US$ 200 cash flow of Project B is generated earlier (in year 1)
- The whole cash flow combination makes NPV of Project B larger than NPV of Project A (US$ 281.10 vs. 60.42)

Discounted Payback Period (DPP) Example 4

An investment of MUS$ 100 generates net cash inflows of MUS$ 20 per year for 7 years. The firm cost of capital is 15%. Without discounting the payback period would be computed as 5 years, because after 5 years the undiscounted cash inflows have repaid the original investment. Recognizing, however, that returns must be paid on the money that was used to fund the project, analysts should discount future cash flows ad compare their present value with the current investment. They find that the project will not be paid back in the 7-year period.

	Cash Flow Economic Analysis					Option 1					
Input						Use as equivalent rate:		15,00%	i = i'+f+i'*f		
Discounted Payback			Clear Input			Option 2					
Project Life (N, yrs)	7		*Max 40 Yrs*			15,0%	is corrected to				
Initial Investment (P_{oINF}, $)	100000						To be added to i				
Interest (I)	15,0%		inflation (f)	0,0%	r corected by f =		15,00%	r = (i - f)/(1 + f)			
Fill Rows 8 & 10		*Yr*	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	
Annty Outflow (C, $ per year)		Ci		0,0	0,0	0,0	0,0	0,0	0,0	0,0	
Annty Inflow (A, $ per year)		Ai		20000,0	20000,0	20000,0	20000,0	20000,0	20000,0	20000,0	
Sunk Cost @ N (S, $)	0										
Salvage Value @ N (Sv, $)	0										
Results											
Profit to Investment Ratio (PIR)	-0,168		NPW/Po								
Discounted Payout (Payback yrs)											
NPVI	-0,168		NPV (@i) / MCO (@i)								
Net Future Worth (FW_{PC}, $)	-44666		$F_1 = P(1+i)^n$		$F_2 = \Sigma(A_x\ (1+i)^{(n-x)})$		$F_3 = \Sigma(C_x\ (1+i)^{(n-x)})$		$F = -F_1 + F_2 - F_3 - S + Sv$		
Net Present Value (NPV, $)	-16792		$P = F/(1+i)^n$		$P = A * \{[(1+i)^n - 1]/(i(1+i)^n)\}$						
Benefit/Cost Ratio (PI)	0,832		ΣPV of net cash inflows/ΣPV of net cash outflows					Net Operating Income to Investment Ratio			
Internal Rate of Return (IRR,%)	9,20%		Click Calc IRR Setting D26=0 Varying B24								

TABLE 44 DPP Not Reached During Project's Life

At the end of year-7, the NPV is negative and short by US$ 83208 (100000-16792). The Discounted Payback Period or the break-even period has not been reached.

In general, the project would be accepted if the DPP is shorter than a specific length of time set by the management´s corporate policy.

Abandonment Occurs Prior to the Discounted Payback Period Example 5

Given a DPP of 4.2 years it may be relevant to know how much would be lost if unforeseen circumstances forced the project to be abandoned after only one or two years.

Figure 7 is the plot of NPV of a project as function of time, assuming abandonment of the project at time t.

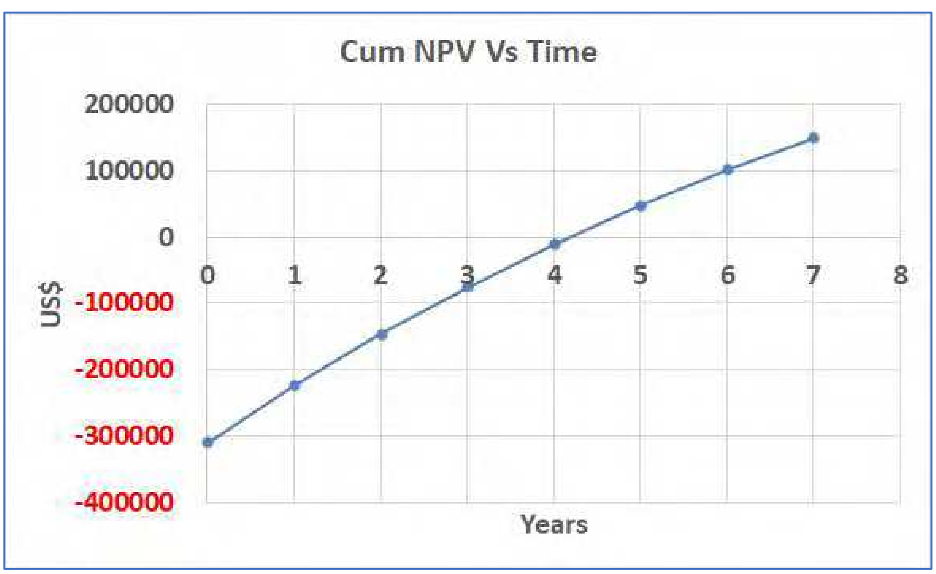


FIGURE 7 CUMULATIVE NPV VS. TIME

Thus, the project shown in Figure 7 has an expected life of 7 years, with an ultimate NPV of US$ 150000. The discounting, project's payback period is 4.2 years and if it is abandoned at that time, the NPV of all cash flows is zero, with the company just breaking even.

However, if unforeseen events force a liquidation after 1 year, the corporation will lose some US$ 225000 in present-value terms. By this time the project has not generated cash inflows.

A curve as provided in Figure 7 shows the project's ultimate NPV, the payback period based on discounted cash flows, and the company's exposure to potential loss throughout the project's life. This way the analysis provides more complete information regarding risk than any single number.

Discounted Payback Period Example 6

A company with a cost of capital of 14% considers an investment project characterized by the following net cash flow over time:

Year	0	1	2	3	4	5	6	7
Net Cash Flow (US$)	-100000	-20000	20000	40000	60000	60000	40000	20000

a) Compute the project's payback period ***<u>without</u>*** discounting cash flows
b) Compute the project's ***<u>discounted</u>*** payback period
c) Draw the project's NPV as a function of time *t*, assuming abandonment at time *t*
d) Given that the project might have to be abandoned early
    - What is the maximum amount that the company stands to lose (in present value terms) if early abandonment comes to the worst possible time?

Solution

i = 14.0%

Year	Net Cash Flow (US$)	Cum NCF (US$)	NPV (US$)	Cum NPV (US$)
(I)	(II)	(III) = Σ(II)	(IV) = (II)/(1+i)^n	(V) = Σ(IV)
0	-100000	-100000	-100000.00	-100000.00
1	-20000	-120000	-17543.86	-117543.86
2	20000	-100000	15389.35	-102154.51
3	40000	-60000	26998.86	-75155.65
4	60000	0	35524.82	-39630.83
5	60000	60000	31162.12	-8468.71
6	40000	100000	18223.46	9754.75
7	20000	120000	7992.75	17747.50

Undiscounted PBP 4.0
Discounted PBP 5.5

TABLE 45 Payback Period Undiscounted and Discounted

a) The payback period (PBP) without discounting is 4.0 years
b) The discounted payback period (DPP) period is 5.5 years
c) The following graph is derived from the Column V of Table 45

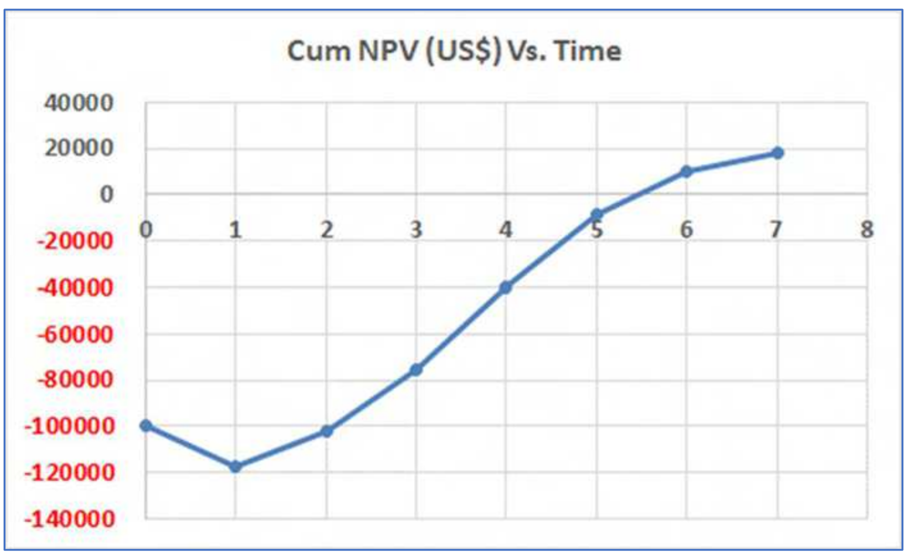


FIGURE 8 CUMULATIVE NPV VS. TIME AND DPP

d) From Figure 8 and Table 45 the worst time for abandonment occurs at the end of year 1, implying a loss of US$ 117543.86 (in present value terms)

Principal Observations to the Payback Period as a Measure of Profitability

- The Discounted Payback Period yardstick permits to discard or disregard some proposed investments before progressing to the next level in the screening analysis
- The method is simple, but it ignores all proceeds after the payback period
- Payback Periods do not consider the possible advantages of a project with a long economic life, way beyond the payout time
- The disadvantage with both Payback Periods (Undiscounted and Discounted) is the selection of the acceptable "time" that could be subjective or arbitrary.
- The yearly amount to be paid-back or to be entered as net income must be agreed to with the funds-provider)
- The payback-time indicator provides a single number or value which can be used as an index or indicator of profitability, at the conditions agreed-to (interest rate and yearly net income)
- Accepting projects with the shortest payback period can lead management to accept higher-risks
- The payout period is a hurdle which investment proposals must clear
- Simply measuring how long it will take to recover the initial investment does not allow to assessing the earning power of a project. The payout period ignores:
    - Differences in the timing of cash flows
    - The difference between the present and future value of money
    - The proceeds after the payback period
- Usually, companies are interested not only in the break-even point as represented by the payback period, but also in the magnitude of its exposure to loss if abandonment occur prior to the Discounted Payback Period (DPP)

Payback period should not be used as the only quantitative tool for making investment decisions, but rather in a supplementary role to others economic hurdles.

Example NPV and DPP as Economic Criteria

An Oil Company considers investing in a new gas compressor. The equipment would cost US$ 100000 and is expected to produce after tax savings of US$ 24716 per year for 10 years, at which point the compressor could be sold for US$ 4500. The company cost of capital is 12%.

a) Based on the project NPV
    - Should the Oil Company proceed?

b) A more careful forecast is prepared by the divisional manager, including possible variations in anticipated cash inflows
    - It is possible that gas sales could vary from expected values to just US$ 15716. Once the sales patterns are established in the first year, this inflow is expected to remain at this level over the 10-year life of the compressor

- It is possible that the compressor may last up to 12 years, allowing savings for another to continue for another 2 years
  The expected salvage value of the compressor at the end of year 10 may vary between zero and US$ 6000
- Perform a sensitivity analysis and shows the effect on the desirability of the project of the worst possible values that may occur for operating savings, length of life, and salvage value, one at the time

c) Where applicable
- Plot the NPV as a function of these three key variables, one at the time. Indicate the break-even values where the compressor's NPV just equal zero

Solution

a) Based on the project NPV

	Cash Flow Economic Analysis					Option 1							
Input						Use as equivalent rate:		12,00%	I = I'+f+I'*f				
Compressor Case			Clear Input			Option 2							
Project Life (N, yrs)	10		Max 40 Yrs			12,0%	is corrected to						
Initial Investment (P_{oINF}, $)	100000						To be added to I						
Interest (I)	12,0%		inflation (f)	0,0%	r corected by f =		12,00%	r = (I - f)/(1 + f)					
Fill Rows 8 & 10		Yr	0	1	2	3	4	5	6	7	8	9	10
Annty Outflow (C, $ per year)		Ci		0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
Annty Inflow (A, $ per year)		Ai		24716,0	24716,0	24716,0	24716,0	24716,0	24716,0	24716,0	24716,0	24716,0	24716,0
Sunk Cost @ N (S, $)	0												
Salvage Value @ N (Sv, $)	4500												
Results													
Profit to Investment Ratio (PIR)	0,411		NPW/Po										
Discounted Payout (Payback yrs)	5,86												
NPVI	0,411		NPV (@i) / MCO (@i)										
Net Future Worth (FW_{PC}, $)	127650		$F_1 = P (1 + i)^n$		$F_2 = \Sigma(A_x \ (1 + i)^{(n-x)})$		$F_3 = \Sigma(C_x \ (1 + i)^{(n-x)})$		$F = - F_1 + F_2 - F_3 - S + Sv$				
Net Present Value (NPV, $)	41100		$P = F / (1 + i)^n$		$P = A * \{[(1 + i)^n - 1]/(i (1 + i)^n)\}$								
Benefit/Cost Ratio (PI)	1,411		ΣPV of net cash inflows/ΣPV of net cash outflows					Net Operating Income to Investment Ratio					
Internal Rate of Return (IRR,%)	21,26%		Click Calc IRR Setting D26=0 Varying B24										

TABLE 46 DPP and NPV for P = US$ 100000, I = 12%, N = 10 Years, S = US$ 4500, A = US$ 24716

Based on the calculated NPV of US$ 41100 with a Discounted Payback Period (DPP) of 5.9 years the Oil Company should accept the project

b) If the inflows are reduced to US$ 15716

	Cash Flow Economic Analysis					Option 1							
Input						Use as equivalent rate:		12,00%	I = I'+f+I'*f				
Compressor Case			Clear Input			Option 2							
Project Life (N, yrs)	10		Max 40 Yrs			12,0%	is corrected to						
Initial Investment (P_{oINF}, $)	100000						To be added to I						
Interest (I)	12,0%		Inflation (f)	0,0%	r corected by f =		12,00%	r = (i - f)/(1 + f)					
Fill Rows 8 & 10		Yr	0	1	2	3	4	5	6	7	8	9	10
Annty Outflow (C, $ per year)		Ci		0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
Annty Inflow (A, $ per year)		Ai		15716,0	15716,0	15716,0	15716,0	15716,0	15716,0	15716,0	15716,0	15716,0	15716,0
Sunk Cost @ N (S, $)	0												
Salvage Value @ N (Sv, $)	4500												
Results													
Profit to Investment Ratio (PIR)	-0,098		NPW/Po										
Discounted Payout (Payback yrs)													
NPVI	-0,098		NPV (@i) / MCO (@i)										
Net Future Worth (FW_{PC}, $)	-30289		$F_1 = P (1 + i)^n$		$F_2 = \Sigma(A_x \ (1 + i)^{(n-x)})$		$F_3 = \Sigma(C_x \ (1 + i)^{(n-x)})$		$F = - F_1 + F_2 - F_3 - S + Sv$				
Net Present Value (NPV, $)	-9752		$P = F / (1 + i)^n$		$P = A * \{[(1 + i)^n - 1]/(i (1 + i)^n)\}$								
Benefit/Cost Ratio (PI)	0,902		ΣPV of net cash inflows/ΣPV of net cash outflows					Net Operating Income to Investment Ratio					
Internal Rate of Return (IRR,%)	9,61%		Click Calc IRR Setting D26=0 Varying B24										

TABLE 47 DPP and NPV for P = US$ 100000, I = 12%, N = 10 Years, S = US$ 4500, A = US$ 15716

In this case the NPV is reduced to US$ -9752 and DPP is not reached

Sensitivity analysis: assuming the worst case with respect salvage value equal to zero and assuming the inflows of US$ 24716

	Cash Flow Economic Analysis					Option 1							
Input						Use as equivalent rate:		12,00%	i = i'+f+i'*f				
Compressor Case			Clear Input			Option 2							
Project Life (N, yrs)	10		*Max 40 Yrs*			12,0%	is corrected to						
Initial Investment (P_{oINF}, $)	100000					To be added to i							
Interest (I)	12,0%		inflation (f)	0,0%	r corected by f =		12,00%	r = (i - f)/(1 + f)					
Fill Rows 8 & 10		*Yr*	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
Annty Outflow (C, $ per year)		Ci		0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
Annty Inflow (A, $ per year)		Ai		24716,0	24716,0	24716,0	24716,0	24716,0	24716,0	24716,0	24716,0	24716,0	24716,0
Sunk Cost @ N (S, $)	0												
Salvage Value @ N (Sv, $)	0												
Results													
Net Cash Flow Prsnt. Val. (NCFP,$)	139650,91		Without Salvage Value or Sunk Cost										
Profit to Investment Ratio (PIR)	0,397		NPW/Po										
Discounted Payout (Payback yrs)	5,86												
NPVI	0,397		NPV (@i) / MCO (@i)										
Net Future Worth (FW_{PC}, $)	123150		$F_1 = P (1 + i)^n$		$F_2 = \Sigma(A_x \ (1 + i)^{(n-x)})$		$F_3 = \Sigma(C_x \ (1 + i)^{(n-x)})$		$F = - F_1 + F_2 - F_3 - S + Sv$				
Net Present Value (NPV, $)	39651		$P = F / (1 + i)^n$		$P = A * \{[(1 + i)^n - 1]/(i (1 + i)^n)\}$								
Benefit/Cost Ratio (PI)	1,397		ΣPV of net cash inflows/ΣPV of net cash outflows					Net Operating Income to Investment Ratio					
Internal Rate of Return (IRR,%)	21,06%		Click Calc IRR Setting D26=0 Varying B24										

TABLE 48 DPP and NPV for P = US$ 100000, I = 12%, N = 10 Years, S = US$ 0, A = US$ 24716

In this case DPP is the same 5.9 years and NPV is US$ 39651. This is a reduction of US$ 1449 relative to case of salvage value of US$ 4500. Therefore, salvage value is not a relevant issue in this case.

As the useful life of the of the compressor is not expected to be less than 10 years, managers see that the only real concern regarding to the project’s economic desirability is the possible decrease in operating savings that may result from declining sales.

c) The plot of these functions

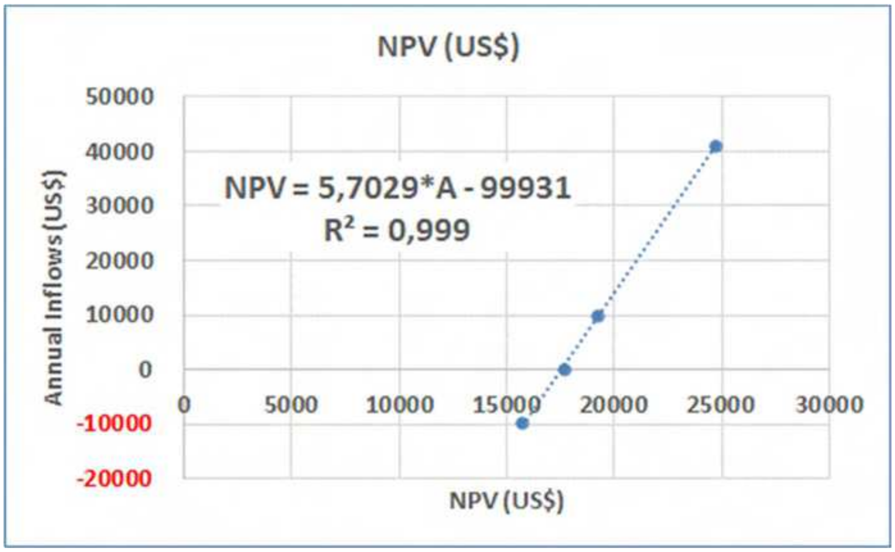


FIGURE 9 NPV SENSITIVITY DUE TO *A* VARIATION

From the plot or by linear interpolation the beak even value for the annual for annual cash inflows is US$ 17699

As the project's DPP is close to 6 years the project is at risk if the compressor lasts less than these 6 years.

Finally given the insignificance of the change in NPV due to changes in salvage value, it is not worthwhile to analyze the NPV function. There is no break-even value, as the NPV remains positive regardless of the salvage value assumed.

Cost per Barrel

In addition to the typical economic hurdles the hydrocarbon industry uses several parameters to indicate the relationship between cost and production or productive capacity. One of these parameters is the cost per barrel. This is normally used to make general statements about cost or comparison and frequently appear with little or no explanation or definition. For example, if it said that oil, from a particular region, costs $XX per barrel without any explanation, the terminology could be misleading and subject to manipulation.

In general, cost per barrel provides a simple and familiar parameter, which can be applied to comparative studies and strategic planning. However, it is not used for specific investment decisions. Two forms of the parameter are commonly used:

a) ***Cost (CAPEX) per daily barrel***, is related to productive capacity This parameter focuses on investment and is assumed to be CAPEX

- It is a measure of the cost of creating a system with the capacity to produce one barrel of oil per day
- In those cases where no reference is made to the time, a medium to long-term period is assumed
- The units applied are $/daily barrel of oil equivalent
- In general, the data is used to estimate the cost of developing. For example, a 100,000 barrel per day field, so the analysis of cost is usually part of a study of larger requirements
- Every field is unique in terms of reserves, peak production and life, so it is advisable to use this parameter as a yardstick

b) ***Cost per (single) barrel***, relating to reserves or production. This measure may be either CAPEX or OPEX (or both). These measures relate cost to barrels produced or reserves. OPEX and/or CAPEX may be included

- OPEX per Single Barrel (Lifting Cost)
  - Relates operating cost to production
  - Since the cost and production data relates to a single year, there is no need to make an adjustment concerning the time value of money
  - There is a requirement to define cost in terms of a specific currency
  - OPEX per barrel is highly variable, depending on the location and technology of the project and the stage in its life
    - Onshore fields may have lifting costs of less than $1 per barrel
    - An offshore field will cost several times more
  - Costs rise over time for all fields, leading to eventual abandonment, when lifting cost exceeds selling price
- CAPEX per Single Barrel
  - Issues concerning costs and production profiles are relevant
  - A way of presenting it is dividing “CAPEX” by “Reserves”
  - To introduce the time value of money, both CAPEX and Barrels produced are discounted at “*I*”
  - Because production is further into the future, the impact is greater and barrels are discounted more than CAPEX
  - Consider the full cost of production (CAPEX plus OPEX)
  - When reserves and cost are both discounted, the result is independent of the date of origin of the discounting

Discounting Production Profile and Reserves

In addition to the example provided in Chapter 3 about Discounted Reserves in here the objective is to discount both the production profile and reserves. As stated earlier the productive capacity of an oil field is variable since production builds to a peak for a period and then diminishes with declining reservoir energy.

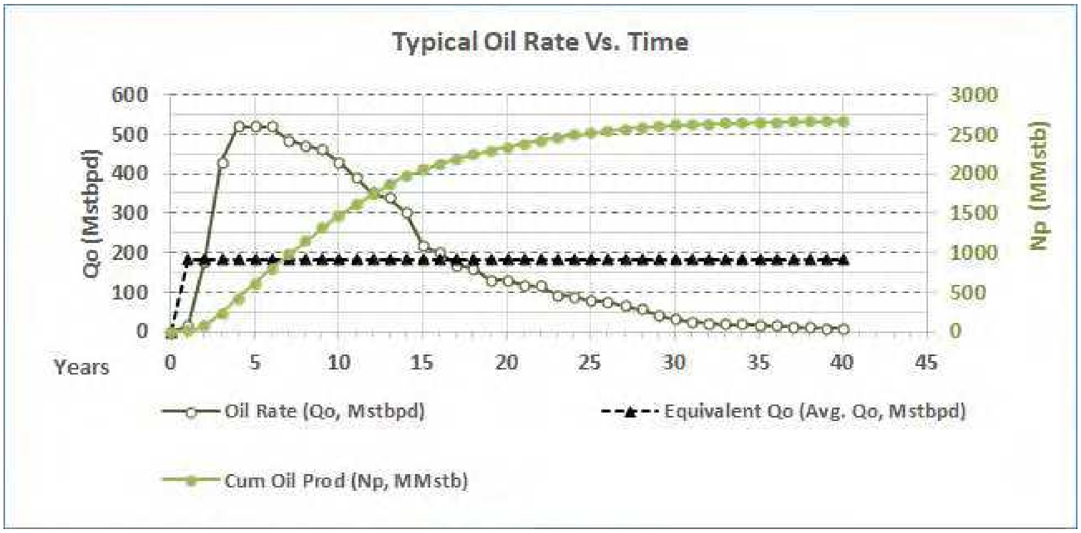


FIGURE 10 OIL RATE, CUMULATIVE OIL PRODUCTION, AND AVERAGE RATE

As an alternative to real production with its “peak”, it could be used an “average” equivalent rate as an indicator of productivity. This involves truncating the peak and generating more production later to create a uniform profile. The simplest form of average is reserves (total cumulative oil production or Np) divided by field life, as illustrated in Figure 10 above.

Reserves (stb) = Np = $\sum_{j=0}^{J=n} Qo * 365$

For example, from Figure 10:

Reserves = Np = 2667.6 MMstb

Qo Equivalent = Np / Time = 2667.6 E06 /(40*365)

Qo Equivalent = 182.7 Mstbpd

If time value of money is considered, this averaging process diminishes project value, because significant production will occur later in time. From the production profile the Recoverable Reserves are 2667.6 million barrels MMstb). When discounted at for example, 10% this generates a Present Volume of Reserves of 1088.6 MMstb, for a 40-year project as it is shown in Table 49:

		$Q_{o\ Dscnt.} = Q_o/(1+i)^n$	Rsrv/ (Base Life*365)	$Q_{o\ avg}$ $/(1+i)^n$ (Mstbpd)	Rsrv/Yr*1000	Rsrv NPV
Year (n)	Oil Rate (Qo) (Mstbpd)	Cum Oil Prod (Np) (MMstb)	Qo Dscnt. Prod	Equiv. Oil Rate ($Q_{o\ avg}$) (Mstbpd)	$Q_{o\ avg}$ Dscnt @ i	$Q_{o\ Avg\ if\ n\ =\ Base}$ (years) (Mstbpd)	Np = (Qo Avg/i) $(1 - 1/(1+i)^n)$
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1	13.0	4.7	.8	182.7	166.1	7308.5	2425.1
2	175.0	68.6	144.6	182.7	151.0	3654.3	2314.9
3	428.0	224.8	321.6	182.7	137.3	2436.2	2211.3
4	519.0	414.3	354.5	182.7	124.8	1827.1	2114.0
5	520.0	604.1	322.9	182.7	113.5	1461.7	2022.5
6	519.5	793.7	293.2	182.7	103.1	1218.1	1936.4
7	484.0	970.4	248.4	182.7	93.8	1044.1	1855.3
8	471.0	1142.3	219.7	182.7	85.2	913.6	1778.9
9	461.0	1310.5	195.5	182.7	77.5	812.1	1707.0
10	428.0	1466.8	165.0	182.7	70.4	730.9	1639.1
11	391.0	1609.5	137.0	182.7	64.0	664.4	1575.1
12	351.0	1737.6	111.8	182.7	58.2	609.0	1514.7
13	338.0	1861.0	97.9	182.7	52.9	562.2	1457.6
14	301.5	1971.0	79.4	182.7	48.1	522.0	1403.7
15	216.0	2049.8	51.7	182.7	43.7	487.2	1352.7
16	199.0	2122.5	43.3	182.7	39.8	456.8	1304.4
17	167.5	2183.6	33.1	182.7	36.1	429.9	1258.7
18	160.0	2242.0	28.8	182.7	32.9	406.0	1215.5
19	129.0	2289.1	21.1	182.7	29.9	384.7	1174.4
20	130.5	2336.7	19.4	182.7	27.2	365.4	1135.5
21	116.5	2379.3	15.7	182.7	24.7	348.0	1098.6
22	115.0	2421.2	14.1	182.7	22.4	332.2	1063.6
23	92.5	2455.0	10.3	182.7	20.4	317.8	1030.3
24	88.0	2487.1	8.9	182.7	18.5	304.5	998.7
25	77.5	2515.4	7.2	182.7	16.9	292.3	968.6
26	75.0	2542.8	6.3	182.7	15.3	281.1	939.9
27	65.0	2566.5	5.0	182.7	13.9	270.7	912.6
28	55.0	2586.6	3.8	182.7	12.7	261.0	886.7
29	42.0	2601.9	2.6	182.7	11.5	252.0	861.9
30	30.0	2612.9	1.7	182.7	10.5	243.6	838.2
31	25.0	2622.0	1.3	182.7	9.5	235.8	815.7
32	21.0	2629.6	1.0	182.7	8.7	228.4	794.1
33	19.0	2636.6	0.8	182.7	7.9	221.5	773.6
34	18.0	2643.1	0.7	182.7	7.2	215.0	753.9
35	16.0	2649.0	0.6	182.7	6.5	208.8	735.1

		$Q_{o\ Dscnt.} = Q_o/(1+i)^n$	Rsrv/ (Base Life*365)	$Q_{o\ avg}$ $/(1+i)^n$ (Mstbpd)	Rsrv/Yr*1000	Rsrv NPV
Year (n)	Oil Rate (Qo) (Mstbpd)	Cum Oil Prod (Np) (MMstb)	Qo Dscnt. Prod	Equiv. Oil Rate ($Q_{o\ avg}$) (Mstbpd)	$Q_{o\ avg}$ Dscnt @ i	$Q_{o\ Avg\ if\ n\ =\ Base}$ (years) (Mstbpd)	Np = (Qo Avg/i) $(1 - 1/(1+i)^n)$
36	14.0	2654.1	0.5	182.7	5.9	203.0	717.0
37	12.0	2658.5	0.4	182.7	5.4	197.5	699.8
38	10.0	2662.1	0.3	182.7	4.9	192.3	683.2
39	8.0	2665.0	0.2	182.7	4.4	187.4	667.4
40	7.0	2667.6	0.2	182.7	4.0	182.7	652.2
Sum *0.365 =			**1088.6**	**2667.6**			

	i	Base (years)	
	10%	40	
Rsrv (MMstb)	**Rsrv Dscnt. (MMstb)**		**Reserves NPV**
*Cum Q_o * 365*	*Cum $Q_{o\ Dscnt}$ * 365*	*Cum $Q_{o\ avg}$ * 365*	*Cum $Q_{o\ avg\ Desc}$ * 365*
2667.6	1088.6	2667.6	652.2

TABLE 49 Reserves NPV

Average production is derived from Reserves and Field Life of 40 years. This is calculated to be 182.7 thousand barrels of oil per day (Mstbpd). When discounted at 10% this produces a Present Volume of Reserves of only 652.2 Mstb.

$$\text{Reserves NPV} = 365^* \sum_{j=0}^{j=n} \left(\frac{Qo\ Equivalent\ Discounted}{(1+i))^n} \right)$$

Reserves NPV = 652.2 Mstb

It is required a uniform profile, which gives the same value for discounted reserves (present volume of reserves) as the field production profile (1088.6 MMstb) instead of the 652.2 MMstb. This must be at a higher rate than the previously calculated average (> 182.7 stbpd), and for a shorter period (< 40 years), to give the same value for total production (2667.6 MMstb).

Annuity Theory Applied to Reserves

Annuity theory is the way of estimating an approximate value as it defines the relationship between a series of uniform annual values and an equivalent present value, as it was presented in

Chapter 3. In this case the units are barrels, rather than dollars, but the principles are the same. Last column values of Table 49 were calculated as follows:

$P = (A/i) * (1 – (1/(1+i)^n))$

In this Case

Np Discounted = $365*((Q_{o\ Prom}/i) * (1 – (1/(1+i)^n)))$

No matter how a profile is averaged, the difficulties are that the parameter does not make reference to project life, so one measurement may relate to an investment with a life of 14 years and another to an investment with a life of 40 years.

Capital Exposure

Creating productive capacity takes several years of investment. Aggregation of CAPEX over time, is an important topic in the derivation of any parameter based on this cost. If a project takes five years to complete, simply adding the individual cash flows ignores the time value of money.

Refreshing concepts, an **asset** is something a company owns which generates money. Therefore, the money invested in an asset has a (***financial or opportunity***) cost and an opportunity cost represents the potential benefits an individual, investor, or business misses when choosing one alternative over another.

Interest During Construction (IDC)

When the project is completed and ready to produce, it has a cost, which is part physical expenditure and part interest. Considering the interest either as cost associated with borrowing, or profit or interest, which would have been earned if the money had been invested elsewhere. This component of cost is commonly known as “Interest During Construction” (IDC).

Assume that MMUS$ 500 is spent over a period of six years, with production start-up in Period 6. The convention is to derive IDC relative to the period, which is one time step before production start-up (Period 5 in this case). See Table 50. Usually, it is called the origin or time reference for the IDC. In this case, CAPEX cash flows are compounded forward to the origin at 10% per annum, the assumed opportunity cost of capital.

	*Reference Year =		5	Interest Rate =	10.0%
	Construction Period	CAPEX (US$)	Adjust Year	C. F. @ i $((1+i)^{(n\ Adjust)}$	CAPEX + IDC (CAPEX* C.F.@i)
	1	50	4	1.464	73.205
	2	100	3	1.331	133.100
	3	100	2	1.210	121.000
	4	100	1	1.100	110.000
*	5	100	0	1.000	100.000
	6	50	-1	0.909	45.455
	Cum	**500**			**582.760**

TABLE 50 Interest During Construction (IDC)

One CAPEX element occurs in Period 6. Any cash flows, which occur later in time than the origin, should be discounted back at 10% per annum. CAPEX plus IDC for the project is calculated to be MMUS$ 582.76.

It is appropriate to calculate IDC in a situation, where the economic cost of a productive facility or system is required. In a conventional discounted cash flow analysis, IDC is not an issue, because the time distribution of all cash flows is acknowledged by the discounting process.

Units and Equivalent Conversion Factors

Hydrocarbon fields produce oil and gas, so it is necessary to consider both as part of “production”. It is normal to apply a conversion factor for the secondary product using energy equivalents. the conversion may use field specific data or, alternatively, apply an average conversion factor, such as 6000 scf per barrel.

In other words, typical oil companies produce both oil and gas, among other petroleum products, but the unit of measure for each is different. Oil is measured in barrels and natural gas is measured in cubic feet. Since energy content of both oil and gas varies, the industry standardized natural gas production into "equivalent barrels" of oil or "equivalent units of gas." One barrel of oil is generally deemed to have the same amount of energy content as 6,000 cubic feet of natural gas. So, this quantity of natural gas is "equivalent" to one barrel of oil.

Example of Discounting CAPEX, Oil and Gas Reserves

	bl	cf					I = 10%			
	1	6000	**Qo/6+Qg**	**Qo/(1+i)^n**	**ΣQo Eq/17**	**ΣQo Eq/n**	**Σ 1/(1+i)^n**	**(Rsrv/n) * Annuity**		**CAPEX /(1+i)^n**
Years (n)	**Qo (Mstbpd)**	**Qg (MM scfpd)**	**Qo Equiv. (Mstbpd)**	**Qo Equiv (Mstbpd) Dscnt. @ i**	**Qo Avg. (Mstbpd)**	**Rsrv/n (MMstb)**	**Annuity @ i**	**Rsrv Dscont. (MMstb)**	**CAPEX (MMUS$)**	**CAPEX + IDC**
-2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	23.6	28.6
-1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	102.5	112.8
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	311.2	311.2
1	4.7	0.0	4.7	4.3	58.0	359.8	0.909	327.1	285.6	259.6
2	70.0	0.0	70.0	57.9	58.0	179.9	1.736	312.2	125.0	103.3
3	82.1	0.0	82.1	61.7	58.0	119.9	2.487	298.3	15.4	11.6
4	87.1	0.0	87.1	59.5	58.0	89.9	3.170	285.1	25.0	17.1
5	81.0	0.0	81.0	50.3	58.0	72.0	3.791	272.8	134.8	83.7
6	89.0	48.0	97.0	54.8	58.0	60.0	4.355	261.2	28.6	16.1
7	77.0	48.0	85.0	43.6	58.0	51.4	4.868	250.2	13.9	7.1
8	62.0	48.0	70.0	32.7	58.0	45.0	5.335	239.9	13.6	6.3
9	55.0	49.0	63.2	26.8	58.0	40.0	5.759	230.2	0.0	0.0
10	48.0	50.0	56.3	21.7	58.0	36.0	6.145	221.1	0.0	0.0
11	43.0	52.0	51.7	18.1	58.0	32.7	6.495	212.4	0.0	0.0
12	38.0	55.0	47.2	15.0	58.0	30.0	6.814	204.3	0.0	0.0
13	35.0	55.0	44.2	12.8	58.0	27.7	7.103	196.6	0.0	0.0
14	32.0	50.0	40.3	10.6	58.0	25.7	7.367	189.3	0.0	0.0
15	29.0	50.0	37.3	8.9	58.0	24.0	7.606	182.4	0.0	0.0
16	26.0	50.0	34.3	7.5	58.0	22.5	7.824	175.9	0.0	0.0
17	26.0	50.0	34.3	6.8	58.0	21.2	8.022	169.8	0.0	0.0
18	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	51	9.2
Cum	**884.9**	**605.0**	**985.7**	**492.9**	**985.7**	**1237.5**			**1130.2**	**966.6**

	ΣQo * 0.365	**Σ ΣQg * 0.365**	**ΣQo Equiv. * 0.365**	**ΣQo Dscnt. * 0.365**		**ΣRsrv/n / Life**
	Np (MMstb)	**Gp (MMMscf)**	**Np Eq (MMstb)**	**Np Dscnt. (MMstb)**		**Rsrv/Life**
Reserves	323.0	220.8	359.8	179.9		72.7956

Example Field CAPEX/(bpd)			
Parameter	**Units**	**Value**	**Comments**
Discount Rate	i (%)	10.0%	
Productive Life	Years	17	
Oil Reserves	MM stb	323.0	
Gas Reserves	MMM scf	220.8	
Equivalent Oil Reserves	MM stb	359.8	Gas @ 6 Mscf/stb
Production Peak	M stbpd	97.0	
Average Qo	M stbpd	58.0	Reserve BOE/productive Life
Discounted Reserves	MM stb	179.9	Discounted Production @ i
Discounted Average (*)	M stbpd	61.4	Uniform Rate with NPV as total field
CAPEX	MM US$	1130.2	
CAPEX + IDC	MM US$	966.6	IDC @ i at zero period
(CAPEX+IDC)/Peak stbpd	US$/stbpd	9965	(CAPEX + IDC) * Production Peak
(CAPEX+IDC)/(Avg. Qo)	US$/stbpd	16670	(CAPEX + IDC) * Average Qo
CAPEX/Discounted	US$/stbpd	15731	(CAPEX + IDC) * Discounted Average (*)

(*) $A = (P*i)/(1-1/(1+i)^n)$

TABLE 51 Discounted Reserves and CAPEX

Additional Discounted Measures of Value

The economic hurdles or yardsticks, which disregard the time value of money like Payback or Profit to Investment Ratio(NPV/P), may distort the perception of project value because revenue earned sooner, or expenditure made later is preferable.

The main hurdles are those which incorporate time value of money concepts. These parameters may be separated into two groups:

- Discount Rate Specified
  These parameters are calculated from cash flows, which have been discounted at an appropriate rate. The choice of discount rate is discussed later
  - Net Present Value
  - Net Present Value Index
  - Annual Capital Charge
- Discount Rate Derived
  Alternatively, a rate of growth, which characterizes the project, may be derived directly from NCF and be used, itself as a measure of value. For example:
  - Internal Rate of Return

Using the cumulative discounted cash flow generates a range of parameters, which are commonly used. Net Present Value and its Index are the most widely used and they will be described with their applications in the following Sections.

Net Present Value (NPV) or Net Present Worth (NPW)

As a reminder, Net Present Value (NPV) is the sum of all project cash flows, discounted back to a common point in time.

For a uniform series of cash flows NPV is:

$NPV = NPW = A \{[(1 + i)^n - 1] / (i (1 + i)^n)\}$

Or

$NPV = (A/i) (1 - 1/(1+i)^n)$

For a variable cash flow, it is

$$NPV = \frac{A_0}{(1+i)^0} + \frac{A_1}{(1+i)^1} + \frac{A_2}{(1+i)^2} + \frac{A_3}{(1+i)^3} + \ldots\ldots \frac{A_n}{(1+i)^n}$$

To compute NPV, the following must be specified:

- Project cash flows defined by project
- Appropriate discount rate (may vary over life of project)
- Time origin n = 0, origin of discounting

As a reminder the calculation of NPV is presented in Table 52:

- For each project year, NCF is listed in constant US$
- The time origin for discounting is 1999, the first year of significant expenditure
- Therefore, 1999 is n = 0
- Discounting is performed at 10% per annum (i = 10%)
- Therefore, discount factors (DF) are based on $1/(1+0.10)^n$
- A Present Value is derived from NCF for each project year
- Present Value (PV) and Discounted Cash Flow (DCF) are equivalent terms
- DCF = NCF * DF

			i =	10.0%
n	Year	NCF (MMUS$)	$1/(1+i)^n$	$A_j/(1+i)^n$
0	1999	-23.6	1.000	-23.6
1	2000	-102.5	0.909	-93.2
2	2001	-315.8	0.826	-261.0
3	2002	-320.8	0.751	-241.0
4	2003	10.4	0.683	7.1
5	2004	244.9	0.621	152.1
6	2005	447.7	0.564	252.7
7	2006	113.0	0.513	58.0
8	2007	222.5	0.467	103.8
9	2008	197.3	0.424	83.7
10	2009	151.6	0.386	58.4
11	2010	146.2	0.350	51.2
12	2011	126.4	0.319	40.3
13	2012	113.9	0.290	33.0
14	2013	101.4	0.263	26.7
15	2014	93.6	0.239	22.4
16	2015	84.5	0.218	18.4
17	2016	76.7	0.198	15.2
18	2017	68.9	0.180	12.4
19	2018	71.8	0.164	11.7
20	2019	-56.3	0.149	-8.4
21	2020	8.7	0.135	1.2
	Cum	**1460.5**	**MMUS$**	**321.1**

TABLE 52 NPV from a Cash Flow

In 2009 (n =10), for example, the NCF of MMUS$ 151.6 is calculated to have a PV or DCF of MMUS$ 58.4 in 1999.

$$PV_{10} = 151.6 * 1/(1+0.10)^{10} = 58.4$$

Therefore, values such as 58.4 and 151.6 are equivalent, separated in time (by 10 years). Assuming a time value of 10% per annum, MMUS$ 58.4 is the amount to be invested at 10% over 10 years to generate a future value of MMUS$ 151.6.

The sum of all such calculated PV's or DCF's is the present value equivalent of all future project NCF's. This total has the name ***Net Present Value (NPV) or Net Present Worth (NPW)***. In this example, the numerical value is MMUS$ 321.1. It is in constant dollars, being derived from NCF's in those terms.

Convenient notation for an NPV, based on discounting at 10% is NPV (0.10). Same example calculated using a spreadsheet is as follows:

Cash Flow Economic Analysis

Input								
Net Present Value		Clear Input			Option 1			
					Use as equivalent rate:	10,00%	$i = i' + f + i' * f$	
					Option 2			
Project Life (N, yrs)	21	Max 40 Yrs			10,0% is corrected to			
Initial Investment ($P_{initial}$, $)	23,6				To be added to i			
Interest (I)	10,0%	Inflation (f)	0,0%	r corrected by f =	10,00%	$r = (i - f)/(1 + f)$		
Sunk Cost @ N (S, $)	0							
Salvage Value @ N (Sv, $)	0							

Fill Rows 8 & 10

Yr	0	1	2	3	4	5	6	7	8	9	10
Annty Outflow (C, $ per year) Ci		102,5	315,8	320,8	0,0	0,0	0,0	0,0	0,0	0,0	0,0
Annty Inflow (A, $ per year) Ai		0,0	0,0	0,0	10,4	244,5	447,7	113,0	222,5	197,3	151,6

Yr	11	12	13	14	15	16	17	18	19	20
Annty Outflow (C, $ per year) Ci	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	56,3
Annty Inflow (A, $ per year) Ai	146,2	126,4	113,9	101,4	93,6	84,5	76,7	68,9	71,8	0,0

Results					
Profit to Investment Ratio (PIR)	13,607	NPW/Po			
Discounted Payout (Payback yrs)	8,52				
NPVI	0,519	NPV (@i) / MCO (@i)			
Net Future Worth (FW_{PC}, $)	2376	$F_1 = P(1+i)^n$	$F_2 = \Sigma[A_t (1+i)^{n-t}]$	$F_3 = \Sigma[C_t (1+i)^{n-t}]$	$F = -F_1 + F_2 - F_3 - S + Sv$
Net Present Value (NPV, $)	321,1	$P = F/(1+i)^n$	$P = A * ([(1+i)^n - 1]/(i(1+i)^n))$		
Benefit/Cost Ratio (PI)	1,512	ΣPV of net cash inflows/ΣPV of net cash outflows		Net Operating Income to Investment Ratio	
Internal Rate of Return (IRR,%)	18,05%	Click Calc IRR Setting D26=0 Varying B24			

Results	
Profit to Investment Ratio (PIR)	13,607
Discounted Payout (Payback yrs)	8,52
NPVI	0,519
Net Future Worth (FW_{PC}, $)	2376
Net Present Value (NPV, $)	321,1
Benefit/Cost Ratio (PI)	1,512
Internal Rate of Return (IRR,%)	18,05%

TABLE 53 NPV (321,1 MMUS$) and DPP (8.5 Years) from Cash Flow

NPV as Measure of Profit

NPV may be considered as a measure of profit. It is a sum of money, in the same units as the NCF, from which it was derived. Once the calculation is completed, the NPV can be converted into another currency unit, e.g., euros or pounds with different purchasing power.

Minimum Attractive Rate of Return (MARR) or Cost of Capital

It is the rate that a corporation expects to earn on its investments. MARR is an interest rate at which the investor can always earn or borrow money. Usually, this selection will be a policy decision by top management. If it is possible, the entity will invest only in proposals with an expected return equal to or higher than MARR.

Relationship Between NPV and Project Net Cash Flow (NCF)

Figure 11 shows a relationship between NPV and project NCF. The plotted data are also presented in Table 54. It is assumed that the project is funded from an account, which starts with zero balance and charges debt interest at 10%. It is assumed that 10% is the appropriate interest rate (cost of capital or Minimum Attractive Rate of Return) for the investor. Later the same account accumulates Net Revenues and pays interest at 10%.

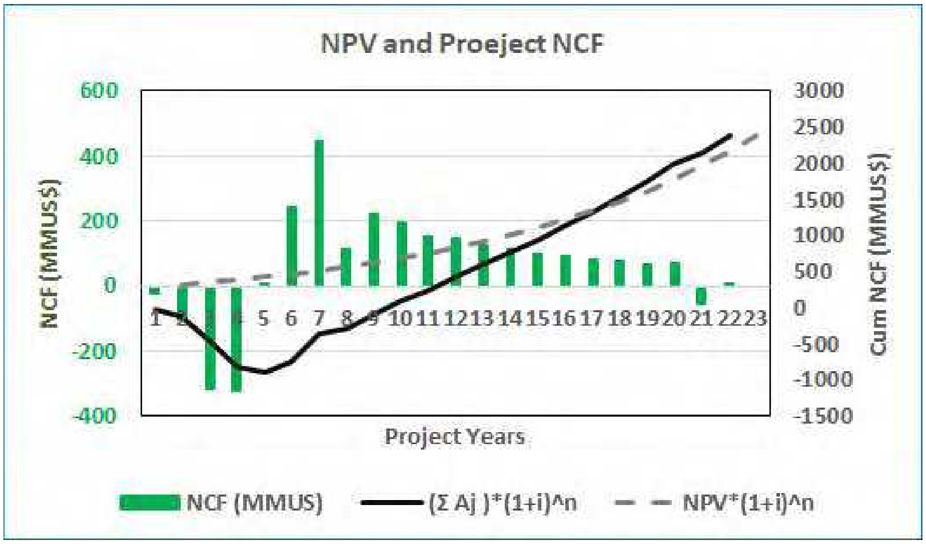


FIGURE 11 NPV AND PROJECT NCF

			i =	10,0%		*NCF*(1+i)*	*NPV*(1+i)^n^*
n	Year	NCF (MMUS$)	$1/(1+i)^n$	$A_j/(1+i)^n$	ΣA_j	$(\Sigma A_j)*(1+i)^n$	$NPV*(1+i)^n$
0	1999	-23.6	1.000	-23.6	-23.6	-23.6	321.1
1	2000	-102.5	0.909	-93.2	-116.8	-128.5	353.2
2	2001	-315.8	0.826	-261.0	-377.8	-457.1	388.6
3	2002	-320.8	0.751	-241.0	-618.8	-823.6	427.4
4	2003	10.4	0.683	7.1	-611.7	-895.6	470.1
5	2004	244.9	0.621	152.1	-459.6	-740.2	517.2
6	2005	447.7	0.564	252.7	-206.9	-366.6	568.9
7	2006	113.0	0.513	58.0	-148.9	-290.2	625.8
8	2007	222.5	0.467	103.8	-45.1	-96.7	688.3
9	2008	197.3	0.424	83.7	38.5	90.9	757.2
10	2009	151.6	0.386	58.4	97.0	251.6	832.9
11	2010	146.2	0.350	51.2	148.2	422.9	916.2
12	2011	126.4	0.319	40.3	188.5	591.6	1007.8
13	2012	113.9	0.290	33.0	221.5	764.7	1108.6
14	2013	101.4	0.263	26.7	248.2	942.6	1219.4
15	2014	93.6	0.239	22.4	270.6	1130.4	1341.4
16	2015	84.5	0.218	18.4	289.0	1328.0	1475.5
17	2016	76.7	0.198	15.2	304.2	1537.5	1623.1
18	2017	68.9	0.180	12.4	316.6	1760.1	1785.4
19	2018	71.8	0.164	11.7	328.3	2007.9	1963.9
20	2019	-56.3	0.149	-8.4	319.9	2152.4	2160.3
21	2020	8.7	0.135	1.2	321.1	2376.3	2376.3
	Total	**1460.5**	**NPV=**	**321.1**			

TABLE 54 NPV and Project NCF

The data, which is labelled *NCF * (1+i) (or* $(\Sigma A_j)*(1+i)^n$*)* traces the history of the bank balance over the life of the project.

During the first few years, the project requires investment capital, which is borrowed from the account and incurs interest charges.

By the 4th year of the project, this had accumulated to a peak value of minus MMUS$ 895.6.

After the 4th year, the revenue earned exceeds all costs, including taxes and interest, such that the balance of the account becomes less negative.

Eventually, between years 8 and 9 the account becomes positive. With a positive balance interest and further revenues and interest accumulate in the account until the end of the project.

The final balance, calculated to be MMUS$ 2376.3 represents the difference between all project revenues and costs, considering that negative balances cost 10% per annum and positive balances earn 10% per annum. This is equivalent of defining the time value of money as 10%.

The second data, labelled NPV*$(1+ i)^n$ plots the growth of calculated NPV at 10% per annum. NPV was calculated as MMUS$ 321.2 and grows to the same terminal value of MMUS$ 2376.3. This is because the NPV is derived from the same NCF's and the same time value of money assumption.

The conclusion is that the NPV is a sum of money, which has the same financial performance as the project cash flows. Both grow to be equal, at the end of the project's life, if compounded at the assumed time value of money.

There is some justification in stating that the sum of money, named NPV, represents the value of the project to the investor at the point in time defined as the origin of discounting.

NPV is often used as a starting point in defining transaction value for a project. In this context, it is important to set the discount origin as close as possible to the transaction date, or to make an adjustment to the calculated NPV to allow for any time discrepancy.

Example of Net Present Value

An oil company is considering the proposed acquisition of a new surface pump.

The required initial investment is US$ 75000 and the projected cash benefits over the 3-years of the project life are as follows:

End of Year	**Cash Flow (US$)**
0	-75000
1	24400
2	27340
3	55760

The production engineers have been asked by management of the company to evaluate the economic merit of the acquisition based on NPV.

The company's MARR (Minimum Attractive Rate of Return) or Cost of capital is 15%.

Should the company acquire the pump?

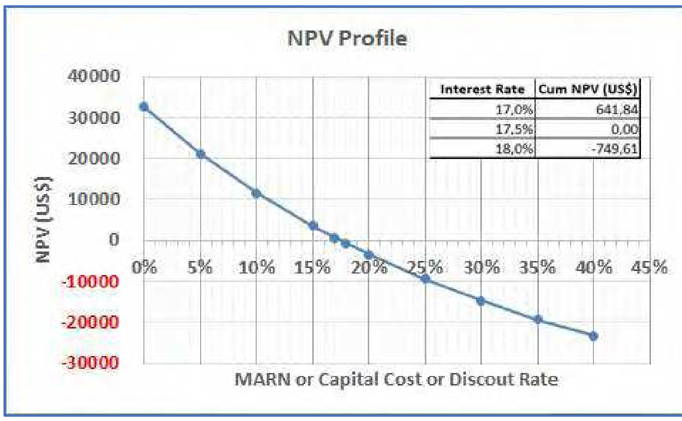
Solution

	i=	15.0%	
	US$		
(I)	(II)	(III) = (II)/(1+i)^(I)	(IV) = Sum (III)
Year	Cash Flow	NPV	Cum NPV
0	-75000	-75000.00	-75000.00
1	24400	21217.39	-53782.61
2	27340	20672.97	-33109.64
3	55760	36663.11	3553.46

Results	
Profit to Investment Ratio (PIR)	0,047
Discounted Payout (Payback yrs)	2,89
NPVI	0,047
Net Present Value (NPV, $)	3553,46
Benefit/Cost Ratio (PI)	1,047
Internal Rate of Return (IRR,%)	17,5%

TABLE 55 NPV Example

Since the project results in an NPV or surplus of US$ 3553.46 the project would be accepted. In this example the engineer computed the NPV of the project at a fixed interest rate. However, additionally the NPV can be calculated as a function of the interest rate (***NPV profile)***



i(%)	NPV (US$)
0,0%	32500,00
5,0%	21203,87
10,0%	11670,17
15,0%	3553,46
17,0%	641,84
18,0%	-749,61
20,0%	-3412,04
25,0%	-9433,28
30,0%	-14673,19
35,0%	-19261,34
40,0%	-23301,75

FIGURE 12 NPV PROFILE

Figure 12 indicates the investment project has a positive NPV if the interest rate is below 17.5% and a negative NPV if the interest rate is above 17.5%. This break-even interest is known as the ***Internal Rate of Return (IRR)***. This concept will be expanded in later chapters.

If the company's MARR is 15% the project has an NPV of US$ 3553.46 and so may be accepted. The figure of US$ 3553.46 measures the immediate gain in present worth to the company following the acceptance of the project.

If MARR is 20%, the NPV(20%) is US$ -3412.04, so the corporation should reject the project.

For this example, it is assumed that the company has such an accurate MARR estimate available for use in investment analysis.

Either accepting or rejecting an investment is influenced by the choice of a MARR or cost of capital. Therefore, it is crucial to estimate the appropriate discount rate correctly.

Discount Origin and Rate

Once NCF, discount rate and origin are defined, the NPV is unique. If the origin is changed, so is NPV. Table 56 indicates the impact of reducing the origin one year of the NCF presented in Table 54. As before, the NCF is at the discount rate of 10%.

	Origen A	i =	10.0%	Origen B			
n	NCF (MMUS$)	$1/(1+i)^n$	$A_j/(1+i)^n$	New Origen	NCF (MMUS$)	$1/(1+i)^n$	$A_j/(1+i)^n$
				0			
0	-23.6	1.000	-23.6	1	-23.6	0.909	-21.5
1	-102.5	0.909	-93.2	2	-102.5	0.826	-84.7
2	-315.8	0.826	-261.0	3	-315.8	0.751	-237.3
3	-320.8	0.751	-241.0	4	-320.8	0.683	-219.1
4	10.4	0.683	7.1	5	10.4	0.621	6.5
5	244.9	0.621	152.1	6	244.9	0.564	138.2
6	447.7	0.564	252.7	7	447.7	0.513	229.7
7	113.0	0.513	58.0	8	113.0	0.467	52.7
8	222.5	0.467	103.8	9	222.5	0.424	94.4
9	197.3	0.424	83.7	10	197.3	0.386	76.1
10	151.6	0.386	58.4	11	151.6	0.350	53.1
11	146.2	0.350	51.2	12	146.2	0.319	46.6
12	126.4	0.319	40.3	13	126.4	0.290	36.6
13	113.9	0.290	33.0	14	113.9	0.263	30.0
14	101.4	0.263	26.7	15	101.4	0.239	24.3
15	93.6	0.239	22.4	16	93.6	0.218	20.4
16	84.5	0.218	18.4	17	84.5	0.198	16.7
17	76.7	0.198	15.2	18	76.7	0.180	13.8
18	68.9	0.180	12.4	19	68.9	0.164	11.3
19	71.8	0.164	11.7	20	71.8	0.149	10.7
20	-56.3	0.149	-8.4	21	-56.3	0.135	-7.6
21	8.7	0.135	1.2	22	8.7	0.123	1.1
Total	**1460.5**		**321.1**	**Total**	**1460.5**		**291.9**

TABLE 56 Change Origin Impact on NPV

In summary, the results are:

Origin A NPV = MMUS$ 321.1

Origin B NPV = MMUS$ 291.9

The relationship is:

321.1 / 291.9 = 1.100

291.9 / 321.2 = 0.909

These are respectively compound and discount factors associated with

“i” = 0.10 and “n” = 1.

Once calculated for a specific discount origin, it is straightforward to compute equivalent NPV’s for any other origin, by compounding forward, or discounting backward in time, at the same rate of interest. The shift in origin will change the magnitude of the NPV, but not its sign.

Effect of Increasing Discount Rate

Increasing Discount Rate reduces individual DCF’s and consequently reduces NPV. There is an inverse relationship between NPV and Discount Rate as indicated by Table 57 and Figure 13:

n	Variable i	i =	10.0%	i =	15.0%	i =	20.0%
	NCF (MMUS$)	$1/(1+i)^n$	$A_j/(1+i)^n$	$1/(1+i)^n$	$A_j/(1+i)^n$	$1/(1+i)^n$	$A_j/(1+i)^n$
0	-23.6	1.000	-23.6	1.000	-23.6	1.000	-23.6
1	-102.5	0.909	-93.2	0.870	-89.1	0.833	-85.4
2	-315.8	0.826	-261.0	0.756	-238.8	0.694	-219.3
3	-320.8	0.751	-241.0	0.658	-210.9	0.579	-185.6
4	10.4	0.683	7.1	0.572	5.9	0.482	5.0
5	244.9	0.621	152.1	0.497	121.8	0.402	98.4
6	447.7	0.564	252.7	0.432	193.6	0.335	149.9
7	113.0	0.513	58.0	0.376	42.5	0.279	31.5
8	222.5	0.467	103.8	0.327	72.7	0.233	51.7
9	197.3	0.424	83.7	0.284	56.1	0.194	38.2
10	151.6	0.386	58.4	0.247	37.5	0.162	24.5
11	146.2	0.350	51.2	0.215	31.4	0.135	19.7
12	126.4	0.319	40.3	0.187	23.6	0.112	14.2
13	113.9	0.290	33.0	0.163	18.5	0.093	10.6
14	101.4	0.263	26.7	0.141	14.3	0.078	7.9
15	93.6	0.239	22.4	0.123	11.5	0.065	6.1
16	84.5	0.218	18.4	0.107	9.0	0.054	4.6
17	76.7	0.198	15.2	0.093	7.1	0.045	3.5
18	68.9	0.180	12.4	0.081	5.6	0.038	2.6
19	71.8	0.164	11.7	0.070	5.0	0.031	2.2
20	-56.3	0.149	-8.4	0.061	-3.4	0.026	-1.5
21	8.7	0.135	1.2	0.053	0.5	0.022	0.2
Cum	**1460.5**		**321.1**		**90.8**		**-44.5**

Rate	i =	0%	i =	10.0%	i =	15.0%	i =	20.0%
NPV @ n = 21	**US$**	**1460.5**	**US$**	**321.1**	**US$**	**90.8**	**US$**	**-44.5**

TABLE 57 NPV at Variable i or NPV Profile

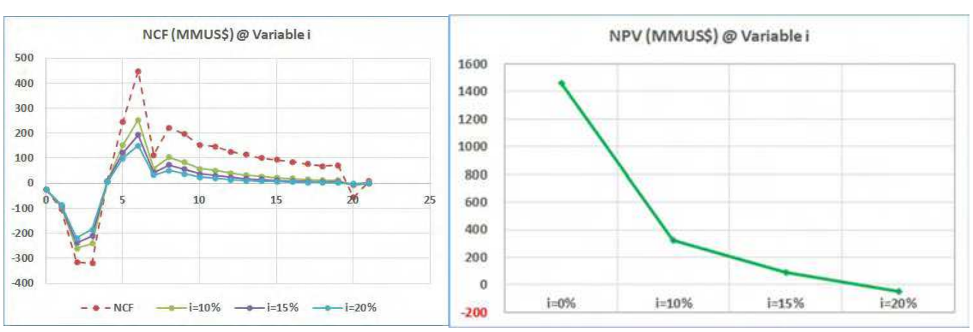


FIGURE 13 NCF AND NPV AT VARIABLE I

In terms of project NCF, the cumulative value at the end of the project's life is the terminal value. It is the sum of all project Revenues, minus the sum of all CAPEX, OPEX and Taxes. This sum is called ***Net Cash, Terminal Cash Surplus (TCS), or Terminal Value of the project***.

Figure 13 (on the right side) shows the characteristic "***NPV Profile***". The intercept with the Y Axis (Discount Rate zero) represents the undiscounted TCS value. The same value is in column i= 0% of Table 57 (MMUS$ 1460.5).

NPV declines with increasing Discount Rate, in this example NPV is zero at around 18.4%. At higher discount rates, NPV becomes increasingly negative.

I (%)	Cum NPV (US$)
15.0	90.77
18.4	0.00
20.0	-44.54

This break-even interest is the ***Internal Rate of Return (IRR)*** of this project NCF.

NPV and Cumulative Discounted Cash Flow (DCF)

NPV is the discounted equivalent of the Terminal Cash Surplus (TCS). This relationship is illustrated in Table 58 and Figure 14, in which cumulative data are shown for NCF, DCF (0.10), DCF (0.15) and DCF (0.20). The terminal values represent respectively, TCS or NPV (0.00), NPV (0.10), NPV (0.15), and NPV (0.20).

n	NCF (US$)	Cum NCF @ i = 0%	Cum NCF @ i=10%	Cum NCF @ i=15%	Cum NCF @ i=20%
0	-23.6	-23.6	-23.6	-23.6	-23.6
1	-102.5	-126.1	-116.8	-112.7	-109.0
2	-315.8	-441.9	-377.8	-351.5	-328.3
3	-320.8	-762.7	-618.8	-562.5	-514.0
4	10.4	-752.3	-611.7	-556.5	-509.0
5	244.9	-507.4	-459.6	-434.7	-410.5
6	447.7	**-59.7**	-206.9	-241.2	-260.6
7	113.0	**53.3**	-148.9	-198.7	-229.1
8	222.5	275.8	**-45.1**	-126.0	-177.3
9	197.3	473.1	**38.5**	-69.9	-139.1
10	151.6	624.7	97.0	-32.4	-114.6
11	146.2	770.9	148.2	**-1.0**	-94.9
12	126.4	897.3	188.5	**22.6**	-80.7
13	113.9	1011.2	221.5	41.1	-70.1
14	101.4	1112.6	248.2	55.5	-62.2
15	93.6	1206.2	270.6	67.0	-56.1
16	84.5	1290.7	289.0	76.0	-51.6
17	76.7	1367.4	304.2	83.1	-48.1
18	68.9	1436.3	316.6	88.7	-45.5
19	71.8	1508.1	328.3	93.7	-43.3
20	-56.3	1451.8	319.9	90.3	-44.7
21	8.7	1460.5	**321.1**	90.8	-44.5

TABLE 58 NCF @ i = 10%, 15%, and 20%

Rate	i =	0%	i =	10.0%	i =	15.0%	i =	20.0%
NPV @ n = 21	US$	1460.5	US$	321.1	US$	90.8	US$	-44.5

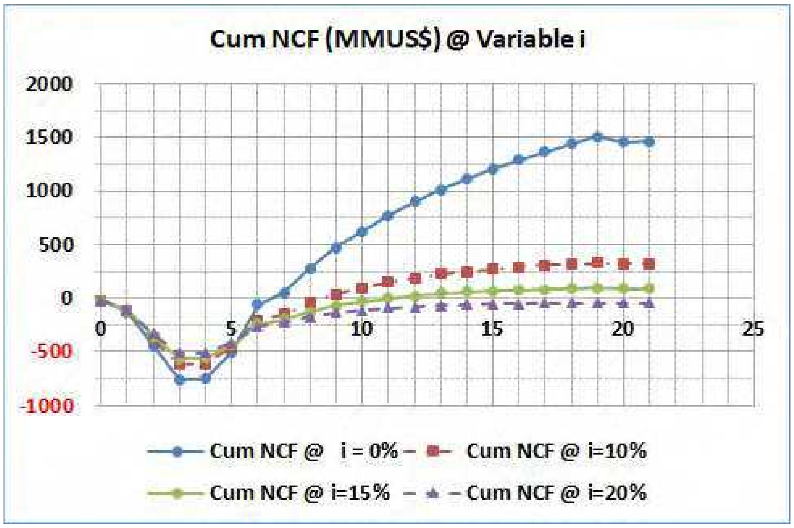


FIGURE 14 CUMULATIVE NCF @ I = 10%, 15%, AND 20%

As Discount Rate increases, NPV diminishes and becomes negative, matching the principles already established.

Maximum Capital Outlay (MCO)

This is the minimum value on the cumulative Net Cash Flow. The MCO represents the worst financial position over the life of the project and it is a useful measure of the financial commitment to the project.

Mathematically, MCO is the sum of all CAPEX, up to the period before the one in which annual Net Revenue (Revenue minus OPEX minus Taxes) first exceeds the annual CAPEX. CAPEX requirements often continue beyond this point and therefore CAPEX values are likely to be higher than MCO values for the same project.

Maximum Capital Outlay (MCO) is defined as the minimum value on the cumulative NCF curve. MCO represents the sum of money which needs to be obtained from outside the project, if the project is approved. As MCO reflects the interaction between positive and negative cash-flows over time, it can be significantly influenced by the timing of the cash flows.

From Table 58 taking year 3 as example, when Discount Rate is increased, MCO diminishes, from MMUS$ -762.7 (zero Discount Rate) to MMUS$ -514 (20% Discount Rate).

The notation used, is MCO (0.00), MCO (0.10), MCO (0.15), and MCO (0.20).

n	NCF (US$)	Cum NCF @ i = 0%	Cum NCF @ i=10%	Cum NCF @ i=15%	Cum NCF @ i=20%
0	-23,6	-23,6	-23,6	-23,6	-23,6
1	-102,5	-126,1	-116,8	-112,7	-109,0
2	-315,8	-441,9	-377,8	-351,5	-328,3
3	-320,8	-762,7	-618,8	-562,5	-514,0

MCO represents the financial commitment to the project.

The basis of discounting is an assumption that investment opportunity exists and that money invested will grow in value over time. With increasing Discount Rate, investment opportunity increases and money invested grows faster over time. Under these circumstances, as Discount Rate increases, a future cash flow or series of cash flows becomes smaller, in Present Value terms.

Figure 14 and Table 58 show the Discounted Project Payback. When increasing discount rate, the Discounted Payback Period is extended, as the periods indicated in colors in Table 58, and eventually becomes infinitely long. In other words, the project has a negative terminal value (NPV) and never returns to cumulative zero.

Discounted Payback Period (Years)			
I = 0%	I = 10%	I = 15%	I = 20%
6.5	8.5	11.0	No

Cash Flow Economic Analysis

Input

NPV and DPP			
Project Life (N, yrs)	21	Max 40 Yrs	
Initial Investment (P_{oiNr}, $)	23,6		
Interest (I)	10,0%	Inflation (f)	0,0%
Sunk Cost @ N (S, $)	0		
Salvage Value @ N (Sv, $)	0		

Clear Input

Option 1

Use as equivalent rate: 10,00% i = i'+f+i'*f

Option 2

10,0% is corrected to

To be added to i

r corected by f = 10,00% r = (i - f)/(1 + f)

Fill Rows 8 & 10

Yr	0	1	2	3	4	5	6	7	8	9	10
Annty Outflow (C, $ per year) Ci		102,5	315,8	320,8	0,0	0,0	0,0	0,0	0,0	0,0	0,0
Annty Inflow (A, $ per year) Ai		0,0	0,0	0,0	10,4	244,9	447,7	113,0	222,5	197,3	151,6

Yr	11	12	13	14	15	16	17	18	19	20	21
Ci	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	56,3	0,0
Ai	146,2	126,4	113,9	101,4	93,6	84,5	76,7	68,9	71,8	0,0	8,7

Results	
Profit to Investment Ratio (PIR)	13,607
Discounted Payout (Payback yrs)	8,52
NPVI	0,519
Net Future Worth (FW_{PC}, $)	2376
Net Present Value (NPV, $)	321,1
Benefit/Cost Ratio (PI)	1,512
Internal Rate of Return (IRR,%)	18,052%

TABLE 59 NPV and DPP Using Spreadsheet

Example of NPV from a Cash Flow Statement

A computerized equipment has been proposed for a small oil company. If the new system costing MUS$ 125 is installed, it will generate annual revenue of MUS$ 100 and requires:

- MUS$ 20 in annual labor
- MUS$ 10 in annual materials
- MUS$ 10 in annual overhead (power and utilities) expenses

This purchase will be classified as property with a depreciation factor of 20%. Where depreciation refers to an accounting method used to allocate the cost of a tangible or physical asset over its useful life or life expectancy.

The company expects to sell the equipment at the end of 5 years for MUS$ 50.

Find the year-by-year, after-tax next cash flow for the project at a 40% tax rate.

Find the after-tax net present value at the company's MARN of 15%.

Discussion

In year 0 there is the investment cost of MUS$ 125 for the equipment. This will be depreciated in years 1-5. The revenues and costs are uniform annual flows in year 1-5.

Once accounts provide with the depreciation and tax figures for each year, analysts can compute results for years 1-4, which have fixed revenue and expense entries along with the corresponding charges.

In year 5 analysts will need to incorporate the salvage value and tax effects from the asset disposal

Once the project's after-tax net cash flows are obtained, analysts can determine the equivalent present value (NPV) at the corporation interest rate (MARN).

The after-tax cash flows series is presented in Table 60:

Income Statement (US$)	

	n	0	1	2	3	4	5
Revenue	(I)	0	100000	100000	100000	100000	100000
Expenses	Labor (II)	0	20000	20000	20000	20000	20000
	Material (III)	0	10000	10000	10000	10000	10000
	Overhead (utilities and others) (IV)	0	10000	10000	10000	10000	10000
	Depreciation **(from accounts)** (V)	0	12500	22500	18000	14400	11520
(VI) = (I)-(II)-(III)-(IV)-(V)	Taxable income (VI)	0	47500	37500	42000	45600	48480
(VII) = (VI) * Tax Rate (40%)	Income tax (VII)	0	19000	15000	16800	18240	19392
(VIII) = (VI) –(VII)	Net Income (VIII)	0	28500	22500	25200	27360	29088

Cash flow Statement (US$)		**0**	**1**	**2**	**3**	**4**	**5**
Cash from operation							
	Net Income (VIII)	0	28500	22500	25200	27360	29088
	Depreciation (given by accounts) (V)	0	12500	22500	18000	14400	11520
Investment (IX)		-125000	0	0	0	0	50000
Disposal tax effect (**given by accounts**) (X)		0	0	0	0	0	-1568
Net Cash Flow (XI)	(XI) = (VIII)+(V)+(IX)+(X)	-125000	41000	45000	43200	41760	89040
Net Present Value (US$) (XII) = Σ (XI)/(1+0.15)^n	**41228.37**	-125000	35652	34026	28405	23876	44269

TABLE 60 NPV from Income Statement

This means that investing US$12500 in this automated equipment would bring in enough revenue to recover the initial investment and the cost of funds ***with a surplus of US$ 41228.37***

NPV Index (NPVI) and The Profit to Investment Ratio (PIR)

NPV Index (NPVI) is the discounted equivalent of the Profit to Investment Ratio (PIR) as PIR is the ratio of the Net Present Value and P_o the initial investment. This is NPV (r) / P_o, where "r" is the real discount rate.

NPV Index (NPVI) is a measure of investment efficiency by optimizing profit earned for every dollar invested. The measure of efficiency is the ratio between Terminal Cash Surplus (TCS), as a measure of profit and Maximum Capital Outlay (MCO), as a measure of investment.

		Discounted Payback Period (DPP)						
		6.5	8.5	11	No			
n	NCF (MMU$)	Cum NCF (MMUS$)	Cum NCF @ i=10%	Cum NCF @ i=15%	Cum NCF @ i=20%	Cum NCF/ MCO @ i=10%	Cum NCF/ MCO @ i=15%	Cum NCF/ MCO @ i=20%
0	-23.6	-23.6	-23.6	-23.6	-23.6	-0.038	-0.042	-0.046
1	-102.5	-126.1	-116.8	-112.7	-109.0	-0.189	-0.200	-0.212
2	-315.8	-441.9	-377.8	-351.5	-328.3	-0.610	-0.625	-0.639
3	-320.8	-762.7	-618.8	-562.5	-514.0	-1.000	-1.000	-1.000
4	10.4	-752.3	-611.7	-556.5	-509.0	-0.989	-0.989	-0.990
5	244.9	-507.4	-459.6	-434.7	-410.5	-0.743	-0.773	-0.799
6	447.7	-59.7	-206.9	-241.2	-260.6	-0.334	-0.429	-0.507
7	113.0	53.3	-148.9	-198.7	-229.1	-0.241	-0.353	-0.446
8	222.5	275.8	-45.1	-126.0	-177.3	-0.073	-0.224	-0.345
9	197.3	473.1	38.5	-69.9	-139.1	0.062	-0.124	-0.271
10	151.6	624.7	97.0	-32.4	-114.6	0.157	-0.058	-0.223
11	146.2	770.9	148.2	-1.0	-94.9	0.240	-0.002	-0.185
12	126.4	897.3	188.5	22.6	-80.7	0.305	0.040	-0.157
13	113.9	1011.2	221.5	41.1	-70.1	0.358	0.073	-0.136
14	101.4	1112.6	248.2	55.5	-62.2	0.401	0.099	-0.121
15	93.6	1206.2	270.6	67.0	-56.1	0.437	0.119	-0.109
16	84.5	1290.7	289.0	76.0	-51.6	0.467	0.135	-0.100
17	76.7	1367.4	304.2	83.1	-48.1	0.492	0.148	-0.094
18	68.9	1436.3	316.6	88.7	-45.5	0.512	0.158	-0.089
19	71.8	1508.1	328.3	93.7	-43.3	0.531	0.167	-0.084
20	-56.3	1451.8	319.9	90.3	-44.7	0.517	0.161	-0.087
21	8.7	1460.5	321.1	90.8	-44.5	0.519	0.161	-0.087
MCO (MMUS$) @ i			-618.8	-562.5	-514.0			
NPVI @ i (=Cum NCF/MCO)			0.519	0.161	-0.087			

TABLE 61 NPVI Calculation at Various Discount Rates

Rate	i =	0%	i =	10.0%	i =	15.0%	i =	20.0%
NPV @ n = 21	US$	1460.5	US$	321.1	US$	90.8	US$	-44.5

Calculation of NPVI in Table 61 uses the same cash flows of Table 58 at a discount rate of 10%, 15% and 20%.

The cumulative of DCFs at these rates are calculated. The minimum values of: US$ MM -618.8, -562.5, and -514.0 represent MCO at various rates (0.10, 0.15, and 0.20) for the project.

The terminal values, MMUS$ 321.2, 90.8, and -44.5 represent NPV (0.10), NPV (0.15), and NPV (0.20) for the project.

NPVI (0.10) = NPV (0.10) / MCO (0.10) = 0.519

NPVI (0.15) = NPV (0.15) / MCO (0.15) = 0.161

NPVI (0.20) = NPV (0.20) / MCO (0.20) = -0.087

The final three columns of Table 61 are the cumulative DCF (@i) divided by the value of MCO (@i). At 10% and 15% they are positive, at 20% the project is not attractive.

Cash Flow Economic Analysis

Input

NPV and DPP		
Project Life (N, yrs)	21	Max 40 Yrs
Initial Investment (P_{add}, $)	23,6	
Interest (I)	10,0%	Inflation (f) 0,0%
Sunk Cost @ N (S, $)	0	
Salvage Value @ N (Sv, $)	0	

Clear Input

Option 1: Use as equivalent rate: 10,00% $i = i' + f + i' * f$

Option 2: 10,0% is corrected to — To be added to i — r corrected by f = 10,00% $r = (i - f)/(1 + f)$

Fill Rows 8 & 10

Yr	0	1	2	3	4	5	6	7	8	9	10
Annty Outflow (C, $ per year) Ci		102,5	315,8	320,8	0,0	0,0	0,0	0,0	0,0	0,0	0,0
Annty Inflow (A, $ per year) Ai		0,0	0,0	0,0	10,4	244,9	447,7	113,0	222,5	197,3	151,6

Yr	11	12	13	14	15	16	17	18	19	20	21
Annty Outflow (C, $ per year) Ci	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	56,3	0,0
Annty Inflow (A, $ per year) Ai	146,2	126,4	113,9	101,4	93,6	84,5	76,7	68,9	71,8	0,0	8,7

Results

Results		
Profit to Investment Ratio (PIR)	13,607	NPW/Po
Discounted Payout (Payback yrs)	8,52	
NPVI	0,519	NPV (@i) / MCO (@i)
Net Future Worth (FW_{PC}, $)	2376	$F_1 = P(1+i)^n$; $F_2 = \Sigma[A_i (1+i)^{n-i}]$; $F_3 = \Sigma[C_i (1+i)^{n-i}]$; $F = -F_1 + F_2 - F_3 - S + Sv$
Net Present Value (NPV, $)	321,1	$P = F/(1+i)^n$; $P = A * ([(1+i)^n - 1]/[i(1+i)^n])$
Benefit/Cost Ratio (PI)	1,512	ΣPV of net cash inflows/ΣPV of net cash outflows ; Net Operating Income to Investment Ratio

Results	
Profit to Investment Ratio (PIR)	13,607
Discounted Payout (Payback yrs)	8,52
NPVI	0,519
Net Future Worth (FW_{PC}, $)	2376
Net Present Value (NPV, $)	321,1
Benefit/Cost Ratio (PI)	1,512
Internal Rate of Return (IRR,%)	18,052%

TABLE 62 NPVI Calculation Using Spreadsheet

This procedure is a means of making comparison between projects, in terms of investment efficiency. NPVI is a very useful measure of investment efficiency, which incorporates time value of money.

Annual Capital Charge Factor (ACCF) _ Capital Recovery Cost (CR)

In investments decisions, where a uniform, cash flow is expected, analysis can be simplified considerably by applying the principles of the annuity. Uniform cash flows may be expected in investment related to oil production during plateau , manufacture or transportation. The present value of an annuity may be represented by:

$$P = \frac{A}{(1+i)^1} + \frac{A}{(1+i)^2} + \frac{A}{(1+i)^3} + \ldots \ldots \frac{A}{(1+i)^n}$$

Where "P" is the Present Value of a series, in which "***A***" represents a uniform annual cash flow. The Annual Capital Charge analysis is based on a modified version of above equation in which "1" (P/P) represents one unit of investment and "ɑ" (**A/P**) represents the ***Annual Capital Charge (ACCF)***.

ACCF is the uniform annual cash flow as a proportion of one, which must be generated during the project ("n" years) to justify the initial unit of investment. The Discount Rate *"i"* represents the relevant time value of money, or required return on investment.

$$a = \frac{A}{P}$$

$$1 = \frac{a}{(1+i)^1} + \frac{a}{(1+i)^2} + \frac{a}{(1+i)^3} + \ldots \ldots \frac{a}{(1+i)^n}$$

Rearranging the equation results as follows:

$$a = \frac{i}{(1-(1+i)^{-n})}$$

- "ɑ" represents the proportion of the initial investment, which must be earned each year (during "n" years) to give the investment a return of "*i*"
- "ɑ" is called the "Annual Capital Charge Factor" and is the inverse of the Annuity Factor

Rearranging the equation:

$$\frac{1}{(1+i)^0} = \frac{a}{(1+i)^1} + \frac{a}{(1+i)^2} + \frac{a}{(1+i)^3} + \ldots \ldots \frac{a}{(1+i)^n}$$

$$0 = \frac{-1}{(1+i)^0} + \frac{a}{(1+i)^1} + \frac{a}{(1+i)^2} + \frac{a}{(1+i)^3} + \ldots \ldots \frac{a}{(1+i)^n}$$

The unit of investment was moved to the right-hand side of the equation and given the denominator $(1+i)^0$. As $(1+i)^0$ equals one, so equality in the equation is maintained. This equation indicates a cash flow with an NPV of zero.

- Zero NPV represents minimum acceptable profitability

The series includes one unit of investment in period zero, the origin of the discounting, followed by a uniform series of "n" cash flows, starting one year later and of magnitude equivalent to the calculated ACCF or ***Capital Recovery Cost (CR)***.

Once the size of the investment has been identified, it is possible to determine the minimum acceptable annual revenue for the project.

A project investment in these situations is represented by a single unit cash flow. In practice, CAPEX may extend during several years. In these cases, do not simply add the components of CAPEX together. The procedure is to keep the concept of time value, as included with the "Interest During Construction" (IDC) indicated in previous chapters.

The term *$(-1 / (1+i)^0)$* in above equation is replaced by a series, representing all components of CAPEX, which will normally precede year zero and may also continue for one or more years after.

$$\frac{1}{(1+i)^0} = \frac{C_{-m}}{(1+i)^{-m}} + \cdots + \frac{C_{-2}}{(1+i)^{-2}} + \frac{C_{-1}}{(1+i)^{-1}} + \frac{C_0}{(1+i)^0} + \frac{C_1}{(1+i)^1}$$

This equation implies that:

- Any historical (before year zero) CAPEX cash flows are *compounded* forward to year zero at "i" per year
- Any future CAPEX cash flows are *discounted* back to year zero at the same rate

These adjusted cash flows represent a valuation of the cost of the productive facility, including an assumed time value of money.

It is important the timing, (year zero) one year before the first positive cash flow.

The time value *"i"* should be constant throughout. Table 63 indicates the relationship between ACC Factor, Annuity and Discount factors at an annual discount rate of 10% and 20%.

	i=	10,0%		i=	20,0%	
	Discount DF (@i)	Annuity AF (@i)	ACC ACCF (@i)	Discount DF (@i)	Annuity AF (@i)	ACC ACCF (@i)
n	$1/(1+i)^n$	$(1-1/(1+i)^n)/i$	$i/(1-1/(1+i)^n)$	$1/(1+i)^n$	$(1-1/(1+i)^n)/i$	$i/(1-1/(1+i)^n)$
0	1.000	0.000		1.000	0.000	
1	0.909	0.909	1.100	0.833	0.833	1.200
2	0.826	1.736	0.576	0.694	1.528	0.655
3	0.751	2.487	0.402	0.579	2.106	0.475
4	0.683	3.170	0.315	0.482	2.589	0.386
5	0.621	3.791	0.264	0.402	2.991	0.334
6	0.564	4.355	0.230	0.335	3.326	0.301
7	0.513	4.868	0.205	0.279	3.605	0.277
8	0.467	5.335	0.187	0.233	3.837	0.261
9	0.424	5.759	0.174	0.194	4.031	0.248
10	0.386	6.145	0.163	0.162	4.192	0.239
11	0.350	6.495	0.154	0.135	4.327	0.231
12	0.319	6.814	0.147	0.112	4.439	0.225
13	0.290	7.103	0.141	0.093	4.533	0.221
14	0.263	7.367	0.136	0.078	4.611	0.217
15	0.239	7.606	0.131	0.065	4.675	0.214
16	0.218	7.824	0.128	0.054	4.730	0.211
17	0.198	8.022	0.125	0.045	4.775	0.209
18	0.180	8.201	0.122	0.038	4.812	0.208
19	0.164	8.365	0.120	0.031	4.843	0.206
20	0.149	8.514	0.117	0.026	4.870	0.205

TABLE 63 ACCF and Annuity

Table 63 is for ACC Factors for i = 0.10 and 0.20 and values on "n" from 1 to 20.

There is no value for n=0, since a project with unit investment, but no positive cash flow, does not return an NPV of zero.

For "n" = 1, ACCF equals (1+i), and as "n" becomes large, ACCF tends to "i".

Considering the case where "*i*" = 0.10, for "*n*" = 1 the project consists of two cash flows, one of unit investment and the second, a positive cash flow, one year later.

To generate an annual return of 10% for the investment, the positive cash flow is required to be 1.10 units (similarly for the 0.20 case, a single cash flow of 1.20 units).

For the case where "*i*" = 0.10 and "*n*" is very large, it is considered a perpetual project. In other words, it has been made a unit of investment and in return, it is received a payment every year. For such an investment, the annual payment required to generate a 10% return is 0.10 units.

In summary, the ***Capital Recovery Cost (CR)*** is the annual equivalent of a capital cost. It allows calculating an annual equivalent cost of capital for ease of itemization and consideration and comparison with the annual operating costs. Capital cost tends to be one-time cost, so in conducting an annual equivalent cost evaluation, analysts must translate this one-time cost into its annual equivalent over the life of the project.

The equivalent annual cost method converts the Present Value into equivalent annual cost using the capital recovery factor. The annual equivalent of a capital cost is given a specific name: Capital Recovery Cost.

Annuity as Capital Recovery Factor and Hurdle

Annual value refers to a uniform annual series of money flows (annuity) for a given period which is equivalent in amount to a specific sequence of annual cash flows.

The annuity is determined by calculating the present value amount of the actual cash flow series and then multiplying this amount by the Capital Recovery Factor or Capital Recovery Cost. Depending on the case, it could be more convenient to evaluate investment proposals in terms of annual value as opposed to present values.

$$CR = P\, [(i\, (1 + i)^n / ((1 + i)^n - 1)]$$

If the annual value is negative, the project is expected to earn less than the interest rate used in the calculations:

- The project should be rejected because the project will represent a cost to the corporation

The yardsticks, present, future, and annual values, are all measures of equivalency and differ only in the times at which they are determined.

For a given value of interest (*i*) and time (*n*) these hurdles will all lead to the same go/no go decision for the evaluated project/investment.

Example Annual Capital Charge Factor (ACCF)

The system is an offshore and onshore pipeline and associated terminal and other facilities. Investment was made during 1992-95, with start-up in 1996. Project life is 20 years, with an average oil rate of 500,000 barrels per day.

The year 1996 is the production start-up, and assumed first revenue and positive cash flow. Therefore, time zero is 1995, one period (year) before that.

Since time zero is 1995, $1995 is used as a convenient money.

The OPEX (annual cost of operating the system) is MMUS$ 30 in terms of 1995.

Questions

a) How much this project must return as net revenue of for every dollar invested?
b) What is the amount required to repay the capital invested (the Capital Recovery Cost)?
c) How much should be charged on every barrel to generate sufficient funds every year during 20 years to pay the annual OPEX, plus sufficient to give the capital invested a return of 15% per annum?

Solution

a) How much this project must return as net revenue of for every dollar invested?

The CAPEX data must be converted to 1995 terms, using the inflation statistics, then these cash flows are compounded forward to 1995, year zero, at 15% per year. The total of MMUS$ 620.5 is in 1995 terms and includes IDC @ 15%. (See Table 64).

The project has 20 productive years and has a return requirement of 15%. Consequently, the Annuity Factor is 6.259

$$AF = (1-1/(1+i)^n)/i$$

$$AF = (1-1/(1+0.15)^{20})/0.15$$

$$AF = 6.259$$

and the Annual Capital Charge Factor is the inverse of the Annuity Factor. It is calculated to be 0.160

$$ACC = (i/(1-1/(1+i)^n)$$

$$ACC = (0.15/(1-1/(1+0.15)^{20})$$

$$ACC = 0.160$$

The Annual Capital Charge Factor in this case means that for every dollar invested, this project must return a net revenue of $0.16 for each of its productive 20 years, to achieve the required 15%.

b) What is the amount required to repay the capital invested (the Capital Recovery Cost)?

Example ACCF		Interest Rate		15,0%	
For the Project n	Year	CAPEX MoD MMUS$	Given Factor RPI Conversion @ 1995	CAPEX $ (1995) MMUS$	CAPEX (IDC) $CAPEX/(1+i)^n$
-3	1992	55	1.573	86.5	131.6
-2	1993	135	1.442	194.7	257.5
-1	1994	120	1.242	149.0	171.4
0	1995	60	1.000	60.0	60.0

Total Years of the Project	20
Sum CAPEX: (IDC) $CAPEX/(1+i)^n$ (MMUS$)	620.4
ACCF @ i = $i/(1-1/(1+i)^n)$	0.160
ACC @ i = CAPEX * ACCF @ i	99.1
OPEX (MMUS$)	30.0

Annual Cash Requirement (ACC+OPEX) (MMUS$)	129.1

TABLE 64 ACCF Calculation

Since the CAPEX is MMUS$ 620.4 (all values in terms of year 1995), the amount required to repay capital invested is 620.5 * 0.160 = MMUS$ 99.1 or the ***Capital Recovery Cost***.

In addition, revenue must also pay annual OPEX, so, there is an ***annual cash requirement*** of:

Equivalent Total Annual Cost = 99.1 + 30.0

Equivalent Total Annual Cost = MMUS$ 129.1

The same result for question b) is presented using a spreadsheet.

Cash Flow Economic Analysis

Input

Annual Capital Charge Factor				
Project Life (N, yrs)	20		Max 40 Yrs	
Initial Investment (P_{oAV}, $)	620,4			
Interest (I)	15,0%		Inflation (f)	0,0%
Annty Outflow (C, $ per year)	Variable	Ci		
Annty Inflow (A, $ per year)	Variable	Ai		
Sunk Cost @ N (S, $)	0			
Salvage Value @ N (Sv, $)	0			

Clear Input

Option 1

Use as equivalent rate: **15,00%** i = i'+f+i'*f

Option 2

15,0% is corrected to

To be added to i

r corected by f = **15,00%** r = (i - f)/(1 + f)

Yr	0	1	2	3	4	5	6	7	8	9	10
Ci		30	30	30	30	30	30	30	30	30	30
Ai		0	0	0	0	0	0	0	0	0	0

Yr	11	12	13	14	15	16	17	18	19	20
Ci	30	30	30	30	30	30	30	30	30	30
Ai	0	0	0	0	0	0	0	0	0	0

Results

Profit to Investment Ratio (PIR)	-1,303	NPW/Po			
Discounted Payout (Payback yrs)		Only for the variable case			
NPVI	-1,000	NPV (@i) / MCO (@i)			
Net Future Worth (FW_{PC}, $)	-13227,53	$F_1 = P(1+i)^n$	$F_2 = \Sigma(A_x\ (1+i)^{(n-x)})$	$F_3 = \Sigma(C_x\ (1+i)^{(n-x)})$	$F = -F_1 + F_2 - F_3 - S + Sv$
Net Present Value (NPV, $)	-808,21	$P = F / (1+i)^n$	$P = A * \{[(1+i)^n - 1]/(i\ (1+i)^n)\}$		
Benefit/Cost Ratio (PI)	0,000	ΣPV of net cash inflows/ΣPV of net cash outflows		Net Operating Income to Investment Ratio	
Eqvlnt Total Annual Cost (A_{PC}, $)	-129,12	$A = F\ [i / ((1+i)^n - 1)]$	Makes Present Worth = Total Present Value of A		
Capital Recovery (CR_{PC}, $)	99,12	$CR = P\ [(i\ (1+i)^n / ((1+i)^n - 1)]$	Makes NPV =0		

TABLE 65 Equivalent Total Annual Cost and Capital Recovery Using Spreadsheet

c) How much should be charged on every barrel to generate sufficient funds every year during 20 years to pay the annual OPEX, plus sufficient to give the capital invested a return of 15% per annum?

The source of the Revenue is a Tariff on the throughput of 365 * 0.5 MM stbpd and the Equivalent Total Annual Cost is MMUS$ 129.1, so the required Tariff is:

129.1 / (365 * 0.5) = 0.71 per stb.

Production Rate. Q_o (MM stbpd))		0.50
Required tariff	**= (ACC+OPEX) / (365 * Q_o)**	0.71

Based on the information given, charging $0.71 on every barrel would generate enough funds every year during 20 years to pay the annual OPEX, plus sufficient to give the capital invested a return of 15% per annum.

Internal Rate of Return (IRR)

This economic hurdle indicates the return per unit investment. Internal rate of Return (IRR) is therefore a measure of investment efficiency. As a ranking parameter when comparing several projects, IRR indicates the investment option that will grow faster.

IRR is also the interest rate to be earned on the unrecovered project balance of investment such that, when the project´s life and execution is completed, the unrecovered project balance will be zero. Since it is a return internal for the project, it is called internal rate of return.

When evaluating different projects, it is advisable to accept them for further consideration if their IRR exceeds their MARR.

For the case of mutually exclusive projects, it is best to use NPW (NPV) or the Annual-Equivalent-Worth (AEW), because these two indicators are absolute measures of investment worth, while the IRR is only a relative indicator and does not indicate absolute net worth or absolute value in any event.

The Internal Rate of Return (IRR) is the discount rate, which reduces the project NPV to zero. It is, therefore, the solution to the following equation:

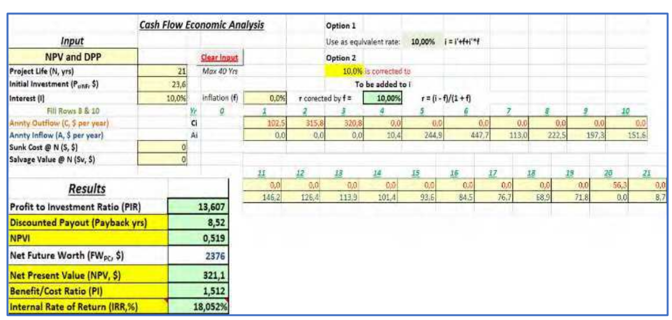
$$NPV \ = 0 = \frac{A_0}{(1+i)^0} + \frac{A_1}{(1+i)^1} + \frac{A_2}{(1+i)^2} + \frac{A_3}{(1+i)^3} + \ldots \ldots \frac{A_n}{(1+i)^n}$$

There is no analytical solution to an equation of this complexity. The following, numerical methods may be applied:

- Trial and error Compute NPV(i) and check for zero
- Graphical Plot NPV Profile and find intercept on x axis
- Extrapolation Iterative calculations to find zero NPV

Usually, spreadsheets and financial calculators usually incorporate an iterative algorithm to compute IRR automatically.

For the same data of Table 59 the numerical solution and the graphical solution using different interest rates are presented in Figure 15. For these data the IRR = 18.05%.



Cash Flow Economic Analysis

Input			
NPV and DPP		Clear Input	
Project Life (N, yrs)	21	Max 40 Yrs	
Initial Investment (P_{oINF}, $)	23,6		
Interest (I)	10,0%	Inflation (f)	0,0%
Fill Rows 8 & 10		Yr	0
Annty Outflow (C, $ per year)		Ci	
Annty Inflow (A, $ per year)		Ai	
Sunk Cost @ N (S, $)	0		
Salvage Value @ N (Sv, $)	0		

Option 1

Use as equivalent rate: 10,00% i = i'+f+i'*f

Option 2

10,0% is corrected to

To be added to i

r corected by f = 10,00% r = (i - f)/(1 + f)

Yr	1	2	3	4	5	6	7	8	9	10
Ci	102,5	315,8	320,8	0,0	0,0	0,0	0,0	0,0	0,0	0,0
Ai	0,0	0,0	0,0	10,4	244,9	447,7	113,0	222,5	197,3	151,6

Yr	11	12	13	14	15	16	17	18	19	20	21
Ci	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	56,3	0,0
Ai	146,2	126,4	113,9	101,4	93,6	84,5	76,7	68,9	71,8	0,0	8,7

Results	
Profit to Investment Ratio (PIR)	13,607
Discounted Payout (Payback yrs)	8,52
NPVI	0,519
Net Future Worth (FW_{PC}, $)	2376
Net Present Value (NPV, $)	321,1
Benefit/Cost Ratio (PI)	1,512
Internal Rate of Return (IRR,%)	18,052%

FIGURE 15 INTERNAL RATE OF RETURN

Significance of IRR

The IRR is an interest or growth rate and is assumed to be a measure of investment efficiency. It is arithmetically analogous to the interest earned on a bank account. In this analogy:

- Negative NCF is equivalent to a Deposit
- Positive NCF is equivalent to a Withdrawal
- IRR is equivalent to the Interest Rate

In Table 66, the same NCF of Table 61 is now presented as a ***bank account***, with deposits, withdrawals, accumulated balances and interest.

The project NCF is split into

- "Deposits" (negative NCF's)
- "Withdrawals" (positive NCF's).

Interest is paid on positive balances and charged on negative balances.

The rate of interest is 18.052% is the same as estimated from Figure 15. This is the calculated value for IRR.

i (%)	NPV (MMUS$)
0,0%	1460,5
5,0%	724,5
10,0%	321,1
15,0%	90,8
18,1%	0,0
25,0%	-125,2
30,0%	-173,3

In reference to Table 66

- In year zero
    - 23.6 is deposited, creating a new balance in the account
- In year 1
    - 102.5 is deposited
    - Interest of 4.3 is added. Coming from the balance of year zero (18.052% * 23.6)
    - The Period Change (change in current year) is the deposit of (102.5 + 4.3 = 106.8)
    - The new Balance is Period Change (106.8) plus previous Balance (23.6), equals 130.4
- In year 2
    - There is interest of 23.5 (18.052% * 130.4)
    - This, added to a Deposit of 315.8 gives a Period Change of 339.3 and Balance of 469.7
- From Period 4 onwards
    - There are withdrawals from the account
    - In most years the amount withdrawn exceeds the interest earned on a positive balance
    - The trend is a declining balance(Column VIII) and becomes negative in Period 19
    - At the end the balance is zero as shown in Column VIII Table 66

	IRR = 18.052%						
	(MMUS$)						
n	NCF	-Cum NCF	Deposit	Wthwls	I = C@$_{n-1)}$ *IRR	(NCF + I)	C = Acum (NCF + I)
(I)	(II)	(III)= -Σ(II)	(IV)	(V)	(VI)=(VIII)$_{(n-1)}$*IRR	(VII) =(II)+(VI)	(VIII)=Σ(VII)
0	-23.6	23.6	-23.6	0.0	0.0	23.6	23.6
1	-102.5	126.1	-102.5	0.0	4.3	106.8	130.4
2	-315.8	441.9	-315.8	0.0	23.5	339.3	469.7
3	-320.8	762.7	-320.8	0.0	84.8	405.6	875.3
4	10.4	752.3	0.0	10.4	158.0	147.6	1022.9
5	244.9	507.4	0.0	244.9	184.7	-60.2	962.6
6	447.7	59.7	0.0	447.7	173.8	-273.9	688.7
7	113.0	-53.3	0.0	113.0	124.3	11.3	700.0
8	222.5	-275.8	0.0	222.5	126.4	-96.1	603.9
9	197.3	-473.1	0.0	197.3	109.0	-88.3	515.6
10	151.6	-624.7	0.0	151.6	93.1	-58.5	457.1
11	146.2	-770.9	0.0	146.2	82.5	-63.7	393.4
12	126.4	-897.3	0.0	126.4	71.0	-55.4	338.0
13	113.9	-1011.2	0.0	113.9	61.0	-52.9	285.2
14	101.4	-1112.6	0.0	101.4	51.5	-49.9	235.3
15	93.6	-1206.2	0.0	93.6	42.5	-51.1	184.1
16	84.5	-1290.7	0.0	84.5	33.2	-51.3	132.9
17	76.7	-1367.4	0.0	76.7	24.0	-52.7	80.1
18	68.9	-1436.3	0.0	68.9	14.5	-54.4	25.7
19	71.8	-1508.1	0.0	71.8	4.6	-67.2	-41.4
20	-56.3	-1451.8	-56.3	0.0	-7.5	48.8	7.4
21	8.7	-1460.5	0.0	8.7	1.3	-7.4	0.0

TABLE 66 IRR Analogy to a Bank Account

The acceptance/rejection based on the IRR hurdle is made by comparing the calculated rate with the cutoff rate established by management:

- If the IRR exceeds the required rate, the decision would be to accept the project

Depending on the annual cash flow, some projects can have mathematically more than one IRR. In these cases, the calculation of the IRR value(s) does not always enable management to accept/reject projects.

Multiple Internal Rates of Return (IRR)

Where an investment has an initial cash outflow followed by cash inflows, the shape of its NPV Profile will have the shape of the blue line in Figure 16.

Other investments, can have more than one reversal of directions in the cash flows as illustrated by the brown curve (Figure 16).

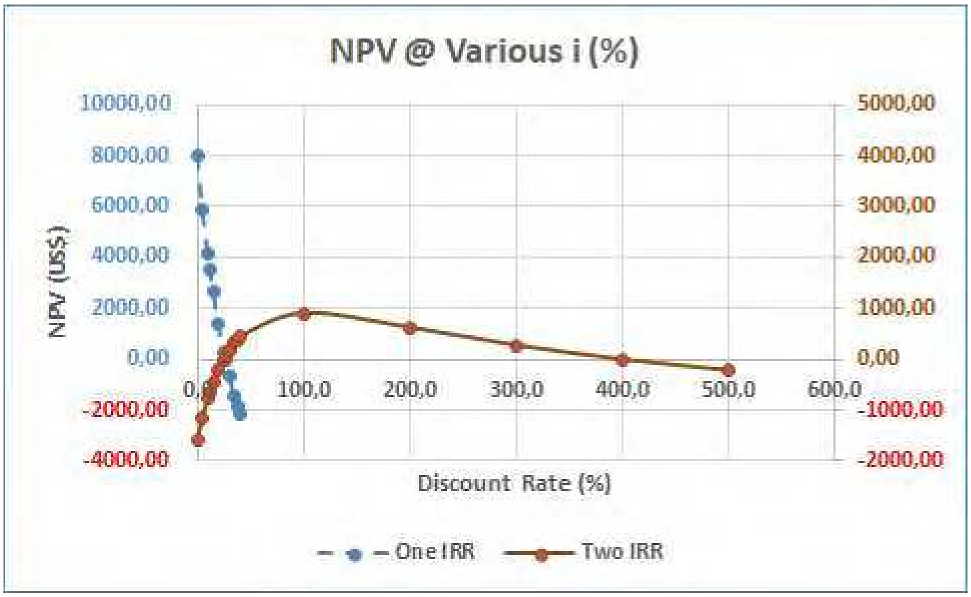


FIGURE 16 SINGLE AND MULTIPLE IRR

In the case of one IRR given that all future cash flows are inflows, the NPV decreases with increasing discount rates and only one value for the IRR is possible. Such projects are characterized by one change in the direction or in the sign of cash flow over time. The initial investment (negative cash flow) is followed by subsequential returns (positive cash flows).

In the case of two or multiple IRR, projects could start with an initial outflow, followed by cash inflows, and at the end of the project's life face cash outflows again such as cost for abandonment. In these cases, a project can have two different IRRs, that comply with the criterion of generating an NPV of zero.

When more than one phase of negative cash flow in the context of IRR calculation and when the second phase becomes significant, there is the issue of multiple roots to consider

- Because IRR is the solution to a complex equation, there is a possibility of obtaining multiple roots when there are multiple changes of signs (positive and negatives cash flows)

IRR as an economic hurdle may be overrated if used as a single yardstick, since it may oversimplify the decision.

The IRR, by itself, does not indicate:

- The order of magnitude of the investment
- The risk involved
- The payout times
- Others

IRR is even more dangerous to use as a "single" indicator when choosing among several projects.

IRR does not discriminate between different projects that may have identical IRR values

- Project size
- Risk
- Duration

Some project features are not considered with a single value of IRR. Therefore, care must be exercised in IRR use. Also, multiple roots implies that more than one value of "i" gives rise to an NPV of zero.

Example of Multiple IRR

Examine an investment in a new pump that is larger than the one currently in use. It would result in a larger immediate oil production but an earlier depletion of the production rate. The net incremental cash flows from the investment are assumed to be as follows:

Year	Net Cash Flow (US$)
0	-1600
1	10000
2	-10000

The US$ 1600 investment in the pump shifts US$ 10000 of cash inflows from year 2 to year 1. The calculation yields the following results:

	Cash Flow Economic Analysis					Option 1			
Input						Use as equivalent rate:		0,00%	i = i'+f+i'*f
Multiple IRR			Clear Input			Option 2			
Project Life (N, yrs)	2		Max 40 Yrs			0,0%	is corrected to		
Initial Investment (P_{oINF}, $)	1600,00					To be added to i			
Interest (I)	0,0%		inflation (f)	0,0%	r corected by f =		0,00%	r = (i - f)/(1 + f)	
Fill Rows 8 & 10		Yr	0	1	2				
Annty Outflow (C, $ per year)		Ci		0,0	10000,0				
Annty Inflow (A, $ per year)		Ai		10000,0	0,0				
Sunk Cost @ N (S, $)	0								
Salvage Value @ N (Sv, $)	0								
Results									
Profit to Investment Ratio (PIR)	-1,000		NPW/Po						
Discounted Payout (Payback yrs)									
NPVI	-1,000		NPV (@i) / MCO (@i)						
Net Future Worth (FW_{PC}, $)	-1600		$F_1 = P(1+i)^n$		$F_2 = \Sigma(A_x\ (1+i)^{(n-x)})$		$F_3 = \Sigma(C_x\ (1+i)^{(n-x)})$		$F = -F_1 + F_2 - F_3 - S + Sv$
Net Present Value (NPV, $)	-1600		$P = F/(1+i)^n$		$P = A * \{[(1+i)^n - 1]/(i\ (1+i)^n)\}$				
Benefit/Cost Ratio (PI)	0,862		ΣPV of net cash inflows/ΣPV of net cash outflows					Net Operating Income to Investment Ratio	
Internal Rate of Return (IRR,%)	25,00%		Click Calc IRR Setting D26=0 Varying B24						
	Calc IRR		0,00 NPV Target			2	IRR Possible solutions		

	Cash Flow Economic Analysis					Option 1			
Input						Use as equivalent rate:		0,00%	i = i'+f+i'*f
Multiple IRR			Clear Input			Option 2			
Project Life (N, yrs)	2		Max 40 Yrs			0,0%	is corrected to		
Initial Investment (P_{oINF}, $)	1600,00					To be added to i			
Interest (I)	0,0%		inflation (f)	0,0%	r corected by f =		0,00%	r = (i - f)/(1 + f)	
Fill Rows 8 & 10		Yr	0	1	2				
Annty Outflow (C, $ per year)		Ci		0,0	10000,0				
Annty Inflow (A, $ per year)		Ai		10000,0	0,0				
Sunk Cost @ N (S, $)	0								
Salvage Value @ N (Sv, $)	0								
Results									
Profit to Investment Ratio (PIR)	-1,000		NPW/Po						
Discounted Payout (Payback yrs)									
NPVI	-1,000		NPV (@i) / MCO (@i)						
Net Future Worth (FW_{PC}, $)	-1600		$F_1 = P(1+i)^n$		$F_2 = \Sigma(A_x\ (1+i)^{(n-x)})$		$F_3 = \Sigma(C_x\ (1+i)^{(n-x)})$		$F = -F_1 + F_2 - F_3 - S + Sv$
Net Present Value (NPV, $)	-1600		$P = F/(1+i)^n$		$P = A * \{[(1+i)^n - 1]/(i\ (1+i)^n)\}$				
Benefit/Cost Ratio (PI)	0,862		ΣPV of net cash inflows/ΣPV of net cash outflows					Net Operating Income to Investment Ratio	
Internal Rate of Return (IRR,%)	400,00%		Click Calc IRR Setting D26=0 Varying B24						
	Calc IRR		0,00 NPV Target			2	IRR Possible solutions		

TABLE 67 Project with Two IRRs

This project has two IRRs: 25% and $400% because there are two reversals in the net cash flows. Using a zero-discount rate, total outflows exceed total inflows, leaving a net balance of US$ -1600. With positive discount rates, cash flows in the most distant period (year 2) will be most heavily reduced.

As these cash flows are negative, the NPV initially rises with increasing discount rates and, at a discount rate of 25%, discount benefits start to exceed discount costs. As the discount rate continues to increase, the cash inflow in year 1 is progressively more affected. The present value declines and again becomes negative. This is illustrated in Figure 17, which shows the project's net present-value profile.

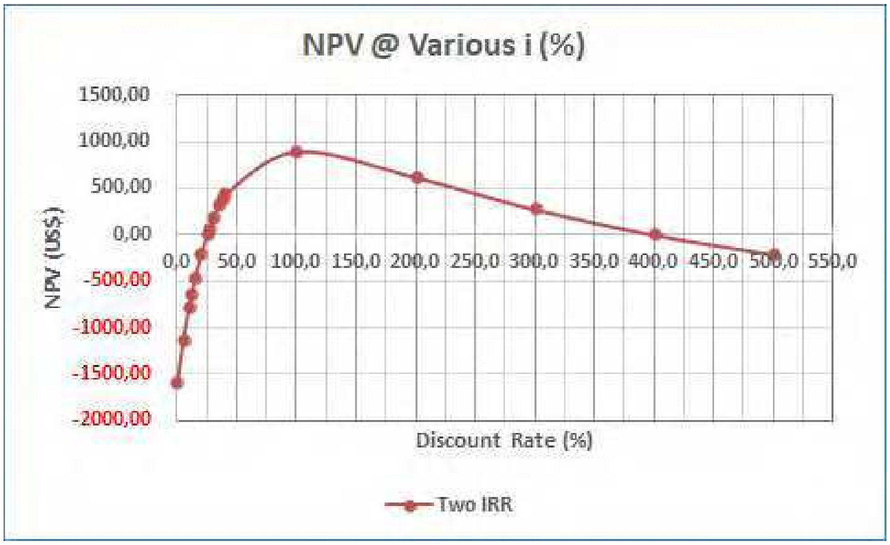


FIGURE 17 TWO IRRs

The interpretation of multiple rates is difficult, and results calculated by software usually are not meaningful economically. The full pattern of cash flows for the pump example is presented in Table 68:

	US$		
Year	**Continue use of old pump**	**Use of new pump**	**Incremental cash flow on replacement**
0	0	-1600	-1600
1	10000	20000	10000
2	10000	0	-10000

TABLE 68 Cash Flow Pump Example

The results do not provide a meaningful rate of return. By investing US$ 1600 in the new pump, the investor receives US$ 10000 one year earlier. The true economic desirability of the project depends on the rate at which the US$ 10000 can be reinvested.

If the investor can earn 23% on the extra US$ 10000 received in year 1. This would provide an additional return of US$ 2300 in year 2. Under such circumstances an economically meaningful IRR would be the discount rate that equates the initial investment of US$ 1600 with the extra return of US$ 2300 received two years hence, which is 19.9%.

P	=	1600
F	=	2300
n	=	2
i	=	19,896%

$i = ((F/P)^{(1/n)})-1$

TABLE 69 Discount Rate that Equates the Initial Investment

It can be concluded that IRR should not be used to evaluate investments that entail more than one change in direction of their cash flows because this may produce multiple solutions.

Conclusion on the Use of IRR as Economic Hurdle

- The proper reinvestment rate for cash flows generated by a project is generally the company's cost of capital or the Minimum Attractive Rate of Return (MARR)
- The NPV hurdle is consistent with MARR, therefore NPV should be the economic yardstick to be used
- Unlike the IRR, the NPV is unaffected by the pattern that cash flows may follow over time

Abandonment Expenditure and the "Extended Yield" method

In the bank account analogy described at the beginning of chapter Significance of IRR (Table 66), the overdraft phase creates a conceptual problem. This is presented in the standard DCF Figure 18 where "i" = IRR, cumulative DCF goes positive during this phase.

FIGURE 18 Cumulative NCF Discounted @ IRR

	IRR = 18,052%								
	(MMUS$)								
n	NCF	-Cum NCF	Deposit	With draw	I = C@$_{n-1)}$ * IRR	(NCF+I)	C = Cum (NCF+I)	Cum NCF	Disc Cum NCF @ IRR
(I)	(II)	(III)= -Σ(II)	(IV)	(V)	(VI) = (VIII)$_{(n-1)}$*IRR	(VII) =(II)+(VI)	(VIII)=Σ(VII)	(IX)=Σ(II)	(X)=(IX) / (1+IRR)^n
0	-23.6	23.6	-23.6	0.0	0.0	23.6	23.6	-23.6	-24
1	-102.5	126.1	-102.5	0.0	4.3	106.8	130.4	-126.1	-107
2	-315.8	441.9	-315.8	0.0	23.5	339.3	469.7	-441.9	-317
3	-320.8	762.7	-320.8	0.0	84.8	405.6	875.3	-762.7	-464
4	10.4	752.3	0.0	10.4	158.0	147.6	1022.9	-752.3	-387
5	244.9	507.4	0.0	244.9	184.7	-60.2	962.6	-507.4	-221
6	447.7	59.7	0.0	447.7	173.8	-273.9	688.7	-59.7	-22
7	113.0	-53.3	0.0	113.0	124.3	11.3	700.0	53.3	17
8	222.5	-275.8	0.0	222.5	126.4	-96.1	603.9	275.8	73
9	197.3	-473.1	0.0	197.3	109.0	-88.3	515.6	473.1	106
10	151.6	-624.7	0.0	151.6	93.1	-58.5	457.1	624.7	119
11	146.2	-770.9	0.0	146.2	82.5	-63.7	393.4	770.9	124
12	126.4	-897.3	0.0	126.4	71.0	-55.4	338.0	897.3	122
13	113.9	-1011.2	0.0	113.9	61.0	-52.9	285.2	1011.2	117
14	101.4	-1112.6	0.0	101.4	51.5	-49.9	235.3	1112.6	109
15	93.6	-1206.2	0.0	93.6	42.5	-51.1	184.1	1206.2	100
16	84.5	-1290.7	0.0	84.5	33.2	-51.3	132.9	1290.7	91
17	76.7	-1367.4	0.0	76.7	24.0	-52.7	80.1	1367.4	81
18	68.9	-1436.3	0.0	68.9	14.5	-54.4	25.7	1436.3	72
19	71.8	-1508.1	0.0	71.8	4.6	-67.2	-41.4	1508.1	64
20	-56.3	-1451.8	-56.3	0.0	-7.5	48.8	7.4	1451.8	53
21	8.7	-1460.5	0.0	8.7	1.3	-7.4	0.0	1460.5	45

TABLE 70 Discounted NCF @ IRR

Within the project, the financial situation is created, because there is a phase of investment at a late stage, relating to abandonment (year 20, MMUS$ 56.3). Usually projects have expenditure for decommissioning, site clearance etc. The implications for cash flow evaluation should be considered because generally abandonment expenditure is a legal obligation on the project and must be paid for out of project revenues.

Project revenues must first provide for abandonment and whatever remains may then be considered as return on investment.

A secondary issue relates to the appropriate time value of money to apply to the "provision for abandonment" phase. This rate is designated "**i***" to differentiate with return on investment "i."

The standard IRR calculation, uses a constant discount rate for all project periods. Therefore, the assumption is that the "provision for abandonment" is invested at the calculated IRR rate. The same interpretation applies to any discounted cash flow analysis, which applies a constant discount rate throughout. This re-investment is an issue, if the time value of money were different during the provision for abandonment period.

Cash reserved for decommissioning might warrant a separate time value of money because such cash may face investment constraints. Usually, projects must provide appropriate guarantees that individual obligations will be met at the time of project abandonment. This may restrict investment opportunity for some or all the relevant cash flow and therefore reduce rates of interest available. It is necessary to consider that "**i***" may be less than "i".

There is an explicit assumption about investment opportunity during this phase of the project. The "***Extended Yield***" method assumes that cash flows, immediately preceding abandonment, are set aside and invested at an explicit rate, to be available to meet the financial obligations associated with abandonment. Such an investment may be called a ***"sinking fund."*** The fund created to meet a future obligation is also related to the ***Sunk Cost.*** As defined in Chapter 2 a sunk cost refers to money that has already been spent and cannot be recovered.

In Table 70 there is an abandonment cost required in Period 20 (MMUS$ 56.3). The ***Extended Yield method*** starts at Period 20 and moves backwards in time seeking cash flows to set aside and cover MMUS$ 56.3. Therefore, it is necessary an explicit assumption about investment opportunity. For example, assume that 8% reflects a reduced investment opportunity as presented in Table 71.

With each backward time-step, cash flow requirement is discounted at 8%, reflecting that the sinking fund will grow over time at 8%. MMUS$ 56.3 in Period 20, becomes MMUS$ 52.1 (56.3/1.08) in Period 19.

In Period 19, there is a cash flow of 71.8, which is more than sufficient. MMUS$ 52.1 is set aside for the sinking fund and MMUS$ 19.7 remains (71.8 - 52.1). The 52.1 within its sinking fund will grow at 8% to become 56.3 in Period 20, to pay for the abandonment. In the project cash flow, the data in Periods 19 and 20 are replaced by 19.7 and zero, as presented in Table 71.

(I)	(II)	(III)	(IV) = (II)*(III)	(V) = Σ(IV)	(VI) = (II)n+1/(1+i)^n	(VII) = (II) + (VI)	(VI)$_{(n-1)}$/(1+i)^n	
	IRR =	**18.1%**			**i = 8.0%**			
n	**NCF (MMUS$)**	**DF@IRR 1/(1+IRR)^n**	**DCF @ IRR**	**Acum DCF @ IRR**	**Reduced Investment Opportunity (OPI)**	**(NCF @n-1) - OPI**	**Growth @ i**	**New NCF (MMUS$)**
0	-23.6	1.000	-23.6	-23.6	22.0	-45.6		-23.6
1	-102.5	0.847	-86.8	-110.4	73.2	-175.7	102.5	-102.5
2	-315.8	0.718	-226.6	-337.0	80.3	-396.1	315.8	-315.8
3	-320.8	0.608	-195.0	-532.0	-2.8	-318.0	320.8	-320.8
4	10.4	0.515	5.4	-526.7	-71.5	81.9	-10.4	10.4
5	244.9	0.436	106.8	-419.9	-141.1	386.0	-244.9	244.9
6	447.7	0.369	165.4	-254.4	-38.5	486.2	-447.7	447.7
7	113.0	0.313	35.4	-219.1	-81.8	194.8	-113.0	113.0
8	222.5	0.265	59.0	-160.1	-78.4	300.9	-222.5	222.5
9	197.3	0.225	44.3	-115.8	-65.0	262.3	-197.3	197.3
10	151.6	0.190	28.8	-87.0	-67.7	219.3	-151.6	151.6
11	146.2	0.161	23.6	-63.4	-63.2	209.4	-146.2	146.2
12	126.4	0.136	17.3	-46.1	-61.5	187.9	-126.4	126.4
13	113.9	0.116	13.2	-33.0	-59.2	173.1	-113.9	113.9
14	101.4	0.098	9.9	-23.0	-59.0	160.4	-101.4	101.4
15	93.6	0.083	7.8	-15.3	-57.5	151.1	-93.6	93.6
16	84.5	0.070	5.9	-9.3	-56.4	140.9	-84.5	84.5
17	76.7	0.060	4.6	-4.8	-54.7	131.4	-76.7	76.7
18	68.9	0.050	3.5	-1.3	-61.6	130.5	-68.9	68.9
19	71.8	0.043	3.1	1.8	52.1	19.7	-71.8	19.7
20	-56.3	0.036	-2.0	-0.3	-8.7	-47.6	56.3	0.0
21	8.7	0.031	0.3	0.0	0.0	8.7	-8.7	8.7

TABLE 71 New NCF @ i = 8%

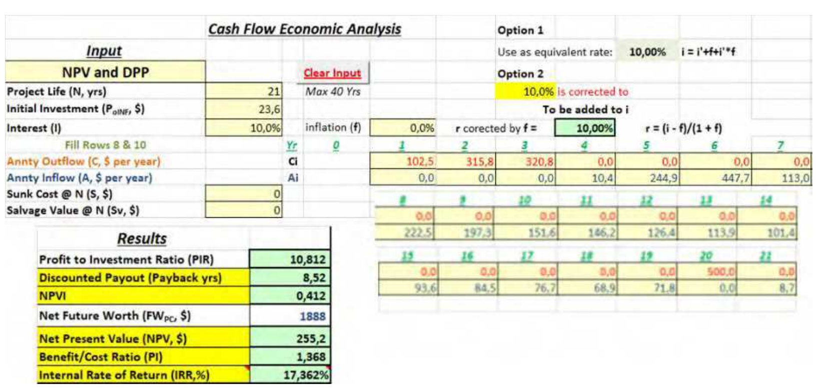
As presented in Table 72, the resulting change in IRR is small reflecting:

- The size of abandonment cost
- Abandonment timing (distance from the origin)
- Difference between computed IRR and sinking fund rate

n	NCF (MMUS$)	Acum DCF @ IRR	NCF @ i (MMUS$)	Cum NCF @ New IRR (MMUS$)
	i =	IRR Original 18.052%	Dscnt @ 8.0%	New IRR 18.044%
0	-23.6	-23.6	-23.6	-23.6
1	-102.5	-110.4	-102.5	-110.4
2	-315.8	-337.0	-315.8	-337.1
3	-320.8	-532.0	-320.8	-532.1
4	10.4	-526.7	10.4	-526.7
5	244.9	-419.9	244.9	-419.9
6	447.7	-254.4	447.7	-254.4
7	113.0	-219.1	113.0	-219.0
8	222.5	-160.1	222.5	-160.0
9	197.3	-115.8	197.3	-115.7
10	151.6	-87.0	151.6	-86.8
11	146.2	-63.4	146.2	-63.3
12	126.4	-46.1	126.4	-46.0
13	113.9	-33.0	113.9	-32.8
14	101.4	-23.0	101.4	-22.9
15	93.6	-15.3	93.6	-15.1
16	84.5	-9.3	84.5	-9.2
17	76.7	-4.8	76.7	-4.6
18	68.9	-1.3	68.9	-1.1
19	71.8	1.8	19.7	-0.3
20	-56.3	-0.3	0.0	-0.3
21	8.7	0.0	8.7	0.0

TABLE 72 Change in IRR Reflecting the Size of Abandonment Cost

The following example expands the economic abandonment concept. In this case the abandonment cost has been increased to MUS$ 500.



Cash Flow Economic Analysis

Option 1
Use as equivalent rate: 10,00% i = i'+f+i'*f
Option 2
10,0% is corrected to
To be added to i

Input	
NPV and DPP	Clear Input
Project Life (N, yrs)	21 (Max 40 Yrs)
Initial Investment (P_{oINF}, $)	23,6
Interest (I)	10,0%
inflation (f)	0,0%
r corected by f =	10,00% (r = (i - f)/(1 + f))
Sunk Cost @ N (S, $)	0
Salvage Value @ N (Sv, $)	0

Fill Rows 8 & 10

Yr	1	2	3	4	5	6	7
Annty Outflow (C, $ per year) Ci	102,5	315,8	320,8	0,0	0,0	0,0	0,0
Annty Inflow (A, $ per year) Ai	0,0	0,0	0,0	10,4	244,9	447,7	113,0

Yr	8	9	10	11	12	13	14
Ci	0,0	0,0	0,0	0,0	0,0	0,0	0,0
Ai	222,5	197,3	151,6	146,2	126,4	113,9	101,4

Yr	15	16	17	18	19	20	21
Ci	0,0	0,0	0,0	0,0	0,0	500,0	0,0
Ai	93,6	84,5	76,7	68,9	71,8	0,0	8,7

Results	
Profit to Investment Ratio (PIR)	10,812
Discounted Payout (Payback yrs)	8,52
NPVI	0,412
Net Future Worth (FW_{PC}, $)	1888
Net Present Value (NPV, $)	255,2
Benefit/Cost Ratio (PI)	1,368
Internal Rate of Return (IRR,%)	17,362%

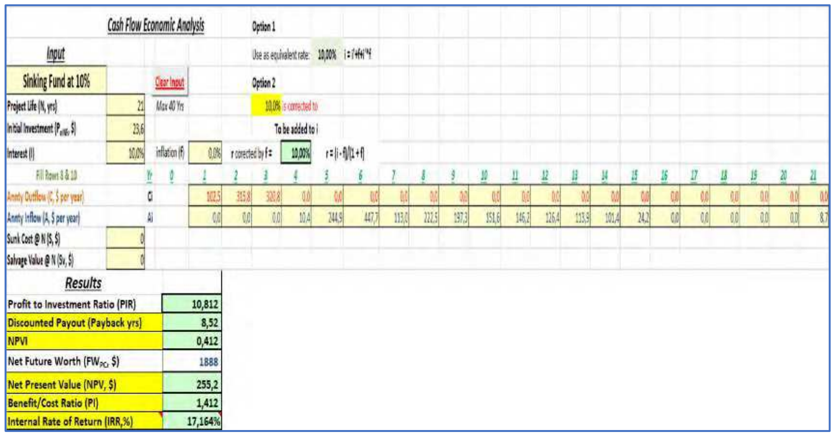
The IRR calculated for this NCF is 17.362%. This compares with an IRR of 18.0519% for the previous NCF with abandonment cost of only MMUS$ 56.3. This indicates how discounted parameters are relatively insensitive to differences at the end of a medium to long project.

The Table 73 is a modified cash flow from Table 72. Now the NCF in Table 73 was normalized over a range of sinking fund rates. A sinking fund rate of 0.0% is used to demonstrate the worst position. It is when the sinking fund receives no interest. The column headed "**Sinking Funds - $NCF_{(n-1)}$**" provides the Extended yield calculation. In this case, there is no discounting and MMUS$ 500 must be allocated from NCF into the sinking fund from Periods 13 to 19. In the 10.0% case, the "**Extended Revenue**" calculation includes discounting at 10% and MMUS$ $500 plus NCF_n is allocated to the sinking fund from Periods 19 to 15.

	(I)	(II)	(III)	(IV)	(V)	(VI)	(VII)	(VIII)	(IX)	(X)
	Values in MMUS$				**$I_{(n-1)}$ - A**	**(I)+(V)**	**$((VII)+(I))_{(n-1)} / (1+i)^1$**	**(I) + (VII)**	**IRR**	
	IRR =	**17.362%**				**0.%**	**10.0%**		**16.580%**	**17.164%**
n	**NCF**	**DF@IRR $1/(1+IRR)^n$**	**DCF @ IRR**	**Acum DCF @ IRR**	**Sinking Fund - $NCF_{(n-1)}$**	**NCF Normaliz. @ i = 0%**	**Extended Revenue**	**NCF Norm. @ i**	**Cum Disc NCF Norm. 0% @ IRR%**	**Cum Disc NCF Norm. 10% @ IRR%**
0	-23.6	1.000	-23.6	-23.6	0.0	-23.6	0.0	-23.6	-23.6	-23.6
1	-102.5	0.852	-87.3	-110.9	0.0	-102.5	0.0	-102.5	-111.5	-111.1
2	-315.8	0.726	-229.3	-340.2	0.0	-315.8	0.0	-315.8	-343.9	-341.1
3	-320.8	0.619	-198.4	-538.7	0.0	-320.8	0.0	-320.8	-546.4	-540.6
4	10.4	0.527	5.5	-533.2	0.0	10.4	0.0	10.4	-540.7	-535.1
5	244.9	0.449	110.0	-423.2	0.0	244.9	0.0	244.9	-427.0	-424.2
6	447.7	0.383	171.3	-251.9	0.0	447.7	0.0	447.7	-248.7	-251.1
7	113.0	0.326	36.8	-215.0	0.0	113.0	0.0	113.0	-210.0	-213.8
8	222.5	0.278	61.8	-153.2	0.0	222.5	0.0	222.5	-144.8	-151.1
9	197.3	0.237	46.7	-106.5	0.0	197.3	0.0	197.3	-95.2	-103.7
10	151.6	0.202	30.6	-75.9	0.0	151.6	0.0	151.6	-62.5	-72.6
11	146.2	0.172	25.1	-50.8	0.0	146.2	0.0	146.2	-35.5	-47.0
12	126.4	0.146	18.5	-32.3	0.0	126.4	0.0	126.4	-15.4	-28.1
13	113.9	0.125	14.2	-18.1	-3.1	110.8	0.0	113.9	-0.3	-13.6
14	101.4	0.106	10.8	-7.3	-104.5	0.0	0.0	101.4	-0.3	-2.6
15	93.6	0.091	8.5	1.2	-198.1	0.0	-69.4	24.2	-0.3	-0.3
16	84.5	0.077	6.5	7.7	-282.6	0.0	-160.9	0.0	-0.3	-0.3
17	76.7	0.066	5.0	12.8	-359.3	0.0	-253.7	0.0	-0.3	-0.3
18	68.9	0.056	3.9	16.6	-428.2	0.0	-348.0	0.0	-0.3	-0.3
19	71.8	0.048	3.4	20.0	-500.0	0.0	-454.5	0.0	-0.3	-0.3
20	-500.0	0.041	-20.3	-0.3	0.0	0.0		0.0	-0.3	-0.3
21	8.7	0.035	0.3	0.0	0.0	8.7		8.7	0.0	0.0

TABLE 73 Modified Cash Flow from Table 72

Despite the large increment in the abandonment cost the sinking fund rate is close to the computed IRR, so the use of the extended yield method leaves the IRR almost unchanged.



The extended yield method enables the analyst to include an explicit assumption about issues such as sinking fund rate, when relevant to a project. However, the impact on IRR is usually less than one percent and consequently, the issue is often ignored.

Acceleration Projects in the Hydrocarbon Business

The main objective of an investment in a hydrocarbon acceleration project is to bring production early on. These projects are common and important because they include activities such as work-overs, infill drilling, gas compression, etc.

Many naturally fractured reservoirs, for example, can be drained efficiently with wide spacing. In some cases, closer spacing can lead to acceleration projects and more attractive economic indicators. However, when production is brought to early times, without significant incremental recovery, more production now is obtained at the expense of lower production later.

In addition to "pure acceleration" to describe an investment where the shift in production time is the only change, other projects include elements such as incremental recovery as well as earlier production.

Acceleration projects must be justified using appropriate economics hurdles, because hydrocarbon prices may or may not rise over time. If a price increase is expected, the economic benefit of accelerated production is diminished.

Taken two production profiles one before and the other after incremental investment, as in the following example (Figure 19).

FIGURE 19 COMPARING PRODUCTION PROFILES

Once the change has been made, there is more oil produced in Project Years 6 to 11, but at the expense of less oil produced in years 12 to 15. The revised profile terminates in Year 12, 3 years early. The difference in oil production, or "Incremental Production" is shown in Figure 20 where the Increment is positive in Years 6 to 11 and negative in Years 12 to 15.

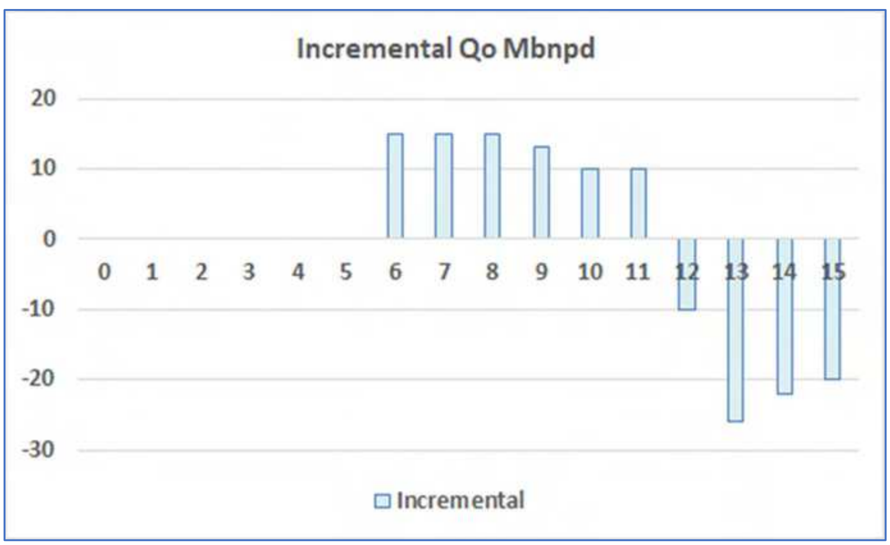


FIGURE 20 INCREMENTAL PRODUCTION

Project Years	Oil Production Rate (Qo (Mbnpd)			New CAPEX	Incremental CF
n	Existent	Proposed	Incremental	MMUS$	
0					
1	25	25	0		
2	70	70	0		
3	115	115	0		
4	130	130	0		
5	130	130	0	-180	-180,0
6	115	130	15		109,5
7	95	110	15		109,5
8	75	90	15		109,5
9	62	75	13		94,9
10	50	60	10		73,0
11	40	50	10		73,0
12	30	20	-10		-73,0
13	26	0	-26		-189,8
14	22	0	-22		-160,6
15	20	0	-20		-146,0

TABLE 74 Incremental Production

When converted into Incremental Revenues, this results in positive Revenues in Years 6 to 11 and negative Revenues in Years 12 to 15. When combined with CAPEX and OPEX, this creates a characteristic cash flow distribution with two phases of negative, with positive in between as it is shown in Table 74.

If the value of incremental production is assumed to be US$ 7.3 per barrel, CAPEX is assumed to be MMUS$ 180 and no other costs are considered. In practice, there would be additional cost savings associated with early abandonment, but also early abandonment CAPEX to consider.

Table 75 contains DCF data for the Acceleration Project for a range of discount rates from zero to 45%. When “i” equals zero, NPV (0.00) equals minus 180, which is the amount of the investment. This confirms that this project is a “pure acceleration”, since the positive Revenue has balanced out the negative revenue.

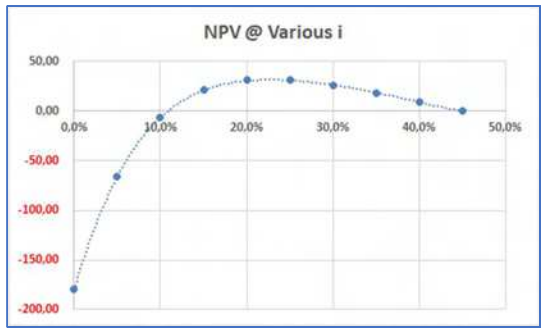
Incremental CF	Years New Project	Discounted Cash Flows @ i NPW = $\sum_{j=0}^{J=n} A_0/(1+i)^n$									
(MMUS$)	n'	0.0%	5.0%	10.0%	15.0%	20.0%	25.0%	30.0%	35.0%	40.0%	45.0%
-180.0	0	-180.0	-180.0	-180.0	-180.0	-180.0	-180.0	-180.0	-180.0	-180.0	-180.0
109.5	1	109.5	104.3	99.5	95.2	91.3	87.6	84.2	81.1	78.2	75.5
109.5	2	109.5	99.3	90.5	82.8	76.0	70.1	64.8	60.1	55.9	52.1
109.5	3	109.5	94.6	82.3	72.0	63.4	56.1	49.8	44.5	39.9	35.9
94.9	4	94.9	78.1	64.8	54.3	45.8	38.9	33.2	28.6	24.7	21.5
73.0	5	73.0	57.2	45.3	36.3	29.3	23.9	19.7	16.3	13.6	11.4
73.0	6	73.0	54.5	41.2	31.6	24.4	19.1	15.1	12.1	9.7	7.9
-73.0	7	-73.0	-51.9	-37.5	-27.4	-20.4	-15.3	-11.6	-8.9	-6.9	-5.4
-189.8	8	-189.8	-128.5	-88.5	-62.0	-44.1	-31.8	-23.3	-17.2	-12.9	-9.7
-160.6	9	-160.6	-103.5	-68.1	-45.7	-31.1	-21.6	-15.1	-10.8	-7.8	-5.7
-146.0	10	-146.0	-89.6	-56.3	-36.1	-23.6	-15.7	-10.6	-7.3	-5.0	-3.6
		NPV = Net Present Value (MMUS$)									
IRR1	10.895%	-180.00	-65.56	-6.74	20.90	30.99	31.29	26.24	18.43	9.35	-0.12
IRR2	44.935%										

TABLE 75 DCF Data for an Acceleration Project for a Range of Discount Rates

	Cash Flow Economic Analysis					Option 1							
Input						Use as equivalent rate:		20,00%	i = i'+f+i'*f				
Incremental Project			Clear Input			Option 2							
Project Life (N, yrs)	10		Max 40 Yrs			20,0% is corrected to							
Initial Investment (P_{oINF}, $)	180						To be added to i						
Interest (I)	20,0%		inflation (f)	0,0%	r corected by f =		20,00%	r = (i - f)/(1 + f)					
Fill Rows 8 & 10		Yr	0	1	2	3	4	5	6	7	8	9	10
Annty Outflow (C, $ per year)		Ci		0,0	0,0	0,0	0,0	0,0	0,0	73,0	189,8	160,6	146,0
Annty Inflow (A, $ per year)		Ai		109,5	109,5	109,5	94,9	73,0	73,0	0,0	0,0	0,0	0,0
Sunk Cost @ N (S, $)	0												
Salvage Value @ N (Sv, $)	0												
Results													
Profit to Investment Ratio (PIR)	0,172		NPW/Po										
Discounted Payout (Payback yrs)	2,17												
NPVI	0,172		NPV (@i) / MCO (@i)										
Net Present Value (NPV, $)	30,99		$P = F / (1 + i)^n$		$P = A * \{[(1 + i)^n - 1]/(i (1 + i)^n)\}$								
Benefit/Cost Ratio (PI)	1,104		ΣPV of net cash inflows/ΣPV of net cash outflows					Net Operating Income to Investment Ratio					
Internal Rate of Return (IRR,%)	10,895%		Click Calc IRR Setting D26=0 Varying B24										
	Calc IRR		0,00 NPV Target			2	IRR Possible solutions						

	Cash Flow Economic Analysis					Option 1							
Input						Use as equivalent rate:	20,00%	i = i'+f+i'*f					
Incremental Project			Clear Input			Option 2							
Project Life (N, yrs)	10		Max 40 Yrs			20,0%	is corrected to						
Initial Investment (P_{oINF}, $)	180						To be added to i						
Interest (I)	20,0%		Inflation (f)	0,0%	r corected by f =		20,00%	r = (i - f)/(1 + f)					
Fill Rows 8 & 10		Yr	0	1	2	3	4	5	6	7	8	9	10
Annty Outflow (C, $ per year)		CI		0,0	0,0	0,0	0,0	0,0	0,0	73,0	189,8	160,6	146,0
Annty Inflow (A, $ per year)		AI		109,5	109,5	109,5	94,9	73,0	73,0	0,0	0,0	0,0	0,0
Sunk Cost @ N (S, $)	0												
Salvage Value @ N (Sv, $)	0												
Results													
Profit to Investment Ratio (PIR)	0,172		NPW/Po										
Discounted Payout (Payback yrs)	2,17												
NPVI	0,172		NPV (@i) / MCO (@i)										
Net Present Value (NPV, $)	30,99		$P = F / (1 + i)^n$		$P = A * \{[(1 + i)^n - 1]/(i (1 + i)^n)\}$								
Benefit/Cost Ratio (PI)	1,104		ΣPV of net cash inflows/ΣPV of net cash outflows				Net Operating Income to Investment Ratio						
Internal Rate of Return (IRR,%)	44,93%		Click Calc IRR Setting D26=0 Varying B24										
	Calc IRR		0,00 NPV Target			2	IRR Possible solutions						

TABLE 76 Base and Incremental Project IRR



As the discount rate increases, the resultant NPV's become less negative and turn positive between 10 and 15% (equal 0 @ 11.2%).

NPV continues to increase to a maximum of MMUS$ 31.29, at a discount rate of 25% and then starts to decline, finally becoming negative again at a discount rate of 45%.

This profile is characteristic of those derived from acceleration projects with the double NPW = 0, indicative of multiple IRRs (10.9 and 44.9% as shown in Table 75).

Interpretation of the results:

- NPV is positive between the two intercepts, around 11 and 45%
- Below 11 percent, investment opportunity is poor and investing to bring revenue forward in time cannot be justified
- Between 15 and 35%, investment opportunities are excellent and superior to this project

Interpretation of IRR is more difficult because in the presence of multiple roots (two or more), none is valid.

The primary benefit of acceleration projects is to bring production, and therefore revenue, forward in time.

Since the real benefit is having this revenue available to invest the question is, how will this revenue be invested earlier? This situation can be solved using the ***extended yield method*** as it allows explicit assumptions about re-investment and has the advantage that multiple roots are removed, thereby allowing the derivation of a unique value for IRR.

In this situation, use of sinking fund as a concept is more theoretical, than in the case of abandonment, since there is no future financial obligation. This means that sinking fund rate is unlikely to be constrained by issues of obligation and risk and therefore may be equivalent to the management's investment opportunity.

The analysis is explained in Table 77 using the Acceleration Project dataset. The objective is to remove late time negatives; therefore, the calculation starts at the end of the project:

Project Years n	**Sinking Funds (SF)** (MMUS$)	**Extended Growth @ i (Sinking Fund Rate)** $SF_{(n)}/(1+i)^1 + SF_{(n-1)}$				
		10.0%	**20.0%**	**30.0%**	**40.0%**	**50.0%**
0	-180.00	-180.00	-180.00	-180.00	-180.00	-180.00
1	109.50	109.50	109.50	109.50	109.50	109.50
2	109.50	89.19	109.50	109.50	109.50	109.50
3	109.50	-22.34	75.51	109.50	109.50	109.50
4	94.90	-145.02	-40.79	21.07	59.03	82.92
5	73.00	-263.91	-162.82	-95.98	-50.22	-17.96
6	73.00	-370.60	-282.99	-219.68	-172.51	-136.45
7	-73.00	-487.96	-427.19	-380.48	-343.72	-314.17
8	-189.80	-456.46	-425.02	-399.73	-379.00	-361.76
9	-160.60	-293.33	-282.27	-272.91	-264.89	-257.93
10	-146.00	-146.00	-146.00	-146.00	-146.00	-146.00

		Normalized Fund (MMUS$)				
0	-180.00	-180.00	-180.00	-180.00	-180.00	-180.00
1	109.50	109.50	109.50	109.50	109.50	109.50
2	109.50	89.19	109.50	109.50	109.50	109.50
3	109.50	0.00	75.51	109.50	109.50	109.50
4	94.90	0.00	0.00	21.07	59.03	82.92
5	73.00	0.00	0.00	0.00	0.00	0.00
6	73.00	0.00	0.00	0.00	0.00	0.00
7	-73.00	0.00	0.00	0.00	0.00	0.00
8	-189.80	0.00	0.00	0.00	0.00	0.00
9	-160.60	0.00	0.00	0.00	0.00	0.00
10	-146.00	0.00	0.00	0.00	0.00	0.00
	IRR	7.1%	31.4%	39.8%	43.7%	45.9%
	IRR - SFR	-2.9%	11.4%	9.8%	3.7%	-4.1%

TABLE 77 Calculation for Sinking Fund Rate (SFR)

As explanation, it is presented the Extended Yield Calculation for Sinking Fund Rate (SFR) for the interest rate of 0.10.

Minus 146 in Period 10 is discounted by 10% and combined with minus 160.6 in Period 9.

- 146.00 / 1.10 = - 132.73

- 132.73 - 160.6 = - 293.33

Minus 293.3 in Period 9 is discounted at 10% and combined with minus 189.8 in Period 8.

- 293.33 / 1.10 = - 266.66

- 266.66 - 189.80 = - 456 46

The calculation continues upwards until it runs into surplus, in this case 89.19 in Period 2.

This 89.19 becomes the project cash flow for that period

All the cash flows in previous years remain as part of the main project.

All the cash flows in later months become part of the sinking fund and show as zeroes in the project cash flow.

n	(MMUS$)	10,0%
0	-180,00	-180,00
1	109,50	109,50
2	109,50	89,19
3	109,50	-22,34
4	94,90	-145,02
5	73,00	-263,91
6	73,00	-370,60
7	-73,00	-487,96
8	-189,80	-456,46
9	-160,60	-293,33
10	-146,00	-146,00
0	-180,00	-180,00
1	109,50	109,50
2	109,50	89,19
3	109,50	0,00
4	94,90	0,00
5	73,00	0,00
6	73,00	0,00
7	-73,00	0,00
8	-189,80	0,00
9	-160,60	0,00
10	-146,00	0,00
	IRR	7,10%
	IRR-SFR	-2,90%

IRR calculated from the normalized cash flows increases with increasing SFR, and IRR is greater than SFR, when in the range 12 to 44.75 percent (interpolated values).

SFR	10.0%	12.0%	20.0%	30.0%	40.0%	44.7%	50.0%
IRR	7.10%	12.03%	31.42%	39.78%	43.69%	44.7%	45.92%
IRR-SFR	-2.90%	0.00%	11.42%	9.78%	3.69%	0.00%	-4.08%

At the limits, they are equal and outside the range, computed IRR is less than the assumed SFR. This confirms that within the defined range (12.0 to 44.7 percent) investment in this project generates a higher return than the company's opportunity rate.

The mathematical reason for the behavior of the NPV Profile relates to the pattern of positive and negative cash flows over time. Previously, the cash flow profiles were predominantly negative cash flows (CAPEX) next to the origin, followed by predominantly positive cash flows (Revenue minus OPEX minus Tax) later. The process of discounting this "conventional "pattern causes more rapid decrease of the more distant and positive cash flows, and consequently results in a progressive decline in the cumulative value (NPV), with increasing discount rate.

The pattern of cash flows for the acceleration project is different, in that there are three phases, negative, positive and negative. With increasing discount rate, the more distant negative cash flows, at the end of the project, are those to be affected most and the decline in these negatives causes the aggregate value (NPV) to increase. Once these later negatives have been substantially eliminated by discounting, the project takes on the appearance of a more conventional pattern, with negatives, followed by positives. At this point, the NPV profile reaches a maximum, turns over and declines through zero in a familiar fashion (Figure 21).

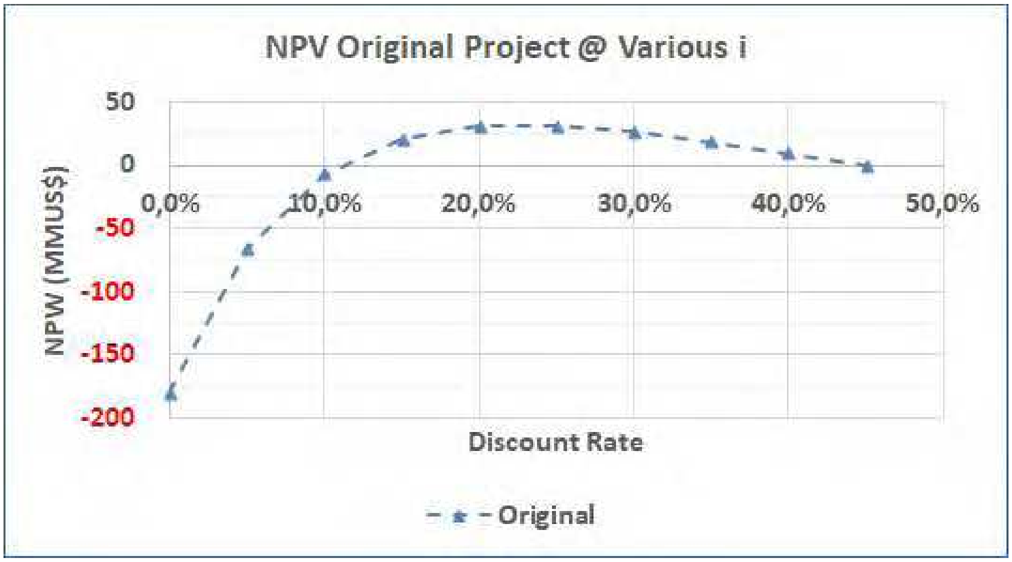


FIGURE 21 ORIGINAL PROJECT @ VARIOUS DISCOUNT RATES

Not all acceleration projects have multiple roots, but if there are significant negative cash flows at the end, this will result in an NPV profile, which increases over some range of increasing discount rate.

With increasing investment, if it occurs in Period zero (discount factor 1.000), the profile simply moves downwards, along its length, by the same amount.

For example, if US$ 50 million were added to the up-front cost (-180-50) the curve is shifted down by 50. In this case there is no intercept on the horizontal axis and therefore no IRR solution (Figure 22).

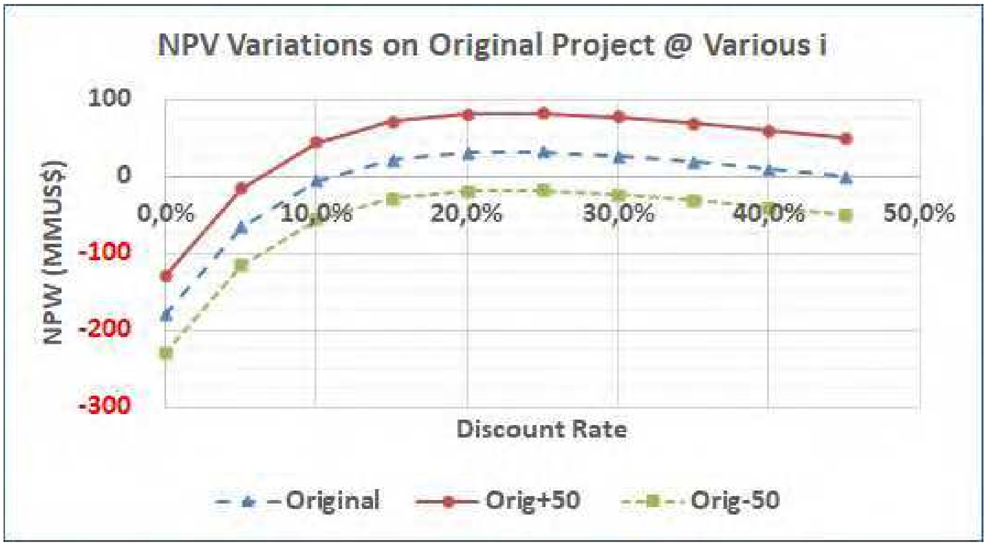


FIGURE 22 NPV VARIATIONS ON INITIAL INVESTMENT

If the investment conveys to increased recovery (in addition to accelerated production), the intercept on the vertical axis moves upwards. At this point, the discount rate is zero (discount factor equals 1.000) so the intercept represents the aggregate of CAPEX and net revenue (Revenue minus OPEX minus Taxes). In this case taxes are ignored.

Since the incremental revenue is distributed over time, the curves will move closer together with increasing discount rate. The relationship will depend on the pattern of production, the further into the future the oil is produced, the more significant will be the effect of discounting.

In the cases presented in Figure 23 the incremental recovery of oil is assumed to be produced at a uniform rate over the remaining life of the project.

- In the case of 2% of increased reserves, the curve has moved upwards, retaining a double intercept on the horizontal axis (at i equal 3.12 and 71.69%)
- In the 4% case, it has moved further, such that the intercept on the vertical axis is now positive
- There is, consequently one intercept on the horizontal axis and one solution to IRR

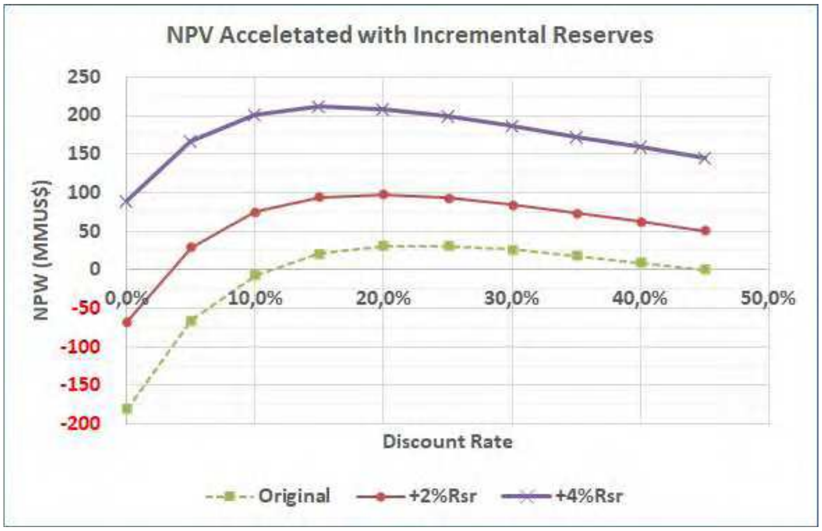


FIGURE 23 NPV ACCELERATED WITH INCREMENTAL RESERVES

In all these cases, the price of oil was kept constant. However, for any project, which involves the shifting of production in time, the assumption, about price changes over time is a critical aspect of the analysis. For example, if price is expected to increase over time, the economic benefit of accelerated production is diminished.

Applications of Cash Flows

Economic parameters, which are derived from project cash flow, have several applications:

A. Transaction valuation
B. Project screening
C. Project ranking

A. Transaction Valuation Based on NPV

Transaction value relates to the process of buying or selling. Measurement here is based on the premise that value of an asset is derived from the anticipated, future stream of positive and negative cash flows.

Considering the timescale of hydrocarbon projects, the concept of time value through the process of discounting must be included.

Recalling that Net-Present-Worth (NPW) or Net Present Value (NPV), is the difference between the present value and the investment at time zero. The equivalent at the end of several years is the future value (worth). Therefore, as NPV incorporates the idea of value it could be used as basis for transaction negotiation. Particularly when NPV is derived from future cash flows and an appropriate discount rate.

The NPV is probably the best economic hurdle by which acceleration projects and projects in general can be evaluated. NPV is a measure of profit.

- Ranking by NPV identifies those investments which will generate the highest profits

The NPV of an investment is calculated by discounting the future net cash flow to time zero, and then adding up all such partial net incomes. The NPV is the difference between the present worth of all cash inflows and the present worth of all cash outflows that are associated with an investment project.

The Net Present Value calculation or estimate is an equivalence method of analysis through which any project’s partial cash flows are discounted to a single net present value.

A positive NPV means that the equivalent worth of all cash inflows is higher than the equivalent worth of all cash outflows, and therefore the project generates a profit and should be considered as a potential project to invest in. However, because of resources constraints, not all positive projects can be pursued.

If the present value of the future net cash flows is higher than the initial or time 0 (zero) investment value, then managers would prefer to make the investment and obtain a future cash stream over the alternative option of keeping their money without investing it.

The following example assumes that the project is on the market and a transaction value is required appropriate to year 0.

Relevant cash flows are those, which would impact on the new owner after that date. Production may decline over the year, or may be irregular because of maintenance shut down. Certain costs may also be non-uniformly distributed throughout the year. Taxes, such as Corporation Tax, are paid partly in arrears, so some of the Tax listed for year 0 may be the responsibility of the previous owner.

Transaction Valuation				Discount Rate		10,0%
Project Years	Revenue	OPEX	CAPEX	Taxes	NCF	NPV
n	(MMUS$)	(MMUS$)	(MMUS$)	(MMUS$)	(MMUS$)	(MMUS$)
(I)	(II)	(III)	(IV)	(V)	(VI) = (II)-(III)-(IV)-(V)	(VII) = $(VI)/(1+i)^n$
-8	0	0	24	0	-24.0	0.0
-7	0	0	102	0	-102.0	0.0
-6	0	5	311	0	-316.0	0.0
-5	21	56	286	0	-321.0	0.0
-4	202	67	125	0	10.0	0.0
-3	337	76	15	0	246.0	0.0
-2	596	78	25	44	449.0	0.0
-1	419	93	135	78	113.0	0.0
0	237	50	17	41	129.3	-129.3
1	358	77	14	69	198.0	180.0
2	294	70	14	59	151.0	124.8
3	265	68	0	50	147.0	110.4
4	236	63	0	46	127.0	86.7
5	216	59	0	42	115.0	71.4
6	196	55	0	39	102.0	57.6
7	183	53	0	37	93.0	47.7
8	168	49	0	34	85.0	39.7
9	155	46	0	32	77.0	32.7
10	142	44	0	29	69.0	26.6
11	142	42	0	28	72.0	25.2
12	0	0	51	5	-56.0	-17.8
13	0	0	0	-9	9.0	2.6
Total	4167,3	1051	1119	624	1373,3	916,9

Cash Flow Economic Analysis

Input					
Transaction Valuation		Clear Input			
Project Life (N, yrs)	13	Max 40 Yrs			
Initial Investment (P_{aInv}, $)	-129,3				
Interest (i)	10,0%	Inflation (f)	0,0%	r corected by f =	10,00%
Sunk Cost @ N (S, $)	0				
Salvage Value @ N (Sv, $)	0				

Option 1
Use as equivalent rate: **10,00%** i = i'+f+i'*f
Option 2
10,0% is corrected to
To be added to i
r = (i - f)/(1 + f)

Fill Rows 8 & 10	Yr	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Annty Outflow (C, $ per year)	Ci		0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	56,0	0,0
Annty Inflow (A, $ per year)	Ai		198,0	151,0	147,0	127,0	115,0	102,0	93,0	85,0	77,0	69,0	72,0	0,0	9,0

Results	
Profit to Investment Ratio (PIR)	-7,091
Net Future Worth (FW_{PC}, $)	3165
Net Present Value (NPV, $)	916,90
Benefit/Cost Ratio (PI)	-7,226

TABLE 78 NPV For a Transaction Valuation of a Project

The transaction date is appropriate as the origin of the discount process. With appropriate financial assumptions, the calculated value for NPV is MMUS$ 916.9.

Buyer and seller might then use this value as a starting point in their negotiation. Also, they may have differences of opinion with respect to some of the assumptions used in the derivation of the NPV. Cash flow, for example, incorporates assumptions about:

- Future levels of production
- Costs
- Oil and gas prices
- Levels of taxation

Two companies may have different perceptions of value based on different incidence of corporate tax and different investment opportunities.

B. Project Screening Based on NPV, NPVI or IRR

In terms of project evaluation, screening is the process of comparing projects against a set of standard criteria or hurdles. For example, project cash flows and measures of profitability. In some cases, screening would include risks, reserves, geology, technology, geography, politics etc.

When a company has several investment proposals, it is necessary to evaluate the attractiveness of each proposal. Any evaluation hurdle or criterion to be used should provide management with a means to identify acceptable projects (or to reject them) in a consistent manner. Later, the criterion provides a ranking of the proposals under consideration in the order of desirability for investment.

Project evaluation criteria only deliver guidelines to managers for making decisions. Management is the entity that makes the actual investment decision after consideration of:

- The engineering and economic analysis
- The relevant qualitative information that has an impact on any major decision
- The risk/uncertainty associated to each investment project/investment

In the hydrocarbon business investment environment, subject to estimating errors, it is required sound managerial judgement. Particularly when there is limited data and when the decision is constrained by corporate strategies and not by an economic criterion.

The treatment of costs during a property evaluation, including the decision to abandon a well or buy an asset should be made by examining the after-tax net cash flow for the company, with and without the project.

The decision to abandon a well should be based on relevant costs and revenues and only revenues and costs that will go away if a well is abandoned should be considered. In other words, compare the revenues and costs for the entire reservoir (or field) with the well minus the revenues and costs for the reservoir(or field) without the well.

The proper method for evaluating a capital investment is to compare the present outflows with anticipated positive net cash flows which will accrue from the project in the future. In making this comparison the timing of the various cash flows are recognized using an appropriate interest rate. In summary, projects should be evaluated based on benefits maximization.

The use of discounting as a basis for screening investments is a typical procedure. Discounting project cash flows at a specific rate could be used as a means of testing whether an investment was growing at that rate. When discounted at "i", aggregate discounted cash flow of zero equates with investment growth of i. Additionally, a positive aggregate would mean a higher growth rate and a negative aggregate, a lower growth rate.

In general, IRR and NPV criteria yield identical accept/reject results for a ***single*** investment proposal. When comparing ***multiple*** projects, IRR and NPV can give contradictory results. In these cases, it is advisable to use NPV in the process of screening projects.

For the investor that considers time value of money the discount factor "i" is the base for any economic hurdle, The three most important economic parameters give rise to identical screening criteria as follows:

NPV @ (i) ≥ 0

NPVI @ (i) ≥ 0

IRR ≥ i

If management decides that it requires a return of 12 percent from its investments, it should discount all project cash flows at 12 percent.

- NPV(0.12) equal to zero
  - It indicates that the project will give a return of precisely 12 percent
  - If NPV[0.12] equals zero, Then, NPVI(0.12) equals zero, since NPVI equals NPV/MCO. Therefore, NPVI ≥ 0 represents the same screening criterion
- If the NPV(0.12) is positive, the return is higher than 12 percent
- If NPV(0.12) is negative, the return is lower than 12 percent
- If NPV(0.12) equals zero, IRR equals 12 percent, since IRR is the discount rate which produces an NPV of zero
- IRR ≥ i therefore represents the same screening criterion. Being the straightforward parameter of the three to calculate, NPV(i) becomes the recommended screening criterion

When comparing investment proposals, this decision criterion is that the proposal with the highest NPV should be selected.

If working with costs, the proposal with the lowest NPV of outflows should be selected, unless management takes the do-nothing option.

NPV is one of the best when used as an accept/reject criterion for project/investment evaluation due to having the following features:

- It takes into account the time value of money based on a specific interest rate
- It is a single number (cash equivalent) or index-indicator for comparison at time zero

- The value is a unique quantity for a given interest rate
- It makes no theoretical difference if projects are evaluated based on future values or present values

Screening, based on discounted cash flow is widely used. However, it may be considered appropriate to incorporate additional, secondary criteria as part of the screening process. In certain circumstances, for example, a maximum Discounted Payback Period may be applied.

The derivation of the discounted parameter is straightforward; however, the main consideration is the choice of an appropriate discount rate as it should reflect the management's perception of investment opportunity and of risk.

C. Project Ranking

The economic hurdles can yield different results when ranking projects in order of priority according to potential profitability. Usually, companies have less capital to invest than projects because there are capital limitations, in these cases the ranking of projects becomes crucial considering that the best projects are those that maximize the company's future worth.

Once calculated the economic hurdles or yardsticks, they are applied to screening and ranking, which are basic steps in the investment decision process.

- Screening
  - It involves a company in the testing of each available project against a set of appropriate criteria, yardsticks, or standards, to determine whether these opportunities are suitable or profitable
  - Standard criteria refer to technical and/or economic hurdles
  - Screening would consider aspects such as:
    - Geology
    - Technology
    - Geography
    - Cash flows
    - Measures of profitability: NPV, IRR, Discounted Payback Period, etc.
- Ranking
  - It follows and requires comparison between suitable projects to determine the best candidates for investment

The screening process generates a list of projects, which have passed the various technical and economic selection criteria. These have been assessed as "good" projects or at worst "acceptable" projects.

Passing the screen test does not necessarily lead to investment. Management may have limited resources or other considerations for wishing to compare projects and to rank them one against another and to select the more suitable project or a combination of projects.

CONSIDERATIONS FOR RANKING PROJECTS

There is no unique solution on how to rank the investment efficiency of various projects. It is in this situation where the analyst who ranks the projects must have a full understanding of the economic hurdles used to make the recommendations. When ranking projects, examples of typical considerations are:

a) Resource Limits

All organizations have finite resources, some of which constrain investment. The most important of these are for example specialist workforce and finance that constrain the investment options

b) Mutual Exclusion

An investment proposal is considered independent when the acceptance of one proposal from the portfolio of projects has no significant effect on the acceptance of any of the other proposals contained in the portfolio. This is the case when the proposals are functionally different.

The more general case is the one of mutually exclusive proposals. The latter is the condition prevailing when a group of proposals are related to one another in such a manner that the acceptance of one proposal from the portfolio of proposals precludes the acceptance of any other proposals in the suite. This disqualification can also be caused by limits to the total capital available to be invested.

Project A and Project B may be alternate development plans for the same hydrocarbon reservoir, in which case ranking and choice are necessary. In some cases, the relationship may not be obvious. Takeover of Company X may be seen as an alternative to initiating exploration in Country Y, in the sense that each would contribute to corporate reserves and possibly increase geographical diversity.

Larger projects will generally have higher NPV's and are likely to demonstrate greater efficiency with respect to nonfinancial resources such as personnel

- If investment requirements are similar, the use of NPV is the best hurdle
- If finance is not a constraint, financial efficiency hurdles are not an issue
- If projects are mutually exclusive, NPV indicates the maximum profit potential from the opportunity (e.g., maximizes profit from a field development or a fracturing job)

c) The "Do-Nothing" Alternative

In many situations, undertaking even a single alternative or project entails making a decision between two options, because implicitly the managers have the "do-nothing" decision, which means continuing to rely on the current system

d) Best First

The goal of identifying which are the best opportunities is to make these investments early. Earning profits early will contribute to corporate growth. Smaller projects may receive a boost from changing technology or may be further displaced by newer and better opportunities

Ranking is not a unique process. Projects rank differently according to the ranking criteria applied.

- The project with the largest profit not necessarily is the most efficient in terms of profit per dollar invested
- The most efficient project in terms of profit per dollar invested could be the worst in terms of workforce efficiency

Ranking Parameters Dilemma Using NPV and IRR

The dilemma is whether to rank by NPV (US$) or by IRR (%). For example, using the concept of Future Value, Terminal Cash Surplus (TCS) or the end point of the cumulative curve (not the highest value achieved during the life of the project).

Project A has an undiscounted TCS of 50 and an IRR of 50%

Project B has an undiscounted TCS of 80 and an IRR of 36%

As shown in Figure 24 a discount rate of about 22%, Project B has a higher NPV, but Project A has a higher IRR. So, which is the better project when these parameters rank differently?

These financial criteria represent only one of several aspects of project rank. Also, cash flow data is someone's perception of the future and accordingly suspect.

The first step in any interpretation of discounted cash flows is to ensure that the discount rate is appropriate. If there is doubt about the actual value, use an appropriate range.

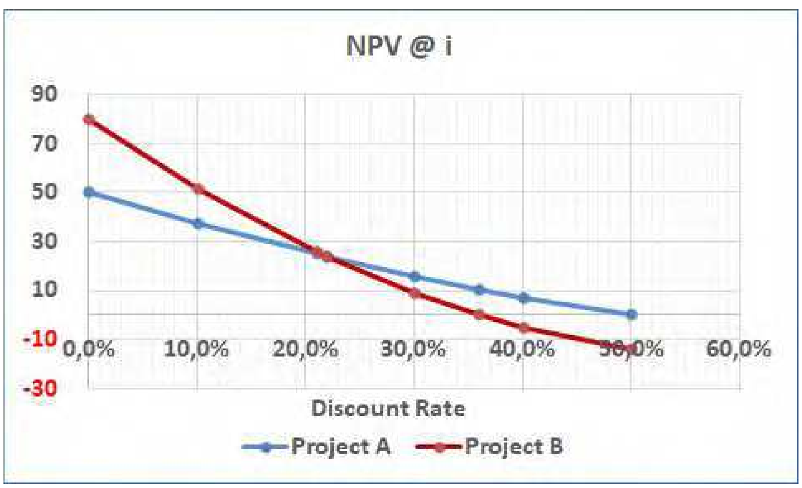


FIGURE 24 NET PRESENT VALUE @ VARIOUS DISCOUNT RATE

Assuming a discount rate of 12% and since the curves intersect at a higher value (about 22%), Project B has a higher NPV and Project A has a higher IRR.

The skill of the managers is an important and difficult factor to consider when determining their relation to the investment decisions of a corporation.

Managers commonly use basic parameters, yardsticks or measures of investment worth as part of the decision process to help compare and rank different options.

Generally, managers use ranking parameters that include the time value of money, such as: Net-Present-Worth or Value (NPW, NPV), Internal Rate of Return (IRR), and Annual-Equivalent-Worth (AEW), among others.

Remember when comparing multiple projects and if IRR and NPV give contradictory results, that it is advisable to use NPV in the process of ranking projects because NPV identifies the investments with the highest profits. If NPV > 0, then managers would prefer to make the investment.

Net Present Value (NPV) as Screening and Ranking Parameter

When screening single projects, NPV of the investment opportunity rate (i_{op} or MARR) leads to the correct decision.

- If NPV is positive, the project is acceptable

If on the other hand, the analysis is on Mutually Exclusive Projects, again NPV is the recommended criteria. Pick the project with the highest NPV.

If the investment opportunity rate is not known, a sensitivity analysis can be run for a range of MARR to help taking the decision.

NPV is a measure of profit and ranking by NPV identifies those investments, which will generate the largest profits.

Example of NPV as Screening and Ranking Parameter

In the following example, two projects are compared at a discount rate of 12%

Project	NPV (MMUS$)
A	34
B	44

Comments:

NPV reveals nothing about the size of the investment or indeed about investment efficiency, it is simply the surplus. In this case, the investment could be MMUS$ 50 or MMUS$ 500.

Larger projects will generally have larger NPV's (doubling all cash flows doubles NPV).

Larger projects usually have greater efficiency with respect to nonfinancial resources such as personnel (one manager per project).

- If investment requirements are similar, use NPV because it is less controversial
- If finance is not a constraint, financial efficiency is not the hurdle

- If projects are mutually exclusive, NPV indicates the maximum profit potential from the opportunity (e.g., maximizes profit from a field development)

Government tends to favor use of NPV as a basis for definition of economic reserves

Internal Rate of Return (IRR) as Screening and Ranking Parameter

Along with the NPV another primary measure of investment worth is the ***internal rate of return (IRR)***.

- The NPV is a measure in dollars and IRR is a method to analyze projects investments in terms of percentage rates of return
- IRR as a percentage, indicates the return per unit investment
- IRR is a measure of investment efficiency
- As a ranking parameter, IRR indicates where investment will grow faster

Example of IRR as Screening and Ranking Parameter

Project	IRR (%)
A	50.0
B	36.0

Using IRR as a ranking criterion requires careful consideration. Project IRR is very sensitive to the timing of cash flows.

- Projects with early revenues often have very high IRR's
- The calculation of project IRR involves discounting the cash flows at the rate, which produces an NPV of zero

The discount rate is determined by the pattern of cash flows, not by any consideration of the corporate investment opportunity.

If a project has an IRR of 50%, that fact has been determined by discounting project cash flow at 50%.

Related to discounting, IRR has the importance of using a rate that was appropriate to the corporate investment opportunity.

- High discount rates place greater emphasis on cash flows closer to the discount origin
- Excessively high discount rates place inappropriate emphasis on early production

IRR is generally used by small companies where an individual investment is likely to represent a larger proportion of the total and therefore have greater proportional impact on investment opportunity.

IRR may also be a useful indicator of the sequence of investment amongst opportunities, which have already been selected.

The problem is where IRR is used to rank in a situation, where one is chosen at the expense of another (mutually exclusive). The project with the high IRR may generate income very quickly; however, in the longer term the project that generates more profit for more years could be a better choice.

The IRR is the rate of discount that equates the present value of future cash receipts to the cost of the project. Therefore, it is the break-even interest rate (***i***) which equates the present worth of a project's cash outflows to the present worth of its cash inflows, making the NPV of the investment equal to zero.

- NPV and IRR yield identical accept/reject results for a single investment proposal
- When ranking projects, IRR and NPV can give contradictory results. In these cases, it is better to use NPV as economic hurdle
- For the case of mutually exclusive projects, it is best to use NPV or AEW, because these two indicators are absolute measures of investment worth, while the IRR is only a relative indicator and does not indicate absolute net worth or absolute value in any event
- When evaluating different projects and if their IRR exceeds management's hurdle, it is advisable to accept them for further consideration

Summary of Net Present Value Index (NPVI) as Screening and Ranking Parameter

Some of the problems associated with using IRR as a ranking criterion have been highlighted. If the requirement is to rank by investment efficiency, it is appropriate to consider using NPVI(i), where "i" is corporate investment opportunity rate.

NPVI(i) is calculated by dividing NPV(i) as a measure of profit, by the Maximum Capital Outlay MCO(i) as a measure of investment. It is therefore a measure of investment efficiency incorporating discounting at an appropriate rate.

It is not always the case that NPV and NPVI ranking coincide. In some cases, it happens because the projects have identical investment and therefore identical MCO(i) for all values of i.

Investment efficiency is a useful criterion to apply, if a company is trying to maximize profit and particularly if a company has limited funds to invest.

In the context of comparing alternate development plans for a single reservoir, the use of NPVI could imply targeting the best part of the reservoir. This is likely to incur the displeasure of the regulatory authority. Use of NPV in this situation is more likely to produce a development plan consistent with maximizing economic recovery.

The continuous use of NPVI as economic hurdle could lead to invest in many small projects, which may have implications if human resources are scarce.

The Profit to Investment Ratio (PIR) and Benefit Cost Ratio (PI) as Screening and Ranking Parameters

A further economic hurdle based on discounted cash-flow analysis that is sometimes used is the Profit to Investment Ratio (PIR).

PIR is particularly common in the public sector, and it is defined as the ratio of project's discount net benefits over the initial investment. As with the NPV, the discount rate applied should reflect the cost of funds to the company.

In other words, PIR is the relation between the net cash flow of the project and the initial investment required to generate such cash flow (P_0).

The Profit to Investment Ratio is a measure of investment efficiency, incorporating the idea of optimizing profit earned for every dollar invested. This is important in a situation, where investment dollars are limited.

PIR, describes an index that represents the relationship between the costs and benefits of a proposed project. It is calculated as the ratio between the present value of future expected net cash flows and the initial amount invested in the project.

- A higher PIR means that a project will be considered more attractive
- Projects with PIR greater than 1 are acceptable, and larger ratios are favored

THE BENEFIT/COST RATIO (PI)

A related indicator is the Benefit/Cost Ratio (**PI**). In computing the PI:

- The initial investment and the subsequent discounted net outflow are placed in the denominator of the ratio
- The subsequent discounted net inflow becomes the numerator

Thus, the calculation of PI from cumulative NCF data is entirely straightforward:

$$PI = \frac{\sum Present\ Value\ of\ Net\ Cash\ Inflows}{\sum Present\ Value\ of\ Net\ Cash\ Outflows}$$

	Cash Flow Economic Analysis			Option 1			
Input				Use as equivalent rate:	15,82%	i = i'+f+i'*f	
Profit to Investment Ratio		Clear Input		Option 2			
Project Life (N, yrs)	22	Max 40 Yrs		15,8% is corrected to			
Initial Investment (P_{oPYO}, $)	26			To be added to i			
Interest (I)	15,8%	inflation (f)	0,0%	r corected by f =	15,82%	r = (i - f)/(1 + f)	
Sunk Cost @ N (S, $)	0						
Salvage Valuet at N (Sv, $)	0						

		Yr	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
Annty Outflow (C, $ per year)	Variable	Ci		0	139	486	839	827	558	65	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Annty Inflow (A, $ per year)	Variable	Ai		0	0	0	0	0	0	0	59	304	521	687	848	987	1112	1224	1327	1420	1504	1580	1659	1597	1607

Results					
Profit to Investment Ratio (PIR)	1,459	NPW/Po			
Discounted Payout (Payback yrs)	21,37	Only for the variable case			
NPVI	0,024	NPV (@i) / MCO (@i)			
Net Future Worth (FW_{PC}, $)	959,35	$F_1 = P(1+i)^n$	$F_2 = \Sigma(A_x\ (1+i)^{(n-x)})$	$F_3 = \Sigma(C_x\ (1+i)^{(n-x)})$	$F = -F_1 + F_2 - F_3 - S + Sv$
Net Present Value (NPV, $)	37,94	$P = F/(1+i)^n$	$P = A * \{[(1+i)^n - 1]/(i(1+i)^n)\}$		
Benefit/Cost Ratio (PI)	1,024	ΣPV of net cash inflows/ΣPV of net cash outflows		Net Operating Income to Investment Ratio	
Internal Rate of Return (IRR,%)	16,08%	Click Calc IRR Setting D26=0 Varying B24			

TABLE 79 Profit to Investment Ratio and Benefit Cost Ratio (PI) Example

Comments on The Profit to Investment Ratio (PIR)

- In Table 79, The Profit to Investment Ratio (PIR) or the net cash flow to the total discount investment is equal to 1.459. This means that the profits are 145.9% of the investment
- The PIR is helpful in ranking various projects because it allows investors quantify the value created per each investment unit
- A Profit to Investment Ratio (PIR) of 1.0 is usually the lowest acceptable measure for the index, as any value lower than that number would indicate that the project's present value (NPV) is less than the initial investment
- As the value of the PIR increases, so does the financial attractiveness of the proposed project
- When using PIR to compare the desirability of projects, consider that projects with larger cash inflows may result in lower profitability index calculations because their profit margins are not as high
- When using PIR as a sole yardstick, ratios greater than 1.0 are ranked based on the highest calculation
- When limited capital is available, and projects are mutually exclusive, the project with the highest PIR is to be accepted as it indicates the project with the most productive use of limited capital

PIR Example 1

Consider a project characterized by the following numbers, where the cost of capital is 12%.

	i =	12,0%	
	US$		
Year	Net Cash Inflow	PV	Cum PV
1	6000	5357,14	5357,14
2	6000	4783,16	10140,31
3	6000	4270,68	14410,99
Po (US$)	12000,000		
NPV (US$)	2410,988		
PIR	0,201		

TABLE 80 PIR 3 Years Project

	Cash Flow Economic Analysis					Option 1					
Input						Use as equivalent rate:		12,00%	i = i'+f+i'*f		
Example Profitability Index (PI)			Clear Input			Option 2					
Project Life (N, yrs)	3		*Max 40 Yrs*			12,0%	is corrected to				
Initial Investment (P_{oINF}, $)	12000,00						To be added to i				
Interest (I)	12,0%		inflation (f)	0,0%	r corected by f =		12,00%	r = (i - f)/(1 + f)			
Fill Rows 8 & 10		*Yr*	*0*	*1*	*2*	*3*					
Annty Outflow (C, $ per year)		Ci		0,0	0,0	0,0					
Annty Inflow (A, $ per year)		Ai		6000,0	6000,0	6000,0					
Sunk Cost @ N (S, $)	0										
Salvage Value @ N (Sv, $)	0										
Results											
Net Cash Flow Prsnt. Val. (NCFP,$)	14410,99		Without Salvage Value or Sunk Cost								
Profit to Investment Ratio (PIR)	0,201		NPW/Po								
Discounted Payout (Payback yrs)	2,41										
NPVI	0,201		NPV (@i) / MCO (@i)								
Net Future Worth (FW_{PC}, $)	3387		$F_1 = P(1+i)^n$		$F_2 = \Sigma(A_x (1+i)^{(n-x)})$		$F_3 = \Sigma(C_x (1+i)^{(n-x)})$		$F = -F_1 + F_2 - F_3 - S + Sv$		
Net Present Value (NPV, $)	2411		$P = F/(1+i)^n$		$P = A * \{[(1+i)^n - 1]/(i(1+i)^n)\}$						
Benefit/Cost Ratio (PI)	1,201		ΣPV of net cash inflows/ΣPV of net cash outflows					Net Operating Income to Investment Ratio			
Internal Rate of Return (IRR,%)	23,38%		Click Calc IRR Setting D26=0 Varying B24								

TABLE 81 Profitability Index (PI) Direct Calculation 3 Years Project

In comparing PIR with NPV, both methods generally produce equivalent results. The PIR is greater than 1 only if the present value of net benefits exceeds the initial investment and this also ensures a positive NPV. Thus, both measures establish the same cutoff level.

Conflicts can only arise when mutually exclusive projects are under consideration in particularly if the investments differ in terms of size.

PIR Example 2

A company considers purchasing a new gas compressor. Two competing, and hence, mutually exclusive, proposals have been received. The cash flows and results are as follows:

Data		Project A	Project B	Difference A-B	
Initial Investment (US$)	(I)	50000	15000	35000	
Present value of future net cash inflows (US$)	(II)	70000	30000	40000	
Results					
Net Present Value (US$)	(III)	20000	15000	5000	(III) = (II) -(I)
Profit to Investment Ratio (PIR)	(IV)	0,4	1,0	-0,6	(IV) = (III)/ (I)

TABLE 82 Cost to Benefit Ratio Data Example 2

Project A is superior according to NPV, whereas Project B has the higher PIR. In these cases, NPV is a better criterion.

In Table 82, the initial investment in Project B is lower by MUS$ 35 (50 – 15) than the amount required to invest in Project A. The absolute economic contribution to the company of choosing Project B depends on how these extra MUS$ 35 can be invested. This marginal investment should yield a return that is close to the company's cost of capital, which implies Net Present Values that are close to zero.

If for example, the remaining MUS$ 35 could only be invested to produce a net present value that is smaller than MUS$ 5, this is the difference between NPV Project A (MUS$ 20) and NPV Project B (MUS$ 15)

The choice of Project B with its high PIR is wrong, the resulting economic contribution to the company would be less than the available from Project A.

Conflict Between NPV and PIR

When there is a conflict between NPV and PIR, the NPV is the better criterion, as it measures the economic contribution of each project to the company in absolute terms.

The PIR like the IRR suffers from the usual limitations of percentages and index numbers in that it conceals absolute magnitudes.

A project may have a high Profit to Investment Ratio (PIR) but be small in terms of the absolute dollar amounts. Therefore, it could be less desirable than another opportunity characterized by a more modest ratio but a larger scale.

In most practical situations, the Profit to Investment Ratio (PIR) will produce the same results as the NPV.

NPV and PIR can be computed and may supplement each other, as they provide information regarding a project's relative and absolute profitability.

The NPV is better in the evaluation of mutually exclusive investments than differ in size. Depend on the PIR under such circumstances may lead to sub-optimal decisions.

Financial Interpretation of the Main Economic Hurdles

Once the analyst has derived an investment's incremental cash flows, the next step in a capital-budgeting process is to decide whether the investment should be accepted or rejected. As stated earlier the parameters commonly used to accept or reject investments recognize the time value of money and are based on the concept of discounting.

The most widespread economic parameters are:

- The net present value (**NPV**)
- The internal rate of return (**IRR**)
- The Benefit Cost Ratio (**PI**)
- Profit to Investment Ratio (**PIR**).

Interpretation of The Net Present Value

The NPV is the sum of all cash flows generated by the project, with each cash flow discounted back to the present. The discount rate used is generally given by the corporation´s after-tax cost of capital. In equation form the NPV is expressed as:

$$NPV = \sum_{t=0}^{n} \frac{A_t}{(1+i)^t}$$

Where:

A_t = The net after-tax cash flow (inflows minus negative outflows) in period ***t***, and is positive for the net inflows and negative for the net outflows

i = time value of money to the firm as requested by its weighted average cost of capital

n = the number of periods comprising the expected life of investment

The rationale for the NPV is that an investment must earn at least the cost incurred in funding the project, if the investment is considered attractive.

- A positive NPV indicates that
  - A project contributes cash flows to the firm more than the cost required to finance it
  - These additional cash flows accrue to the benefit of the stake holders of the company
  - Projects with a positive NPV should be accepted because they enhance the value of the corporation
- The amount of the NPV indicates the increase in shareholder capital that accrues because of the investment

Interpretation of NPV Using an Example

Consider a project that requires an initial investment of US$ 10000 and it generates net cash inflows of US$ 4000 US$ 5000, and US$ 6000 assumed to occur at the end of years 1, 2, and 3. Given a cost of capital (i) = 10%.

Initial Investment (US$)	10000				
Interest Rate	10.0%				
		US$			
		(I)	**(II) = $(1+i)^n$**	**(III) = (I)/(II)**	**(IV) = Sum (III)**
	Year	**Cash Flow**	**Present Value Factor**	**Present Value of Cash Flow**	**Cum NPV**
	0	-10000	1.000	-10000.00	-10000.00
	1	4000	1.100	3636.36	-6363.64
	2	5000	1.210	4132.23	-2231.40
	3	6000	1.331	4507.89	**2276.48**

TABLE 83 NPV Interpretation Example

	Cash Flow Economic Analysis						**Option 1**				
Input							Use as equivalent rate:	**10,00%**	**i = i'+f+i'*f**		
Ejemplo NPV			Clear Input				**Option 2**				
Project Life (N, yrs)	3		*Max 40 Yrs*				10,0%	is corrected to			
Initial Investment (P_{oINF}, $)	10000						**To be added to i**				
Interest (I)	10,0%		inflation (f)	0,0%	r corected by **f =**		**10,00%**	**r = (i - f)/(1 + f)**			
Fill Rows 8 & 10		*Yr*	*0*	*1*	*2*	*3*					
Annty Outflow (C, $ per year)		**Ci**		0,0	0,0	0,0					
Annty Inflow (A, $ per year)		**Ai**		4000,0	5000,0	6000,0					
Sunk Cost @ N (S, $)	0										
Salvage Value @ N (Sv, $)	0										
Results											
Profit to Investment Ratio (PIR)	**0,228**		**NPW/Po**								
Discounted Payout (Payback yrs)	**2,47**										
NPVI	**0,228**		**NPV (@i) / MCO (@i)**								
Net Future Worth (FW_{PC}, $)	**3030**		$F_1 = P (1 + i)^n$		$F_2 = \Sigma(A_x\ (1 + i)^{(n-x)})$		$F_3 = \Sigma(C_x\ (1 + i)^{(n-x)})$		$F = - F_1 + F_2 - F_3 - S + Sv$		
Net Present Value (NPV, $)	**2276**		$P = F / (1 + i)^n$		$P = A * \{[(1 + i)^n - 1]/(i\ (1 + i)^n)\}$						
Benefit/Cost Ratio (PI)	**1,228**		**ΣPV of net cash inflows/ΣPV of net cash outflows**					**Net Operating Income to Investment Ratio**			
Internal Rate of Return (IRR,%)	**21,65%**		Click Calc IRR Setting D26=0 Varying B24								

Results	
Profit to Investment Ratio (PIR)	**0,228**
Discounted Payout (Payback yrs)	**2,47**
NPVI	**0,228**
Net Future Worth (FW_{PC}, $)	**3030**
Net Present Value (NPV, $)	**2276**
Benefit/Cost Ratio (PI)	**1,228**
Internal Rate of Return (IRR,%)	**21,65%**

TABLE 84 NPV Interpretation Example Direct Calculation

The resulting NPV is US$ 2276. The economic interpretation of this NPV is that the project is expected to generate a gain of US$ 2276 over and above the required return of 10% to cover financing charges that are charged each year on the unrecovered portion of the investment.

Assume that the initial amount of US$ 10000 with a return of 10% on any outstanding balance, were to be repaid from cash flows subsequently generated (Table 85).

Initial Investment US$)	10000				
Interest Rate	10.0%				
	US$				
	(I)	**(II) = i% * (I)**	**(III)**	**(IV) = (III) - (II)**	**(V) = (I) - (IV)**
Year	**Investment Outstanding at Beginning of Year**	**Return on Investment Outstanding**	**Amount of Inflow**	**Repayment of Investment**	**Investment Outstanding at Year End**
1	10000	1000	4000	3000	7000
2	7000	700	5000	4300	2700
3	2700	270	6000	5730	-3030

TABLE 85 NPV as the Unrecovered Portion of The Investment

This leaves an additional amount of US$ 3030 in year 3

Thus, the corporation can recover the initial investment, pay 10% each year on funds still tied up in the project, and in year 3, given the US$ 6000 inflow, be left with a gain of US$ 3030. The present value of this gain is US$ 2276

$$Gain = F/(1+i)^n$$

$$\text{Gain} = 3030/(1+0.1)^3$$

$$\text{Gain} = 2276$$

US$ 2276 equals the NPV of the project as computed on Table 84. This represents the amount by which the value of the company increases because of undertaking the investment, or the increase in shareholders wealth.

Interpretation of The Internal Rate of Return

The effective yield of capital investment is also called a project's internal rate of return (IRR). The IRR is the rate of discount that, when applied to the cash flows of an investment, will yield an NPV value oz zero.

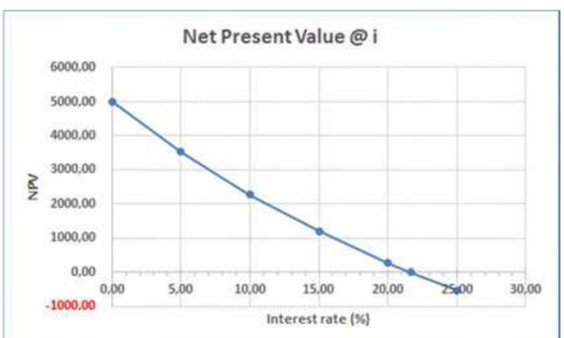
Arithmetically, it is the discount rate, *i*, that satisfies the equation:

$$NPV = \sum_{t=0}^{n} \frac{A_t}{(1+i)^t} = 0$$

- If this yield exceeds the company's after-tax cost of capital, the investment will make a positive economic contribution to the company and to its shareholders and should be accepted

Interpretation of The Internal Rate of Return Using an Example

Consider again the NPV example (Table 83 and Table 84) that requires an initial investment of US$ 10000, and was expected to generate net cash inflows of US$ 4000, 5000, and 6000 in years 1 through 3. Repeating the previous calculation at different yields it is possible to calculate the NPV presented in Figure 25



	i (%)	NPV
	0.00	5000.00
	5.00	3527.70
Cost of Capital	10.00	2276.48
	15.00	1204.08
	20.00	277.78
IRR	**21.65**	**0.00**
	25.00	-528.00

FIGURE 25 IRR GRAPHICAL ESTIMATION FROM NPV @ VARIOUS I

- The shape of the ***NPV profile*** will be similar for all investments that involve an initial expenditure and subsequent inflows
- The initial investment is not affected by discounting
- The present value of future cash inflows decreases as the discount rate increases
- The NPV of a project typically decreases as the discount rate rises

If the cash flows produce a positive NPV at a discount rate of zero; then, this is the total undiscounted cash inflows exceed the initial outflows

By incrementing the discount rate, the NPV is decreased until the NPV becomes negative. In this example, the IRR is 21.65% because this is the rate at which the NPV profile begins to cross the horizontal axis as it is shown in Figure 25.

IRR of 21.65% was also calculated in the Table 84.

Results	
Profit to Investment Ratio (PIR)	0,228
Discounted Payout (Payback yrs)	2,47
NPVI	0,228
Net Future Worth (FW_{PC}, $)	3030
Net Present Value (NPV, $)	2276
Benefit/Cost Ratio (PI)	1,228
Internal Rate of Return (IRR,%)	21,65%

The IRR is the effective yield to be carried over time on the unrecovered portion of the investment. This is, on any portion of the initial investment still tied up in the project.

Using the same example and the same procedure to compute the NPV in Table 85, it is calculated the NPV discounted at IRR in Table 86:

Initial Investment (US$)		10000				
Interest Rate	21.65%					
		US$				
		(I)	**(II) = i% * (I)**	**(III)**	**(IV) = (III) - (II)**	**(V) = (I) - (IV)**
	Year	**Investment Outstanding at Beginning of Year**	**Return on Investment Outstanding**	**Amount of Inflow**	**Repayment of Investment**	**Investment Outstanding at Year End**
	1	10000.00	2164.78	4000.00	1835.22	8164.78
	2	8164.78	1767.49	5000.00	3232.51	4932.27
	3	4932.27	1067.73	6000.00	4932.27	0.00

TABLE 86 IRR as the Effective Yield of Capital Investment

The total net cash inflows in each period (Column (II)) can be thought of as being made up of a 21,65% return on the outstanding investment (Column (IV)), and some recovery of the original investment (Column (V)).

In year 1 there is a cash inflow of US$ 4000. The US$ 2164.78 represents a 21.64% return on the original US$ 10000 investment, and the balance of US$ 1835.22 as a reduction in the capital committed to the project. This leaves a net investment of US$ 8164.78 at the beginning of year 2. Following the same procedure through year 2 and 3, the cash inflows from the project provide for the full recovery of the amount originally invested, plus a return of 21.65% on any funds tied up to the project.

Generally, the NPV and IRR produce equivalent results. If, for a desirable project, the NPV is positive using as a discount rate the cost of capital, this normally implies that the IRR exceeds the cost of capital. Because the project returns more than the cost of financing accrue to the shareholders, the IRR criterion also supports the objective of maximizing the company wealth.

NPV and IRR can often be viewed as equivalent ways of looking at the same problem. In the NPV case it is discounted at the cost of capital and focuses on total benefits to be achieved after meeting the cost of funds invested. In the case of IRR, it is ensured that the effective yield of the project exceeds its cost of financing. Hence, the criterion chosen could be either NPV or IRR.

- As each economic hurdle represents information about the project in a different manner, most companies compute the NPV, the IRR, and the PIR in addition to non-economic parameters

Conflict Between Net Future Worth (F) and IRR Example

When choosing between projects of different scale and/or duration, it is better to use NPV as economic yardstick.

- NPV measures actual economic gain
- IRR calculates the effective percentage yield on monies in the project

For example, if someone can invest US$ 1000 in one of two projects:

- Project A provides a yield of 25% over 1 year
- Project B provides a yield of 20% over 10 years

Assume that normal investments yield 15%.

Project A

P		=	1000	
i		=	25%	
n		=	1	
F		=	1250	**$F = P/(1 + i)^n$**
Project A Reinvestment				
P		=	1250	
i		=	15%	
n		=	9	
F		=	4397	

Total in 10 years	US$ 4397

Project B

P	=	1000
i	=	20%
n	=	10
F	=	6192

TABLE 87 Comparing Projects A and B with Reinvestment

It is better to choose Project B to obtain 20% over 10 years. This means a future value of US$ 6192 whereas Project A yields US$ 1250 that must be reinvested at the prevailing rate of 15% for a total future value of US$ 4397.

Considering that the higher the future value the higher the NPV, this means that the 10-year Project B has the higher NPV and hence leaves the investor better off, even, if it provides the lower initial yield or IRR.

A parallel argument applies if the investor must choose between investing US$ 100 in a 25% project or US$ 1000 in one that yields 20%.

Project A

P	=	100
i	=	25%
n	=	10
F	=	931

$F = P/(1 + i)^n$

Project B

P	=	1000
i	=	20%
n	=	10
F	=	6192

TABLE 88 Select the Project with Higher Future Value

It is better to choose Project B to obtain 20% over 10 years.

NPV Project Comparison _ Choice of Project Example

Even with revenue projects that have different lives, they can be comparable ***if they require only a one-time investment.*** This is a typical case in the extraction of a fixed quantity of natural resource such as oil or coal.

Consider two mutually exclusive processes:

- One that requires 10 years to produce some oil
- A second that can accomplish the task in only 8 years

In this example, the two projects can be compared even though the cash flows cover a different time span.

The revenues must be included in the analysis even if the price of oil is constant, because of the time value of money. While the total revenue could be equal for either process, the one for the faster process has a higher present value.

- The two projects should be compared using the NPV of each over its own life
- In these cases, the analysis period is determined by and coincides with the life of the selected project

Considerations on Project Screening When NPV is Similar _ Example

A small Private Company (PC) has made an oil discovery and has decided to extract the oil. PC can either:

a) Lease the necessary equipment and extract and sell the oil itself
    i) If PC chooses option (a)
        (1) The net annual cash flow after taxes from operations will be MUS$ 500 at the end of each year for the next 5 years
        (2) PC can sell the field for a net cash flow of MMUS$ 1 in 5 years when the oil is depleted
b) Lease the field to an oil-production group (OPG)
    i) If PC chooses option (b)
        (1) The production group (OPG) can extract all the oil in only 3 years and the company PC can sell the field for a net cash flow of MUS$ 800 in 3 years
        (2) The difference in resale value of the field is due to the increasing rate of land appreciation anticipated for the property
        (3) The net cash inflow from the lease payments to PC will be MUS$ 630 at the beginning of each year for the next 3 years. Therefore, the first payment is at the end of year zero

All benefits and costs associated with the two options have been accounted for in the numbers listed above.

Which option should the company select at $\boldsymbol{i}$ = 15%?

Solution

Option (a)

	Cash Flow Economic Analysis					Option 1			
Input						Use as equivalent rate:		15,00%	i = i'+f+i'*f
NPV Option A			Clear Input			Option 2			
Project Life (N, yrs)	5		*Max 40 Yrs*			15,0%	is corrected to		
Initial Investment (P_{oINF}, $)	0					To be added to i			
Interest (I)	15,00%		inflation (f)	0,0%	r corected by f =		15,00%	r = (i - f)/(1 + f)	
Fill Rows 8 & 10		Yr	0	1	2	3	4	5	
Annty Outflow (C, $ per year)		Ci		0,0	0,0	0,0	0,0	0,0	
Annty Inflow (A, $ per year)		Ai		500	500	500	500,0	500,0	
Sunk Cost @ N (S, $)	0								
Salvage Value @ N (Sv, $)	1000								
Results									
Profit to Investment Ratio (PIR)	#¡DIV/0!		NPW/Po						
Discounted Payout (Payback yrs)									
NPVI	#¡DIV/0!		NPV (@i) / MCO (@i)						
Net Future Worth (FW_{PC}, $)	4371		$F_1 = P\ (1 + i)^n$		$F_2 = \Sigma(A_x\ \ (1 + i)^{(n-x)})$		$F_3 = \Sigma(C_x\ \ (1 + i)^{(n-x)})$		$F = - F_1\ + F_2 - F_3 - S + Sv$
Net Present Value (NPV, $)	2173,254		$P = F / (1 + i)^n$		$P = A * \{[(1 + i)^n - 1]/(i\ (1 + i)^n)\}$				
Benefit/Cost Ratio (PI)	#¡DIV/0!		ΣPV of net cash inflows/ΣPV of net cash outflows					Net Operating Income to Investment Ratio	
Internal Rate of Return (IRR,%)	###########		Click Calc IRR Setting D26=0 Varying B24						

TABLE 89 NPV Option (a) Lease the Equipment and Extract and Sell the Oil

Option (b)

	Cash Flow Economic Analysis					Option 1			
Input						Use as equivalent rate:		15,00%	i = i'+f+i'*f
NPV Option B			Clear Input			Option 2			
Project Life (N, yrs)	3		*Max 40 Yrs*			15,0%	is corrected to		
Initial Investment (P_{oINF}, $)	-630					To be added to i			
Interest (I)	15,00%		inflation (f)	0,0%	r corected by f =		15,00%	r = (i - f)/(1 + f)	
Fill Rows 8 & 10		Yr	0	1	2	3			
Annty Outflow (C, $ per year)		Ci		0,0	0,0				
Annty Inflow (A, $ per year)		Ai		630	630				
Sunk Cost @ N (S, $)	0								
Salvage Value @ N (Sv, $)	800								
Results									
Profit to Investment Ratio (PIR)	-3,461		NPW/Po						
Discounted Payout (Payback yrs)									
NPVI	#¡DIV/0!		NPV (@i) / MCO (@i)						
Net Future Worth (FW_{PC}, $)	3316		$F_1 = P\ (1 + i)^n$		$F_2 = \Sigma(A_x\ \ (1 + i)^{(n-x)})$		$F_3 = \Sigma(C_x\ \ (1 + i)^{(n-x)})$		$F = - F_1\ + F_2 - F_3 - S + Sv$
Net Present Value (NPV, $)	2180,210		$P = F / (1 + i)^n$		$P = A * \{[(1 + i)^n - 1]/(i\ (1 + i)^n)\}$				
Benefit/Cost Ratio (PI)	-2,461		ΣPV of net cash inflows/ΣPV of net cash outflows					Net Operating Income to Investment Ratio	
Internal Rate of Return (IRR,%)	###########		Click Calc IRR Setting D26=0 Varying B24						

TABLE 90 Option (b) Lease the Field to an Oil-Production Group

After depletion of the oil, the project will terminate

Option	(a)	(b)	Difference (b)-(a)
NPV (US$)	2173.254	2180.210	6.955

TABLE 91 Comparing Options A and B

The small difference between the two projects' NPV (US$ 6.955) suggests that the actual decision between operating and leasing might be decided on non-economic issues.

Even if the operating option or Option (a) were slightly better, the company might prefer to forego the small amount of additional income and select the lease option rather than undertake a new business venture and run their other own operation.

A variable that might also have a critical effect on this decision is the sale value of the field in each option. The value of fields is often difficult to forecast over any long period of time, and the company may feel some uncertainty about the accuracy of its guesses.

Chapter 7 it will cover the sensitivity analysis as a method by which analysts can factor in the uncertainty about the accuracy of project cash flows into the analysis.

Additional Comments Comparing the NPV and IRR

The NPV and IRR are two of the most common criteria for capital budgeting. The two economic hurdles usually provide equivalent results. However, conflicting project ranking by the two yardsticks may occur under special circumstances, as described below:

Comparing NPV and IRR _ Example of two mutually projects

Consider two mutually projects. The company cost of capital or MARR is 12%. Each project requires an initial investment of US$ 1000 and generates subsequent after-tax cash inflows as follows:

	US$	
Year	**Project A**	**Project B**
1	1200	0
2	0	0
3	0	1643

	Cash Flow Economic Analysis					**Option 1**			
Input						Use as equivalent rate:		**12,00%**	**i = i'+f+i'*f**
Project A			**Clear Input**			**Option 2**			
Project Life (N, yrs)	3		*Max 40 Yrs*			12,0%	is corrected to		
Initial Investment (P_{oINF}, $)	1000,00						**To be added to i**		
Interest (I)	12,0%		inflation (f)	0,0%	r corected by **f =**		**12,00%**	**r = (i - f)/(1 + f)**	
Fill Rows 8 & 10		*Yr*	*0*	*1*	*2*	*3*			
Annty Outflow (C, $ per year)		**Ci**		0,0	0,0	0,0			
Annty Inflow (A, $ per year)		**Ai**		1200,0	0,0	0,0			
Sunk Cost @ N (S, $)	0								
Salvage Value @ N (Sv, $)	0								
Results									
Profit to Investment Ratio (PIR)	**0,071**		**NPW/Po**						
Discounted Payout (Payback yrs)									
NPVI	**0,071**		**NPV (@i) / MCO (@i)**						
Net Future Worth (FW_{PC}, $)	**100**		**$F_1 = P (1 + i)^n$**		**$F_2 = \Sigma(A_x\ (1 + i)^{(n-x)})$**		**$F_3 = \Sigma(C_x\ (1 + i)^{(n-x)})$**		**$F = - F_1 + F_2 - F_3 - S + Sv$**
Net Present Value (NPV, $)	**71**		**$P = F / (1 + i)^n$**		**$P = A * \{[(1 + i)^n - 1]/(i\ (1 + i)^n)\}$**				
Benefit/Cost Ratio (PI)	**1,071**		**ΣPV of net cash inflows/ΣPV of net cash outflows**					**Net Operating Income to Investment Ratio**	
Internal Rate of Return (IRR,%)	**20,00%**		Click **Calc IRR** Setting D26=0 Varying B24						

TABLE 92 Cash Flow and Results Project A

	Cash Flow Economic Analysis					**Option 1**			
Input						Use as equivalent rate:		**12,00%**	**i = i'+f+i'*f**
Project B			**Clear Input**			**Option 2**			
Project Life (N, yrs)	3		*Max 40 Yrs*			12,0%	is corrected to		
Initial Investment (P_{oINF}, $)	1000,00						**To be added to i**		
Interest (I)	12,0%		inflation (f)	0,0%	r corected by **f =**		**12,00%**	**r = (i - f)/(1 + f)**	
Fill Rows 8 & 10		*Yr*	*0*	*1*	*2*	*3*			
Annty Outflow (C, $ per year)		**Ci**		0,0	0,0	0,0			
Annty Inflow (A, $ per year)		**Ai**		0,0	0,0	1643,0			
Sunk Cost @ N (S, $)	0								
Salvage Value @ N (Sv, $)	0								
Results									
Profit to Investment Ratio (PIR)	**0,169**		**NPW/Po**						
Discounted Payout (Payback yrs)	**2,84**								
NPVI	**0,169**		**NPV (@i) / MCO (@i)**						
Net Future Worth (FW_{PC}, $)	**238**		**$F_1 = P (1 + i)^n$**		**$F_2 = \Sigma(A_x\ (1 + i)^{(n-x)})$**		**$F_3 = \Sigma(C_x\ (1 + i)^{(n-x)})$**		**$F = - F_1 + F_2 - F_3 - S + Sv$**
Net Present Value (NPV, $)	**169**		**$P = F / (1 + i)^n$**		**$P = A * \{[(1 + i)^n - 1]/(i\ (1 + i)^n)\}$**				
Benefit/Cost Ratio (PI)	**1,169**		**ΣPV of net cash inflows/ΣPV of net cash outflows**					**Net Operating Income to Investment Ratio**	
Internal Rate of Return (IRR,%)	**18,00%**		Click **Calc IRR** Setting D26=0 Varying B24						

TABLE 93 Cash Flow and Results Project B

Comparing Results

	Project A	Project B
NPV (US$)	71,43	169,45
IRR (%)	20,00%	18,00%

Project A is preferred according to the IRR, whereas Project B has a higher NPV. The conflict emerges because the cash flows generated by the two projects have different time patterns, and the two evaluation hurdles imply different assumptions regarding the rates at which cash inflows can be reinvested.

The nature of the problem is clearer if the Future Wealth is calculated at the end of year 3 for each of the projects. If Project A is chosen, the terminal wealth depends on the rate at which the early cash inflow of US$ 1200 in year 1 can be reinvested. Given that this cash inflow will be invested for two years, the terminal values of Project A for various reinvestment rates become:

	$F = P * (1+i)^2$
Reinvestment Rate (%)	**Future Values for Project A**
12.0%	1505
17.0%	1643
20.0%	1728

These future values must be compared with the cash inflow of US$ 1643 at the end of year 3 for Project B. It is observed that Project B is superior if the reinvestment yield is below 17%.

When applying other economic hurdles Project B also results superior

	Project A	Project B
PI	1.07	1.17
Profit to Investment Ratio (PIR)	0.071	0.169
Discounted Payout (Payback yrs.)		2.8
NPVI	0.07	0.17
F (US$)	100.35	238.07
NPV (US$)	71.43	169.45
Benefit/Cost Ratio (PI)	1.071	1.169
IRR (%)	20.00%	18.00%

TABLE 94 Additional Hurdles Comparing A and B Projects

Selecting based on the IRR implies that funds released from any project can be reinvested at the project's IRR. If in this example reinvestment is possible at 20% (Project A's IRR), an investor wealth will be maximized by choosing Project A.

The NPV hurdle on the other hand, implies that cash flows released from any project are reinvested at the discount rate that was used in calculating the NPV.

Applied to this example, Project B becomes the preferred choice if the reinvestment rate equals the company's cost of capital of 12%.

Comparing NPV and IRR _ Example of two mutually projects with initial investment

Assume two mutually exclusive projects (A and B). The company's cost of capital is 12%. Each projects requires an initial investment of US$ 10000 and generates subsequent after-tax cash inflows as follows:

	US$	
Year	Project A	Project B
1	2000	10000
2	4000	3000
3	12000	3000

Results with i = 12%

	Project A	Project B
PI	1.35	1.35
Profit to Investment Ratio (PIR)	0.352	0.346
Discounted Payout (Payback yrs.)	2.56	1.42
NPVI	0.352	0.346
F (US$)	4940	4855
NPV (US$)	3516	3455
Benefit/Cost Ratio (PI)	1.352	1.346
IRR (%)	26.55%	37.63%

TABLE 95 Comparing Economic Hurdles Two Projects

Figure 26 shows the NPV as function of the discount rate applied for both projects. The IRR values are given by the intersection of the two curves with the horizontal axis.

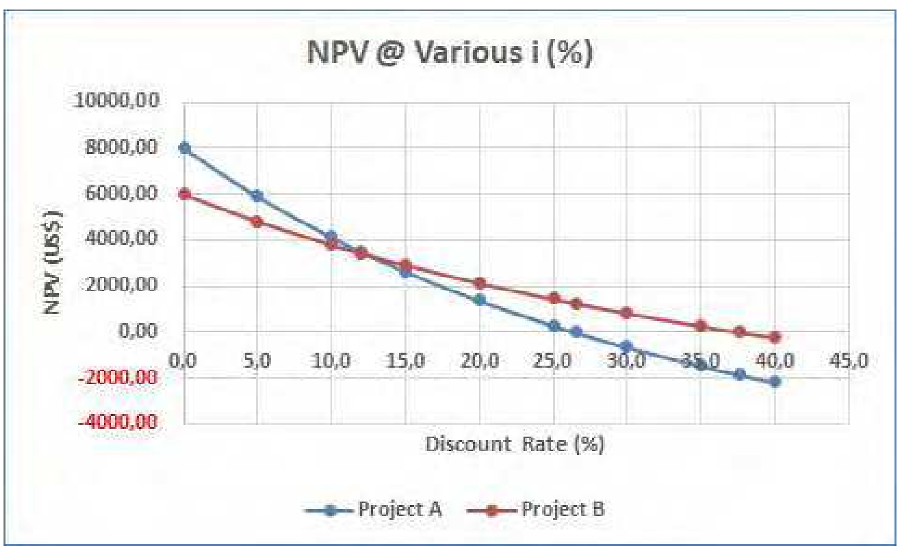


FIGURE 26 NPV AS FUNCTION OF THE DISCOUNT RATE APPLIED FOR BOTH PROJECTS

Contradicting ranking between the NPV and IRR criteria occurs in those cases where the ***NPV profiles*** of two projects cross over at some rate higher, or to the right of, the cost of capital.

In the example, the crossovers are caused by differences in the time patterns of the project's cash flows, as shown in Figure 26.

Conflicts may also occur because of a difference in scale between projects

- The NPV as an absolute measure of investment worth varies with the size of the investment
- The IRR as a relative measure remains unaffected

The NPV of Project A, with its more distant cash inflows, will be the more sensitive to changes in the discount rate. In applying a discount factor ($A/(1 + i)^n$), any increase in i is magnified by a large exponent n, and the resulting NPV is exaggerated.

The relatively immediate cash returns of Project B, on the other hand, are much less sensitive to changes, which implies that the NPV profile has a lower slope. Given the different slopes of the two curves, it is easy to observe why a crossover occur.

The desirability of Project B will depend on the rate at which its early cash returns can be reinvested.

- If this reinvestment rate is low, the higher but distant cash returns of Project A will be preferred
- If the reinvestment rate increases, Project B becomes progressively more attractive

If a choice between NPV and IRR is to be made, NPV is generally a better choice over IRR.

An inconsistency of the IRR approach is the assumption that cash inflows from different projects can be reinvested at different rates depending on the project's effective yield.

The NPV value approach, on the other hand, implies reinvestment of any cash inflows at one consistent rate for all projects.

The measures of economic value should be additive. This means that, if two independent projects A and B have been approved, the economic value of A and B must equal the sum of the individual economic values, or:

$$\text{Value } (A + B) = \text{Value } (A) + \text{Value } (B)$$

- NPV possess the property of additivity
- NPV measures the economic gain that an investment contributes to shareholders
- IRR (which measures percentage returns) does not measure the economic gain

NPV additivity is useful when the issue is to select between projects with different lives, as it is explained in the following example.

COMPARING PROJECTS WITH DIFFERENT LIVES USING NPV INSTEAD OF IRR EXAMPLE

Consider three EOR (Enhanced Oil Recovery) projects (A, B, and C), assume these three methods as stand-alone investments that cannot be done at the same time.

However, project B can be done after Project A.

The discount rate is 10%.

	Project A	Project B	Project C
NPV (US$)	15000	20000	30000
Repetitions every (years)	2	4	6

Solution

The evaluation of these options requires NPV of various sequential options to be discounted back to the present.

The NPV of the Project B that would follow the Project A must also be discounted back to time zero.

The following calculations illustrates the proposal:

<u>Option with only Project C</u>

Total NPV Project C = US$ 30000

<u>Option Project A + Project B</u>

Option Project A plus Project B to cover the same 6 years as Project C

	Cash Flow Economic Analysis					**Option 1**					
Input						Use as equivalent rate:	**10,00%**	**i = i'+f+i'*f**			
Option A and B			**Clear Input**			**Option 2**					
Project Life (N, yrs)	6		*Max 40 Yrs*			10,0%	is corrected to				
Initial Investment (P_{oINF}, $)	-15000,00						**To be added to i**				
Interest (I)	10,0%		inflation (**f**)	0,0%	r corected by **f =**		**10,00%**	**r = (i - f)/(1 + f)**			
Fill Rows 8 & 10		***Yr***	***0***	***1***	***2***	***3***	***4***	***5***	***6***		
Annty Outflow (C, $ per year)		**Ci**		0,0	0,0	0,0	0,0	0,0	0,0		
Annty Inflow (A, $ per year)		**Ai**		0,0	20000,0	0,0	0,0	0,0	0,0		
Sunk Cost @ N (S, $)	0										
Salvage Value @ N (Sv, $)	0										
Results											
Profit to Investment Ratio (PIR)	-2,102		**NPW/Po**								
Discounted Payout (Payback yrs)											
NPVI	#¡DIV/0!		**NPV (@i) / MCO (@i)**								
Net Future Worth (FW_{PC}, $)	55855		**$F_1 = P\ (1 + i)^n$**		**$F_2 = \Sigma(A_x\ (1 + i)^{(n-x)})$**		**$F_3 = \Sigma(C_x\ (1 + i)^{(n-x)})$**		**$F = - F_1 + F_2 - F_3 - S + Sv$**		
Net Present Value (NPV, $)	31529		**$P = F / (1 + i)^n$**		**$P = A * \{[(1 + i)^n - 1]/(i\ (1 + i)^n)\}$**						
Benefit/Cost Ratio (PI)	-1,102		**ΣPV of net cash inflows/ΣPV of net cash outflows**					**Net Operating Income to Investment Ratio**			
Internal Rate of Return (IRR,%)	###########		Click **Calc IRR** Setting D26=0 Varying B24								

TABLE 96 NPV Project B after Project A

NPV Project A + NPV Project B = US$ 31529

<u>Option Project A   Repeated</u>

Option Project A repeated to cover the six-year span

	Cash Flow Economic Analysis					**Option 1**			
Input						Use as equivalent rate:	**10,00%**	**i = i'+f+i'*f**	
Option A Every 2 Years			**Clear Input**			**Option 2**			
Project Life (N, yrs)	6		*Max 40 Yrs*			10,0% is corrected to			
Initial Investment (P_{oINF}, $)	-15000,00						**To be added to i**		
Interest (I)	10,0%		inflation (f)	0,0%	r corected by **f =**		**10,00%**	**r = (i - f)/(1 + f)**	
Fill Rows 8 & 10		*Yr*	*0*	*1*	*2*	*3*	*4*	*5*	*6*
Annty Outflow (C, $ per year)		**Ci**		0,0	0,0	0,0	0,0	0,0	0,0
Annty Inflow (A, $ per year)		**Ai**		0,0	15000,0	0,0	15000,0	0,0	0,0
Sunk Cost @ N (S, $)	0								
Salvage Value @ N (Sv, $)	0								
Results									
Net Cash Flow Prsnt. Val. (NCFP,$)	**0,00**		**Without Salvage Value or Sunk Cost**						
Profit to Investment Ratio (PIR)	**-2,509**		**NPW/Po**						
Discounted Payout (Payback yrs)									
NPVI	#¡DIV/0!		**NPV (@i) / MCO (@i)**						
Net Future Worth (FW_{PC}, $)	**66685**		**$F_1 = P (1 + i)^n$**		**$F_2 = \Sigma(A_x\ (1 + i)^{(n-x)})$**	**$F_3 = \Sigma(C_x\ (1 + i)^{(n-x)})$**		**$F = - F_1 + F_2 - F_3 - S + Sv$**	
Net Present Value (NPV, $)	**37642**		**$P = F / (1 + i)^n$**		**$P = A * \{[(1 + i)^n - 1]/(i\ (1 + i)^n)\}$**				
Benefit/Cost Ratio (PI)	**-1,509**		**ΣPV of net cash inflows/ΣPV of net cash outflows**					**Net Operating Income to Investment Ratio**	
Internal Rate of Return (IRR,%)	###########		**Click Calc IRR Setting D26=0 Varying B24**						

TABLE 97 Project A Repeated to Cover the Six-Year Span

Total NPV Project A repeated = US$ 37642

Six Years Span Projects	
i =	10%
Project	**NPV (US$)**
A Repeated	37642
A + B	31529
C	30000

Hence, the sequential productions of two-years of Project A with a total NPV of US$ 37642 is optimal because NPV of Project B after Project A is US$ 31529.

Only NPV analysis with its property of additive value allows a meaningful comparison and appropriate recommendation for mutually exclusive projects with different lives.

For the IRR hurdle no discount rate needs to be specified in advance of any project evaluation. However, even if projects are ranked based on IRR it is necessary to establish a yardstick rate that specifies the minimum return that a company finds acceptable.

Comparing NPV and IRR _ Analysis of mutually exclusive project Example

Under NPV analysis the mutually exclusive project with highest value is preferred. The analogy is not the same for IRR analysis because the project with the highest IRR may not be the preferred alternative.

Suppose an analyst has two mutually exclusive projects, each with a one-year service life.

- Option A requires an investment of US$ 1000 with a return of US$ 2000
- Option B requires US$ 5000 with a return of US$ 7000

The resulting NPV and IRR for MARR of 10% is as follows:

Year (n)	Option A	Option B	Units
0	-1000	-5000	US$
1	2000	7000	US$

i	10,00	10,00	%
NPV	818	1364	US$
IRR	100,0	40,0	%

TABLE 98 Input and Results Projects A And B

Would the analysts prefer Option A simply because they expect a higher rate of return?

- As stated in previous example, the answer is to select Option B because the NPV of this option is higher
- NPV is an absolute (dollar) measure of investment and the IRR is a relative (percentage) measure and cannot be applied in the same way
- The IRR measure ignores the scale of investment

Option B with the lower rate of return but higher NPV would be selected unless the investor cannot afford Option B.

COMPARING NPV AND IRR _THREE OPTIONS OF INVESTMENT EXAMPLE

Consider the cash flows of three mutually exclusive projects:

Years	Option A	Option B	Option C
(n)	(US$)		
0	-2000	-1000	-3000
1	1500	800	1500
2	1000	500	2000
3	800	500	1000

TABLE 99 Cash Flow Projects A, B, and C

Which project would an analyst select assuming a MARR = 15%?

Solution

The calculated economic hurdles are as follows:

Calculated Economic Hurdles	Option A	Option B	Option C	Selection
Net Present Value (NPV, US$)	586,50	402,48	474,15	A
Internal Rate of Return (IRR,%)	34,37%	40,76%	24,81%	B
Discounted Payout (Payback yrs.)	1,91	1,78	2,25	B
NPVI	0,29	0,40	0,16	B
Net Future Value (NFV, $)	892,00	612,13	721,13	A
Benefit/Cost Ratio (PI)	1,29	1,40	1,16	B

TABLE 100 Calculated Economic Hurdles Projects A, B, and C

Even though some economic hurdles are better for Option B, the higher NPV is for Option A and this should be the preferred option

If the company can only afford Option B, then in these cases the concept of ***Capital Rationing*** and ***Capital Budgeting*** applies.

Capital Budgeting _ Capital Rationing

In this section it will be summarized most of the concepts presented before and it will be examined a corporation's capital-expenditure decisions, considering that investment in a project is expected to generate returns for more than one year. For example, addition to a plant or equipment, incorporation of new hydrocarbon field, or expenditures for exploration and discoveries.

As cash flows of hydrocarbon projects accrue over an extended period, so the time value of money plays an important role in establishing the economic feasibility of long-term investments. Special emphasis must be placed on risk analysis, because of the uncertainties inherent in any long-term production forecast and cash flow forecast, because a significant percentage of the corporation's resources could be retained for large periods.

Capital budgeting decisions are crucial to establishing long-term profitability and hence the economic value of a hydrocarbon company. A long-term investment in a new oil field, or in new production facilities require additions to materials, equipment or wells to support the operation.

The economic evaluation of capital expenditure decisions done by companies should be related to the valuation process that takes place in financial markets. Only such relation will ensure that the company maximizes shareholders wealth.

Capital investments are evaluated by discounting their cash flows to calculate a net present value, with the discount rate and risk adjustment determined by conditions that are commonly used in financial markets. Usually, this evaluation is done in phases such as:

a) Generation of projects ideas
b) Estimation of cash flows
c) Economic evaluation and project selection
d) Implementation of investment decision including controls and periodic reevaluations

a) Generation of Projects Ideas

Generating project ideas is critical for success in capital budgeting. Investment ideas ranging from overall strategic initiatives to operational improvements on the production wells are generated at all levels of the company.

Corporate strategy formulated by top management is usually based on two questions:

- What projects should the company be in?
    - The answer to this question depends on the combination of the opportunities offered by the market place and the resources the company can commit to exploit them
- How can the corporation create a sustainable competitive advantage?
    - In here, the company should only pursue projects where it can create sustainable advantages. One way is offering something that is not readily available elsewhere and becoming low-cost producers by operating more efficiently than the competitors

The company´s strategy, as approved by top management sets the overall framework within which capital budgeting decisions are made. Thus, investments in new projects, facilities, maintenance of equipment, EOR, or research and development programs all derive from the necessity of having to create sustainable competitive advantage.

Once the organization's overall decision has been set it is important to maintain open channels of communication that facilitates the free flow of ideas and to provide incentives for everyone in the company to contribute. It is a balance between new investment proposals and the company's ability to provide funding.

b) Estimating Cash Flows

Next to the generation of good project ideas, an accurate assessment of the cash flows that result from new investments is probably the most critical prerequisite for capita budgeting. These forecasts of cash flows over a long period are complex. It involves predicting variables that will take place several years into the future. For example:

- Expenditures on new production facilities
- Hydrocarbon production
- Oil and gas price
- Additional investment required in CAPEX and / or in OPEX
- Shift in foreign-exchange rates or taxes

With each forecast, uncertainties must be considered and quantified because risk affects economic value. All these individual projections must be combined into overall cash flows that determine the attractiveness of the investment to the company.

Others issues may appear because cash flows should include the funds committed to the acquisition of a long-term assets, and any additions to current facilities that may be required to support the new proposal investment.

The period over which the forecasted cash flows are to be included in the analysis must be determined, considering that future cash flows that are discounted over a long time-spans have low present values.

Other issues affecting the estimation of cash flows are the external effects such as environmental, political or legal actions by parties who may suffer because of corporate activities or negligence.

Financial charges incurred in funding investments generally should not be included in cash-flow forecast. However, all cash flows should be on after-tax basis, as only after-tax dollars accrue to the investors.

Forecasting cash flows are based on diversity assumptions: about the general environment and about the specific project. It is important that general assumptions that affect several projects be made in a consistent manner; otherwise, projects cannot be compared in a reliable condition.

Key assumptions are analyzed using sensitivity as a normal way for assessing the risk inherent to the investments.

Finally, forecasting the cash flows to be generated by an investment can be a difficult task requiring sound judgement and may depend on numerous assumptions.

In reviewing various techniques for investment evaluation, analysts should consider that even the most sophisticated approaches ultimately depend on the quality of subjective inputs.

- Regardless of the evaluated criteria used, poor cash flow forecasts will generate poor investment decisions

c) Economic Evaluation and Project Selection _ The Discount Rate or Cost of Capital

The returns a company's investments generate must be proportionate with the returns managers expect when they provide funds to finance these projects.

Managers are risk averse and usually trade off risk against expected return. Managers are willing to fund projects only if these investments promise higher expected returns. Hence, when managers evaluate a new project, they should decide whether the returns it adds to the company's cash flows are sufficient to compensate for the added risk investors must bear. The questions to be answered are, what return would be required in today's financial markets to make money available, given the risk of the project? What is the extra return for bearing additional risk?

This required return is the ***corporation's cost of capital*** to fund the project. It is the discount rate managers should use to derive the present value of cash flows the investment is expected to generate.

If a project produces more than the return managers require, the project should be accepted; otherwise, the project should be postponed, discarded or revaluated under new conditions if it is realistic to do so.

The cost of financing is reflected in the discount rate; however, it should not be included in the forecast of a project's cash flows. The process is:

- Create the cash flows without regard to any financing charges that may be incurred in funding the project
- Apply the cost of financing as a discount rate to determine if the project can support itself

The average cost of capital represents the overall cost of financing the corporation's new investments, and it is commonly used to discount rate in capital-budgeting evaluations. It is calculated on after tax basis, and is applied to project's forecasted after-tax cash flows, as company and investors are interested in after-tax wealth.

d) Implementation of investment decision including controls and periodic reevaluations

This phase is related to the peculiarities of application of risk analysis methods when implementing projects. In economic analysis, different methods are used for risk management. Methods for risk assessment of an investment project help to assess the feasibility of the project, initial period after which it will generate revenue, as well as the probabilistic range of future profits

Capital Rationing

Under normal circumstances, a company should accept all projects that are technically feasible and with cash flows that, when discounted at the cost of capital (interest rate) yield net benefits to shareholders as reflected in positive NPV.

At any moment a company may have limited resources such as time, money or labor and be constrained in the activities it pursues. Although, the following paragraphs focus on economic constraints, analysts recognize that other constraints such environment, politics, or qualified personnel are limitations as well.

When constraints prevent a company from accepting new investments, it faces ***capital rationing*.** This is, money must be shared among projects that compete for the same funds, particularly when it is not possible to raise additional capital. The raising of new capital involves time-lags, so it may not be possible to overcome the constraints.

Also, the budgetary process, which is a customary control procedure, particularly in large and decentralized companies, usually sets limits on capital spending for each operating unit.

Financial constraints may be in part a strategic policy for maintaining an overall balance in the company's operation. It is also possible that in a corporation the control and the use of debt could be of concern and external financing may be limited by policy.

A company faced with capital rationing can no longer afford all investment proposals that produce returns more than their cost of capital. Therefore:

- The company will accept the most attractive investment projects until its ability to fund are exhausted
- The corporation will reject or postpone less attractive opportunities even if their returns exceed the cost of capital

Companies that operate under capital rationing will impose a discount rate that exceeds the cost of capital, because opportunity costs are now the main yardstick against which new investments must be measured.

- Any accepted project should yield at least as much as the most desirable opportunity that has to be rejected or postponed

Capital Rationing Theoretical Example

For example, consider a corporation with an overall cost of capital of 12%. This corporation has more investments opportunities than it can afford with the limited capital currently available, and several investment projects promising returns of 16% must be discarded.

The opportunity cost of 16% is therefore the standard against which new investment proposals must be measured, because this is the return that could be earned on any not assigned funds.

Even with the appropriate adjustment to the discount rate, as the company is operating under capital constraint, the corporation cannot rank projects on an NPV basis.

Giving limited funds, it becomes important that the feasible combination of projects yielding the highest NPV be accepted. From this consideration, capital rationing can be viewed as special sort of project interdependency. Going with one project may make another project inaccessible.

CAPITAL RATIONING BASED ON NPV AND PROFIT TO INVESTMENT RATIO (PIR) EXAMPLE

Assume capital constraint of MUS$ 10 and consider these three investment projects:

Project	Initial Investment (P) (MUS$)	NPV (MUS$)
A	10	5
B	6	4
C	4	2

Considering the constraint of MUS$ 10, the company can either accept:

Project A (P_A = 10)

or

Projects B and C ((P_B = 6) + (P_C = 4)) = $P_{(B+C)}$ = 10

While Project A has the highest NPV (NPV_A = 5), it must be rejected because the combination of Project B and Project C, have a higher NPV. The combined NPV of B and C ((NPV_B = 4) + (NPV_C = 2) = 6) exceeds that of NPV_A = 5.

In situation of this type, companies can rank projects based on NPV per dollar invested. This equals the Profit to Investment Ratio (PIR) discounted and reflects the "productivity of capital" per project. When capital is constrained, this becomes a reasonable criterion for the ranking of projects.

Project	Initial Investment (MUS$)	NPV (MUS$)	PIR = NPV/P
A	10	5	0,50
B	6	4	0,67
C	4	2	0,50

Investment Pool and Borrowed Funds

In present value analysis it is assumed that all funds in a company's treasury can be placed in investments yielding a return equal to the MARR (Minimum Attractive Rate of Return). These funds can be viewed as an ***Investment Pool***. Alternatively, if no funds are available for investment, it can be assumed that the company can borrow the funds at the MARR from the capital market.

These two concepts, ***Investment Pool and Borrowed Funds***, will be used in explaining the meaning of MARR in NPV calculation.

Investment Pool Concept

An investment pool is equivalent to a company's treasury where all fund transactions are administered and managed by the corporation's comptroller. The company may withdraw funds from the investment pool for other investment purposes, but if left in the pool the funds will earn at the MARR.

Revisiting a previous example (Table 55) where an oil company is considering the acquisition of a new subsurface pump. The required initial investment is US$ 75000. The production engineers have been asked by management of the company to evaluate the economic merit of the acquisition based on NPV. The company's MARR or Cost of capital is 15%.

In solving this problem, the results obtained (Table 55) are as follows:

	i=	15.0%	
	US$		
(I)	**(II)**	**(III) = (II)/(1+i)$^{(I)}$**	**(IV) = Sum (III)**
Year (n)	**Cash Flow**	**NPV**	**Cum NPV**
0	-75000	-75000.00	-75000.00
1	24400	21217.39	-53782.61
2	27340	20672.97	-33109.64
3	55760	36663.11	3553.46

Discussion

If the company did not invest in the project and left US$ 75000 in the investment pool for 3 years, these funds would grow as follows:

P	=	75000
i	=	15%
n	=	3
F	=	114065.63

$F = P/(1+i)^n$

TABLE 101 Grow of Funds Left in Investment Pool

If the company decided instead to invest US$ 75000 in the project, then, the company would receive a stream of cash inflows during the project life of 3 years in the amount of:

Year	Cash Flow
0	-75000
1	24400
2	27340
3	55760

Since the funds that return to the investment pool earn interest at the rate of 15%, it would be of interest to see how much the company would benefit from this investment. For this alternative, the future value or returns after investment are:

	i=	15,0%	
	US$		
(I)	(II)	(III) = (II)*$(1+i)^{(3-n)}$	(IV) = Sum (III)
Year (n)	Cash Flow	FV	Cum FV
0	0	0,00	0,00
1	24400	32269,00	32269,00
2	27340	31441,00	63710,00
3	55760	55760,00	119470,00

TABLE 102 Returns after Investment

The return generated by the project is US$ 119470.00 that is higher than the US$ 114065.63 generated by the growth of funds if left in the investment pool.

	Cash Flow Economic Analysis				Option 1		
Input					Use as equivalent rate:	15,00%	i = i'+f+i'*f
Investment Pool		Clear Input			Option 2		
Project Life (N, yrs)	3	Max 40 Yrs			15,0% is corrected to		
Initial Investment (P_{oINF}, $)	75000				To be added to i		
Interest (I)	15,00%	inflation (f)	0,0%	r corected by f =		15,00%	r = (i - f)/(1 + f)
Fill Rows 8 & 10		Yr 0	1	2	3		
Annty Outflow (C, $ per year)		CI	0,0	0,0	0,0		
Annty Inflow (A, $ per year)		AI	24400	27340	55760		
Sunk Cost @ N (S, $)	0						
Salvage Value @ N (Sv, $)	0						

The additional cash accumulation at the end of 3 years from investing in the project is:

NFV = US$ 119470.00 – US$ 114065.63

NFV = US$ 5404.38

Results	
Profit to Investment Ratio (PIR)	0,047
Discounted Payout (Payback yrs)	2,89
NPVI	0,047
Net Future Value (NFV, $)	5404,38
Net Present Value (NPV, $)	3553,464
Benefit/Cost Ratio (PI)	1,047
Internal Rate of Return (IRR,%)	17,46%
	Calc IRR
Eqvlnt Total Annual Cost (A_{PC}, $)	1556,34
Capital Recovery (CR_{PC}, $)	32848,27

If the analyst compares the equivalent present value of this net cash surplus at time 0 it is obtained:

$P = F/(1+i)^n$ = 5403.38/(1+0.15)^3 = US$ 3553.46

This is the same as the NPV of the project computed above. Therefore, based on its positive NPV, the option of purchasing a new pump should be preferred to simply leaving the funds in the investment pool at the MARR.

Thus, in NPV analysis, any investment is assumed be returned at the MARR. If there is a surplus at the end of the project, then NPV/MARR > 0.

Borrowed Funds Concept

In the previous example, If the company does not have US$ 75000 or does not even have an investment pool, it can be assumed that the company obtains all its capital by borrowing from a bank at the interest rate of 15%, invests in the project and uses the proceeds from the investment to pay off the principal and interest on the bank loan.

The question is, how much is available for the firm at the end of the project loan?

Solution

At the end of first year, the interest on the use of the bank loan for the project would be:

US$ 75000 * (0,15) = US$ 11250.00

Therefore, the total loan balance grows to:

US$ 75000.00 * (1 + 0.15) = US$ 86250.00

Then, the company receives US$ 24400.00 from the project and applies the entire amount to repay the portion of the loan. This repayment leaves a balance due of:

US$ 86250.00 – US$ 24400.00 = US$ 61850.00

This amount becomes the net amount the project is borrowing at the beginning of year 2. This net amount is known as project balance.

At the end of period 2 the bank debt grows to:

US$ 61850.00 * (1 + 0.15) = US$ 71127.50

However, with the receipt of US$ 27340, the project balance reduces to:

US$ 71127.50 – US$ 27340.00 = 43787.50

Similarly, at the end of year 3, the project balance becomes:

US$ 43787.50 * (1 + 0.15) = US$ 50355.63

However, with the receipt of US$ 55760 from the project, the company should be able to pay off the remaining balance and come out with a surplus in the amount of:

US$ 5404.38 (55760 - 50355.63)

In other words, the company fully repays its initial bank loan and interest at the end of period 3, with a resulting profit of US$ 5404.38. If analysts compute the equivalent present worth of this net profit at time 0, they obtain

$$NPV = US\$\ 5404.38/(1+0.15)^3$$

$$NPV = US\$\ 3553.46$$

The result is identical to the case where analysts computed the NPV of the project at i = 15%. The process can be summarized as follows:

	US$					
	(I)	(II) = (I)*i	(III)	(IV) = (I)+(II)+(III)	(V) = (I)*(1+i)	(VI) = $(IV)/(1+i)^n$
Year (n)	Principal Outstanding at the beginning of the year	Interest on Principal Outstanding	Cash Receipt (payment)	Project Balance	Bank Debt	NPV (US$) Project Balance
0	0.00	0.00	0.00	-75000.00		-75000.00
1	-75000.00	-11250.00	24400.00	-61850.00	-86250.00	-53782.61
2	-61850.00	-9277.50	27340.00	-43787.50	-71127.50	-33109.64
3	-43787.50	-6568.13	55760.00	5404.38	-50355.63	3553.46

TABLE 103 Using the Proceeds from the Investment to Pay Off the Principal and Interest on the Bank Loan

Another way to view the process is using the same format for the amortization concept including the payment receipt from the project, and calculating the annuity to repay the loan.

Loan	75000		Results		
Interest Rate (i)	15.0%		Annual Repayment	32848.27	$A = P / ((1-(1+i)^{-n})/i)$
Years	3		Total Future Value	114065.63	$F = A (((1+i)^{n} - 1)/i)$

Total		98544.82	23544.82	75000.00	OK				
	(I)	(II)	(III) = i*(I)	(IV) = (II)-(III)	(V) = (I) - (IV)	(VI)	(VII) = (VI)-(II)	(VIII)	(IX) = Sum (VIII)
Year (n)	Principal Outstanding at the beginning of the year	Annual Repayment	Interest on Principal Outstanding	Repayment of Principal	Principal Outstanding at the end of the year	Cash Receipt (payment)	Profit	NPV Profit	Cum NPV
1	75000.00	32848.27	11250.00	21598.27	53401.73	24400.00	-8448.27	-7346.32	-7346.32
2	53401.73	32848.27	8010.26	24838.01	28563.71	27340.00	-5508.27	-4165.05	-11511.37
3	28563.71	32848.27	4284.56	28563.71	0.00	55760.00	22911.73	15064.83	3553.46

TABLE 104 Using the Amortization Concept to Pay Off the Principal and Interest on the Bank Loan

Additional Examples of Capital Rationing

Capital rationing refers to situations where the funds available for capital investment are not sufficient to cover all the projects having either positive NPV, IRR or return on investment capital greater than the MARR.

Under capital rationing, analysts enumerate all investment options and eliminate from consideration any mutually exclusive options exceeding the budget.

The most efficient way to proceed in capital rationing situation is to select the group of projects that maximizes the total NPV of future cash flows over required investment expenditures.

Three projects under budget constraint Example

A private oil company is considering the following independent short projects and their cash flows. Suppose the firm has only MUS$ 7000 to spend initially.

Formulate all mutually exclusive investment options under the MUS$ budget constraint indicate the feasible and the optimal investment(s) at MARR = 15%

Project	Year (n)			
	0	1	2	3
A (MUS$)	-1000	800	800	800
B (MUS$)	-4000	2000	2000	2000
C (MUS$)	-6000	3000	3000	3000

Solution

The possible decisions including the do-nothing are:

Option	Project					Cash Flow @ Year				NPV (MUS$) @MARR
						0	1	2	3	$\Sigma A_i/(1+MARR)^n$
1						0	0	0	0	0.00
2	A					-1000	800	800	800	**826.58**
3	A	B				-5000	2800	2800	2800	1393.03
4	A	B	C			-11000	5800	5800	5800	2242.71
5		B				-4000	2000	2000	2000	**566.45**
6		B	C			-10000	5000	5000	5000	1416.13
7			C			-6000	3000	3000	3000	**849.68**
8	A		C			-7000	3800	3800	3800	1676.26

Ordering results including initial investment:

				NPV @MARR	Funds Required	
Option	**Project**			**(MUS$)**		**Notes**
4	A	B	C	2242.71	-11000.00	Out of budget
8	A		C	1676.26	-7000.00	Best Option
6		B	C	1416.13	-10000.00	Out of budget
3	A	B		1393.03	-5000.00	
7			C	849.68	-6000.00	
2	A			826.58	-1000.00	
5		B		566.45	-4000.00	
1				0.00	0.00	

TABLE 105 Ordered Results Considering NPV and Investment Constraint

Of the eight possible choices, only five would be feasible because of the budget constraint:

Option 8 (AC)

Option 3 (AB)

Option 7 (C)

Option 2 (A)

Option 5 (B)

In summary, Option 8 (AC) would be the best combination of projects because of its maximum NPV.

FOUR PROJECTS UNDER BUDGET CONSTRAINTS EXAMPLE

Consider these four energy-efficiency projects.

- Project 1 (electrical)
  - This project requires replacing the existing standard efficient motors in the air conditioners and exhaust blowers of a control room with high-efficiency motors
- Project 2 (building coating)
  - This project involves coating the inside surface of existing design and placement of windows in a flow station with low-emissivity solar film
- Project 3 (air conditioning)
  - This project is for the installation of heat exchangers between the office building's existing ventilation and relief air ducts
- Project 4 (lighting)
  - This project requires the installation of led reflectors and light poles for the warehouse's storage area and the external shipping dock

These projects require capital expenses in the MUS$ 15 to MUS$ 50 range and have useful lives of about 8 years.

The facilities department's first task was to estimate the annual savings that can be realized by these energy-efficiency projects.

The company pays 7.68 cents per kilowatt-hour (kWh) for electricity and US$ 150 per thousand cubic meters (m^3) of air conditioned.

If the current energy prices would continue for the next 8 years, the company has estimated the cash flows and the NPV (at 15%) for each project as follows:

	US$				US$	
Project	**Investment**	**Annual O&M Cost**	**Annual Savings (Energy)**	**Units**	**Annual Savings**	**NPV**
1	15000	400	51120	kWh	3926	222
2	34950	350	173700	kWh	13340	23340
3	45160	450	72207	m^3	10831	1423
4	31410	314	130666	kWh	10035	12211
Total	**126520**	**1514**			**38132**	**37196**

As each project could be adopted by itself in isolation, there were at least as many alternatives as there were projects.

With US$ 100000 approved for energy improvement funds during the current fiscal year, the department did not have sufficient capital on hand to undertake all four projects (Total US$ 126520) without any additional allocation from headquarters.

Enumerate the total number of decision options and select the best decision combination of projects.

Solution

Since the NPV for each project is positive (NPV>0), all four projects pass the initial project screening.

With four projects, there are 16 decision options as presented in Table 106. The table was ordered based on investment.

Option	Description							
	1	2	3	4	Total Investment (US$)	Total Annual Cost (US$)	Total Annual Savings (US$)	Total NPV (US$)
8	Accept	Accept	Accept	Accept	126520.00	1514.00	38132.00	37196.00
10	Reject	Accept	Accept	Accept	111520.00	1114.00	34206.00	36974.00
12	Accept	Accept	Reject	Accept	81360.00	1064.00	27301.00	35773.00
15	Reject	Accept	Reject	Accept	66360.00	664.00	23375.00	35551.00
7	Accept	Accept	Accept	Reject	95110.00	1200.00	28097.00	24985.00
9	Reject	Accept	Accept	Reject	80110.00	800.00	24171.00	24763.00
6	Accept	Accept	Reject	Reject	49950.00	750.00	17266.00	23562.00
3	Reject	Accept	Reject	Reject	34950.00	350.00	13340.00	23340.00
14	Accept	Reject	Accept	Accept	91570.00	1164.00	24792.00	13856.00
11	Reject	Reject	Accept	Accept	76570.00	764.00	20866.00	13634.00
16	Accept	Reject	Reject	Accept	46410.00	714.00	13961.00	12433.00
5	Reject	Reject	Reject	Accept	31410.00	314.00	10035.00	12211.00
13	Accept	Reject	Accept	Reject	60160.00	850.00	14757.00	1645.00
4	Reject	Reject	Accept	Reject	45160.00	450.00	10831.00	1423.00
2	Accept	Reject	Reject	Reject	15000.00	400.00	3926.00	222.00
1	Reject	Reject	Reject	Reject	0.00	0.00	0.00	0.00

TABLE 106 Based on Investment 16 Decision Options Ordered

The department cannot select options 8 and 10 because these options exceed the budget. Option 8 means accept all four projects and Option 10 is reject project 1 and accept projects 2, 3, and 4.

Option 12 (reject project 3, and accept projects 1, 2, and 4) has the higher NPV in spite that other options require lower investment.

Therefore, option 12 is the best decision option.

The best rejected project is the best opportunity foregone, and its value is called the ***opportunity cost***.

- As a reminder the opportunity cost is the potential benefits management misses when choosing one alternative over another.

When there is a limit on capital, the **MARR** (Minimum Attractive Rate of Return) is assumed to be equal to this opportunity cost, which is higher than the cost of capital.

Replacement Decision

Economic analysis is frequently performed on projects related to existing facilities and equipment. These are called profit-maintaining projects whose purposes are:

- To maintain ongoing operations instead of to increase production
- Comparison on new equipment

Management must decide whether to buy new and more efficient equipment or to continue using existing equipment. These decision problems are known the ***replacement problem*** and are classified as:

- Comparison between defender and challenger
- Determination of service life
- Replacement analysis when the required service period is long

Comparison Between Defender and Challenger

Replacement projects are those required to replace existing obsolete or worn-out assets. If these replacements are not implemented, the results will be a slowdown or shutdown operations. The question raised is whether existing equipment or system (***the defender***) should be replaced with more efficient equipment (***the challenger***).

Determination of Service Life, Current Market Value and Sunk Cost

An existing equipment will be removed, either when the task it performs is no longer necessary or when the task can be performed better by different equipment. Therefore, questions are:

- When will the equipment be removed?
- Why the company should replace existing equipment at this time rather than postponing replacement by repairing or overhauling the existing equipment?
- What is the best alternative to replace the defender by the challenger?

Current Market Value

In all defender analysis, the relevant cost is the current market value for the equipment. The ***current market value*** is related to an asset's liquidity, which is the ease by which an asset can be converted from an investment into cash. It offers interested parties a price for which they can make a transaction.

Book Value

A related concept is the ***book value***: As a reminder the book value of a company is the net difference between that company's total assets and total liabilities, where book value reflects the total value of a company's assets that shareholders of that company would receive if the company were to be liquidated.

Sunk Cost

Another concept used in the defender-challenger analysis is the ***sunk cost*** briefly described in previous chapter

A sunk cost refers to money that has already been spent and cannot be recovered. A hydrocarbon company, for example, may have several sunk costs, such as the cost of facilities, equipment, and the lease expense on the corporation. If for an equipment the current book value is US$ 14400, and the market value is US$ 10000. Therefore, a loss of US$ 4400 will occur if the equipment was taken out of service.

- The value of the defender that should be used in replacement analysis is the current market value adjusted for the disposal tax effects, not the cost when it was originally purchased and not the cost of any repairs that may have been made

Basic Replacement Analysis

Although replacement projects are a subcategory of the mutually exclusive projects decisions, they do possess unique characteristics that allow analysts to use specialized concepts and analysis techniques in their evaluation.

There are two basic approaches to analyzing this type of problems:

- The cash flow approach
- The opportunity cost approach

The Cash Flow Approach

In this case both the defender and challenger have the same useful life, which begins now. Presumably the defender has been around for a while, and the required service period for the defender is relatively short. In this situation analysts can use the cash flow approach, if the analysis period is the same for all options.

Replacement Analysis Cash Flow Approach Example

Consider that a company has been offered a chance to purchase a machine and that the old machine will be sold rather than exchanged for the new machine.

Suppose that the company will need either machine (old or new) for 3 years and that it does not expect a superior new machine to become available on the market during the required service period. Assuming that the firm's interest rate is known to be 12%, the actual cash flow for both machines is as follows:

Year	Defender (US$)	Challenger (US$)
0	0	-3240
1	-3648	-3000
2	-3878	-2520
3	86	4320

Decide whether replacement is justified now

Solution

		i =	12.0%	
	(I)	(II)	(III) = (I)/(1+i)^n	(IV) = (II)/(1+i)^n
Year	**Defender (US$)**	**Challenger (US$)**	**Defender NPV (US$)**	**Challenger NPV (US$)**
0	0	-3240	0.00	-3240.00
1	-3648	-3000	-3257.14	-2678.57
2	-3878	-2520	-3091.52	-2008.93
3	86	4320	61.21	3074.89
		NPV(US$) =	**-6287.45**	**-4852.61**

TABLE 107 Defender and Challenger NPV

There is a difference in NPV of -1434.84 in favor of the challenger; therefore, the replacement should be made now.

In terms of annuity the results are:

Equivalent Annual Defender

P	=	-6287.4
i	=	12.0%
n	=	3
A	=	-2617.77

A = P /1- (((1 + i)$^{-n}$)/i)

Equivalent Annual Challenger

P	=	-4852,6
i	=	12,0%
n	=	3
A	=	-2020.38

There is an annual difference of US$ 597.39 in favor of the challenger.

The Opportunity Cost Approach

Opportunity costs are easily overlooked. Understanding the potential missed opportunities foregone by choosing one investment over another allows for better decision-making.

Opportunity Cost Approach Example

In the previous example assume that US$ 11760 in net salvage value was foregone by not selling the defender. Therefore, another way to analyze such a problem is to charge the US$ 11760 as an ***opportunity cost*** of keeping the asset.

Under these circumstances the cash flows are as follows:

Year	Defender (US)	Challenger (US)
0	-11760	-15000
1	-3648	-3000
2	-3878	-2520
3	86	4320

For the Challenger

= -3240-11760

		i =	12.0%	
	(I)	(II)	(III) = (I)/(1+i)^n	(IV) = (II)/(1+i)^n
Year	Defender (US)	Challenger (US)	Defender NPV (US	Challenger NPV (US
0	-11760	-15000	-11760.00	-15000.00
1	-3648	-3000	-3257.14	-2678.57
2	-3878	-2520	-3091.52	-2008.93
3	86	4320	61.21	3074.89
		NPV(US$) =	**-18047.45**	**-16612.61**

ΔNPV (US$) = 1434.84

Equivalent Annual Defender

P	=	-18047.4
i	=	12.0%
n	=	3
A	=	-7514.04

A = P /1- (((1 + i)$^{-n}$)/i)

Equivalent Annual Challenger

P	=	-16612.6
i	=	12.0%
n	=	3
A	=	-6916.64

ΔA (US$) 597.39

TABLE 108 Defender/Challenger NPV and Annuity Analysis Including Opportunity Cost

The result is the same as in the previous example since both the challenger and defender cash flows were adjusted by the same amount -US$ 11760 at time 0.

Usually, old equipment has relatively short remaining life compared with new equipment, so this assumption is simplistic.

Economic Service Life

In engineering economic studies analysts are more interested in knowing what an asset's practical service life is, rather than what is its remaining physical life. Economic life is the period of useful life that minimizes the equivalent annual cost of an asset.

Economic Service Life Example

Assume that a company is considering the replacement of an equipment that was purchased 3 years ago. The following are the cash flows including maintenance, salvage value, overhauling, and market value.

The company MARR is 15%.

Compute the economic service life based on the following cash flows when the equipment is retained for 1,2, 3, 4, or 5 years.

n (year)	Retained for 1 years	Retained for 2 years	Retained for 3 years	Retained for 4 years	Retained for 5 years
0	-7956	-7956	-7956	-7956	-7956
1	4056	-1109	-1109	-1109	-1109
2		2465	-1547	-1547	-1547
3			832	-1838	-1838
4				-330	-2346
5					-4465

Solution

Retained						MARR	15,0%			
n (year)	1 year	2 years	3 years	4 years	5 years	NPV for 1 year	NPV for 2 years	NPV for 3 years	NPV for 4 years	NPV for 5 years
0	-7956	-7956	-7956	-7956	-7956	-7956.00	-7956.00	-7956.00	-7956.00	-7956.00
1	4056	-1109	-1109	-1109	-1109	3526.96	-964.35	-964.35	-964.35	-964.35
2	0	2465	-1547	-1547	-1547	0.00	1863.89	-1169.75	-1169.75	-1169.75
3	0	0	832	-1838	-1838	0.00	0.00	547.05	-1208.51	-1208.51
4	0	0	0	-330	-2346	0.00	0.00	0.00	-188.68	-1341.33
5	0	0	0	0	-4465	0.00	0.00	0.00	0.00	-2219.89
$NPV = A/(1+i)^n$				NPV (US$)		-4429.04	-7056.45	-9543.05	-11487,30	-14859,84
$A = P / (((1 + i)^{-n} - 1)/i)$				A (US$)		-5093.40	-4340.54	-4179.64	-4023.60	-4432.92

TABLE 109 Economic Service Life Based on Cash Flow

If the equipment were sold after 4 years, it would have a minimum annual cost of US$ 4023.60 per year, and that is the is the most favorable for comparison purposes.

Therefore, the economic life for the equipment is 4 years.

The rationale for finding the economic life of an asset is that, by replacing perpetually according to economic life, analysts obtain the minimum infinite annuity (A) cost stream and they should be envisioning a long period of required service of the asset. Consider that the service life is influenced by taxes and depreciation.

Replacement Analysis Considerations

Once it has been determined the economic service lives for defender and challenger, the next question is to decide whether ***now*** is the time to replace the defender based on these assumptions:

- Planning Horizon
  - Planning horizon is the service period required by the defender and a sequence of future challengers
  - The infinite planning horizon is used when analysts are unable to predict when the activity under consideration will be terminated
  - Finite planning horizon, if the project will have a definite and predictable duration. In these cases, replacement policy should be formulated more realistically based on a finite planning horizon
- Technology
  - Predictions of technology patterns over the planning horizon refer to the development of types of challengers that may replace the defenders under study
  - An infinite variety of predictions might be made of purchase cost, salvage value, and operating costs dictated by the efficiency of the machine over the life of an asset
  - If analysts assume that all future equipment will be the same as those now in service; then, there will be no technological progress in the area
  - In other cases, analysts explicitly recognize that equipment that may become available in the future will be significantly more efficient, reliable, or productive than those now on the market. This situation leads to a recognition of technological change or obsolescence
  - If the best available system or equipment gets better all the time, the best decision may be to delay replacement in contrast to the situation where no technological change is likely
- Revenue and cost pattern over asset life (revenue cash flow) information
  - There are many variables of predictions possible in estimating patterns of revenue, cost, and salvage value over the life of an asset
  - Revenue is increasing, but costs and salvage value are constant over the life of the equipment
  - A decline in revenue over equipment life can be expected
    - This will determine whether the replacement analysis is directed towards:
      - Cost optimization (with constant revenue)
      - Profit maximization (with varying revenue)

Replacement Decision Using the Annuity (A) Method

The annuity method is frequently used in replacement analysis; however, the present value approach is equally applicable. Therefore, here it is only presented the general work flow for the analysis based on the annual equivalent approach.

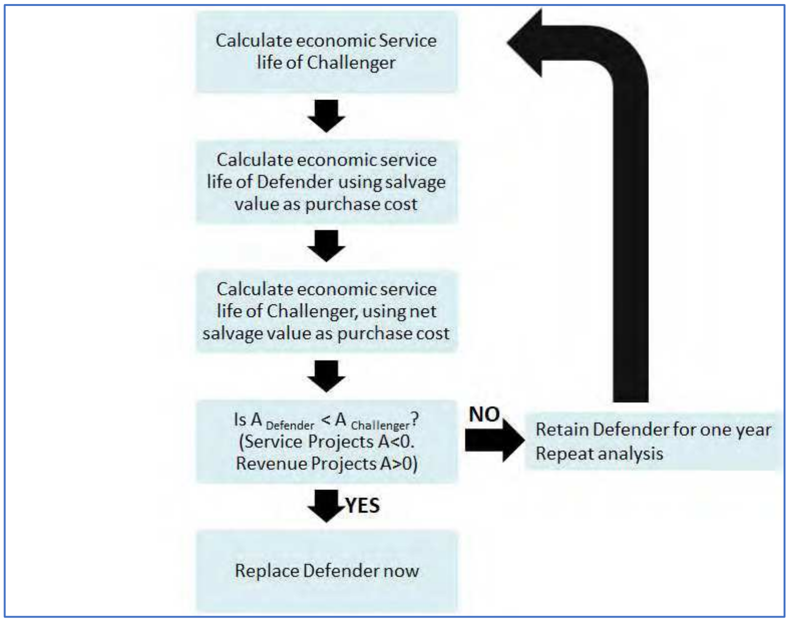


FIGURE 27 WORK FLOW FOR REPLACEMENT DECISION USING THE ANNUITY METHOD

Replacement Decision Using the NPV Method

The NPV method is applicable to a more general class of replacement decisions than the Annuity method because the NPV method provides a more direct solution to a replacement problem with either an infinite or a finite planning horizon or a technological change in the potential challenger.

INFINITE PLANNING HORIZON

The problem here is when, if at all, the defender should be replaced with the challenger. When there is no technological change, an asset of type ***j*** can be purchased at any time ***n*** with a purchase price, revenues, and salvage value that do not depend on time, only on age. In this case let NPV stand for the present value of all future cash flows associated with retaining the defender for ***n*** years facing an infinite sequence of identical challengers, each of which is replaced after n_j years. Here n_j represents the economic service life of the challenger.

NPV =	**NPV of retaining the defender for "*n*" additional years**	**+**	**NPV of repeated replacements of the challenger**

The objective is to find the value of ***n*** that maximizes the NPV.

Finite Planning Horizon

If the planning period is finite a comparison based on the annuity method over its economic service life does not generally apply. The procedure for solving such a problem with a finite planning horizon is to set all "reasonably" replacement patterns and then use the NPV method for the planning period to select the most economical pattern.

Considerations of Technological Change

When defining the ***challenger,*** the question is not only to select the best available replacement for the defender. It is more realistic to recognize that often the replacement decision involves an asset now in use versus a candidate for replacement that is in some way an improvement on the present asset. This reflects the technological progress that is continually going on.

The future models of equipment and systems are likely to be more effective than the current model. In most areas, this technological change appears as a combination of gradual advances in effectiveness, with an occasional technological breakthrough that revolutionizes the character of the equipment or system.

The prospect of improved future challengers makes the current challenger a less desirable alternative. By retaining the defender, managers may have a later opportunity to acquire an improved challenger. If this is a fact, the prospect of improved future challengers may affect the present decision between the defender and the challenger. It is difficult to forecast future technological trends. However, in developing a long-term replacement policy, analysts need to take technological change into consideration.

Summary Decision Procedure for Evaluating Multiple Investment Options

This section expands the basic framework of capital budgeting which involves investment decisions related to fixed assets considering that the ***capital budget*** outlines the planned expenditures on fixed assets.

The ***capital budgeting*** includes the entire process of analyzing projects and deciding whether they should be included in the capital budget. Once analysts evaluate and compare investments projects the next step is the budgeting aspect. Proper capital budgeting decisions require an estimate of minimum attractive rate of return (***MARR***). Therefore, the analyses examine the relationship between capital budgeting and the MARR.

Evaluation of Multiple Investment Options

In this section it is examined the process of deciding whether projects should be included in the capital budget. The decision procedures that should be applied when analysts must evaluate a set of multiple investment options for which companies have limited capital budget will be considered.

In previous sections it was discussed how to compare two or more mutually exclusive projects. Now in this section, the comparison technique is to complement and expand the case of multiple decision options which are not mutually exclusive. Here the distinction is between a ***project*** and an ***investment alternative***, which is a decision option.

For a single project there are two investment options:

1. To accept the project
2. To reject the project

For two independent projects, analysts have four investment options:

1. To accept both projects
2. To reject both projects
3. To accept only the first project
4. To accept only the second project

As analysts add interrelated projects, the number of investment options to consider grows exponentially.

Independent Versus Dependent Projects

For a proper capital budgeting analysis, a company must group all projects being considered into decisions options. This grouping requires the company to distinguish between projects that are dependent of one another and those that are independent to formulate options correctly.

As a reminder an ***Independent Project*** is one that may be accepted or rejected without influencing the acceptance-reject decision of another independent project. Only projects that are economically independent of one another can be evaluated separately. However, budget constraints may prevent analysts from selecting one or more of several independent projects, this external constraint does not alter the fact that the projects are independent.

On the other hand, there are ***Dependent Projects***. In many decision problems, several investment projects are related to one another such the acceptance or rejection of one project influences the probability of acceptance of others. There are two such types of dependencies:

- Mutually exclusive projects
- Contingent projects

Two or more projects are contingent if the acceptance of one requires the acceptance of another. For example, the purchase of a printer is dependent upon the purchase of a computer, but the computer may be purchased without considering the purchase of the printer.

Formulating Mutually Exclusive Projects

The selection of investment projects can be seen as a problem of selecting a single decision alternative from a set of options. Each investment project is an investment alternative, but a single alternative may entail a whole group of investment projects.

The common method of handling various project relationships is to arrange the investment projects so that the selection decision involves only mutually exclusive options. To obtain this set of mutually exclusive options, analysts need to enumerate all the feasible combinations of the projects under consideration.

In enumerating the number of investment options, an additional useful way is to describe the project relationships in graphic form, for example:

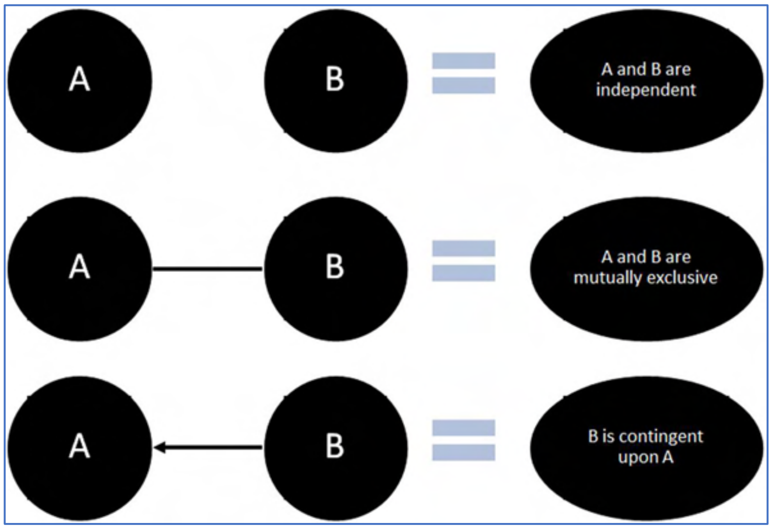


FIGURE 28 INVESTMENT OPTIONS

This symbolism will be used in developing the mutually exclusive options for three different investment situations:

- Independent projects
- Mutually exclusive projects
- Contingent projects

Independent Projects

With a given number of independent investment projects, analysts can enumerate mutually exclusive options. For example, in considering two projects, ***A*** and ***B*** analysts have four decision options, including a "do nothing" option (reject both projects).

Option	Description		Decision Option	
	A	B	X_A	X_B
1	Reject	Reject	0	0
2	Accept	Reject	1	0
3	Reject	Accept	0	1
4	Accept	Accept	1	1

In this notation X_j is a decision variable associated with investment project j.

- If X_j = 1 project j is Accepted
- If X_j = 0 project j is Rejected

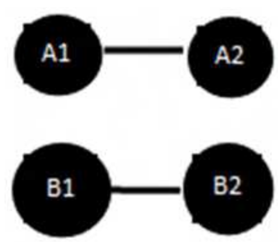
If the acceptance of one of the projects will exclude any other, the projects are ***mutually exclusive***.

Mutually Exclusive Projects

For two independent sets of mutually exclusive projects, each pair of projects (***A1, A2***) and (***B1***, ***B2***), is mutually exclusive.

The selection of either ***A1*** or ***A2***, however, is also independent of the selection of any project from the set (***B1***, ***B2***). For these investment projects the mutually exclusive options are:

Option	Description				Decision Option			
	A1	*A2*	*B1*	*B2*	X_{A1}	X_{A2}	X_{B1}	X_{B2}
1	Reject	Reject	Reject	Reject	0	0	0	0
2	Accept	Reject	Reject	Reject	1	0	0	0
3	Reject	Accept	Reject	Reject	0	1	0	0
4	Reject	Reject	Accept	Reject	0	0	1	0
5	Reject	Reject	Reject	Accept	0	0	0	1
6	Accept	Reject	Accept	Reject	1	0	1	0
7	Reject	Accept	Accept	Reject	0	1	1	0
8	Accept	Reject	Reject	Accept	1	0	0	1
9	Reject	Accept	Reject	Accept	0	1	0	1



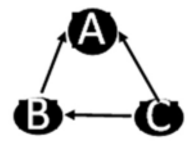
With two independent sets of mutually exclusive projects, analysts can have nine different options.

Contingent Projects

If ***C*** is contingent on the acceptance of both **A** and *B*, and acceptance of ***B*** is contingent on acceptance of A; then, the possible number of decision options can be formulated as follows:

Option	Description			Decision Option		
	A	*B*	*C*	X_A	X_B	X_C
1	Reject	Reject	Reject	0	0	0
2	Accept	Reject	Reject	1	0	0
3	Accept	Accept	Reject	1	1	0
4	Accept	Accept	Accept	1	1	1

TABLE 110 Decision Options Contingent Projects



Thus, analysts can formulate a set of mutually exclusive investment options with a limited number of projects that are independent, mutually exclusive, or contingent merely by arranging the projects in a logical sequence.

One difficulty with the enumerating approach is that, as the number of projects increases, the number of mutually exclusive alternatives increases exponentially. Fortunately, in real-world business, the number of engineering projects to consider at any one time is usually manageable, so the enumeration approach is a practical option.

Formulating Mutually Exclusive Options Example

Consider the following investment projects with estimated cash flows and conditions:

- Projects A and B are mutually exclusive
- Project C is independent
- Project D is contingent on Project C
- Project E is contingent on Project B

Project	Year (n)			
	0	**1**	**2**	**3**
A (US$)	-100	50	50	50
B (US$)	-200	50	100	200
C (US$)	-100	30	80	120
D (US$)	-300	100	100	100
E (US$)	-200	100	100	150

Formulate the total number of investment options and calculate their cash flows

Solution

There are two sets of independent project groups:

- (***A, B, BE***)
- (***C, CD***)

In the first group, there are four mutually exclusive options, counting the "do-nothing" option. In the second group, there are another three.

- First group (***A, B, BE***)

Option	Description			Option		
	A	***B***	***BE***	X_A	X_B	X_{BE}
1	Reject	Reject	Reject	0	0	0
2	Accept	Reject	Reject	1	0	0
3	Reject	Accept	Reject	0	1	0
4	Reject	Reject	Accept	0	0	1

- Second Group (***C, CD***)

Option	Description		Option	
	C	***CD***	X_C	X_{CD}
1	Reject	Reject	0	0
2	Accept	Reject	1	0
3	Reject	Accept	0	1

By combining the two groups, the total number of mutually exclusive options that can be obtained from the group of projects in this example is 12.

Total Number of Mutually Exclusive Options and cash flows in US$. The cash flows for any option are simply the sum of the cash flows of the included projects.

Option	Description					Decision Option					Year			
	A	*B*	*C*	*D*	*E*	X_A	X_B	X_C	X_D	X_E	0	1	2	3
1	Reject	Reject	Reject	Reject	Reject	0	0	0	0	0	0	0	0	0
2	Accept	Reject	Reject	Reject	Reject	1	0	0	0	0	-100	50	50	50
3	Reject	Reject	Accept	Reject	Reject	0	0	1	0	0	-100	30	80	120
4	Reject	Accept	Reject	Reject	Reject	0	1	0	0	0	-200	50	100	200
5	Accept	Reject	Accept	Reject	Reject	1	0	1	0	0	-200	80	130	170
6	Reject	Accept	Accept	Reject	Reject	0	1	1	0	0	-300	80	180	320
7	Reject	Accept	Reject	Reject	Accept	0	1	0	0	1	-400	150	200	350
8	Reject	Reject	Accept	Accept	Reject	0	0	1	1	0	-400	130	180	220
9	Accept	Reject	Accept	Accept	Reject	1	0	1	1	0	-500	180	230	270
10	Reject	Accept	Accept	Reject	Accept	0	1	1	0	1	-500	180	280	470
11	Reject	Accept	Accept	Accept	Reject	0	1	1	1	0	-600	180	280	420
12	Reject	Accept	Accept	Accept	Accept	0	1	1	1	1	-800	280	380	570

TABLE 111 Cash Flows and Decision Options for the Five Investment Projects

Once analysts have the list of options to consider the next step is apply a specific decision criterion to measure the investment worth associated to each alternative.

Multiple-Options Comparison by NPV Method

The first step for proper use of any selection criterion is to formulate mutually exclusive decision option before applying any investment criterion. To select projects, follow this procedure:

1) Remove
   a) Remove any individual project that fails to meet the NPV (or an equivalent criterion)
2) Combine
   a) Develop a list of all possible project combinations
   b) The resulting list contains all the mutually exclusive options to be chosen
3) Order
   a) Order the options by increasing order of investment
   b) The ordering is normally done by the investment required at time 0
   c) This step is not critical but it helps to organize the decision problem
   d) If there is a tie, it is broken arbitrarily
4) Check Budget
   a) If budget constraint exists, remove any project that exceeds the budget
5) Filter
   a) Remove any option that is dominated by another option
   b) Project domination means that one option requires less investment yet generates more revenue for every period
   c) This step may be omitted, but it makes the analysis more efficient by reducing the number of investment options to be considered

Applying NPV to more than two options Example

Use the NPV measure to select the best option at MARR = 10% for the list of project combination developed in the previous example. Table 111 is repeated here.

Option	Description					Year			
	A	**B**	**C**	**D**	**E**	**0**	**1**	**2**	**3**
1	Reject	Reject	Reject	Reject	Reject	0	0	0	0
2	Accept	Reject	Reject	Reject	Reject	-100	50	50	50
3	Reject	Reject	Accept	Reject	Reject	-100	30	80	120
4	Reject	Accept	Reject	Reject	Reject	-200	50	100	200
5	Accept	Reject	Accept	Reject	Reject	-200	80	130	170
6	Reject	Accept	Accept	Reject	Reject	-300	80	180	320
7	Reject	Accept	Reject	Reject	Accept	-400	150	200	350
8	Reject	Reject	Accept	Accept	Reject	-400	130	180	220
9	Accept	Reject	Accept	Accept	Reject	-500	180	230	270
10	Reject	Accept	Accept	Reject	Accept	-500	180	280	470
11	Reject	Accept	Accept	Accept	Reject	-600	180	280	420
12	Reject	Accept	Accept	Accept	Accept	-800	280	380	570

Solution

There are 12 options, including the “do-nothing” option (reject all).

Organize and analyze them in order of increasing initial investment. Since the mutually exclusive option in Table 111 are already listed in increasing order of initial investment, simply apply the procedure to this list.

Compute the NPV for each option at MARR = 10%.

Option	Project					Cash Flow @ Year				NPV @MARR (US$)	NPV of Cash Flow @ Year $A_i/(1+MARR)^n$			
						0	1	2	3		0	1	2	3
1						0	0	0	0	0,00	0,00	0,00	0,00	0,00
2	A					-100	50	50	50	24,34	-100,00	45,45	41,32	37,57
3			C			-100	30	80	120	83,55	-100,00	27,27	66,12	90,16
4		B				-200	50	100	200	78,36	-200,00	45,45	82,64	150,26
5	A		C			-200	80	130	170	107,89	-200,00	72,73	107,44	127,72
6		B	C			-300	80	180	320	161,91	-300,00	72,73	148,76	240,42
7		B			E	-400	150	200	350	164,61	-400,00	136,36	165,29	262,96
8			C	D		-400	130	180	220	32,23	-400,00	118,18	148,76	165,29
9	A		C	D		-500	180	230	270	56,57	-500,00	163,64	190,08	202,85
10		B	C		E	-500	180	280	470	**248,16**	-500,00	163,64	231,40	353,12
11		B	C	D		-600	180	280	420	110,59	-600,00	163,64	231,40	315,55
12		B	C	D	E	-800	280	380	570	196,84	-800,00	254,55	314,05	428,25

With no budget constraint imposed, option 10 (Project B, C, and E) best maximizes the present worth at NPV = US$ 248.16

Option	Project					NPV @MARR (US$)
10		B	C		E	248,16
12		B	C	D	E	196,84
7		B			E	164,61
6		B	C			161,91
11		B	C	D		110,59
5	A		C			107,89
3			C			83,55
4		B				78,36
9	A		C	D		56,57
8			C	D		32,23
2	A					24,34
1						0,00

TABLE 112 Investment Option Selection Based on NPV

Summary of Concepts _ Chapter 4

Models Used to Determine Profit

There are three common schemes or models used to determine profit:

- Cash flow
- Financial
- Tax

This handbook is focused on the cash flow model because it is the most common method used in the upstream business for screening and ranking single and multi-projects.

Single Project

The first test any project must pass is the "go or no-go" criteria established by management. That is, each project must be able to stand alone. In following this process, management uses several yardsticks to measure profitability (NPV, IRR, or PIR).

Mutually Exclusive Projects

This means that the acceptance of one project implies the rejection of any other. Only one project may be undertaken. All the projects being considered must first meet the single project go or no-go decision criteria established by management. In cases where there are more than two projects, the "winner" of the first test is then compared (in the same manner) to each one of the remaining projects. This process is continued until only one project remains.

Service Projects and Revenue Projects

In comparing mutually exclusive projects (options), generally there are two types to be considered: Service Projects and Revenue Projects.

Service Projects

They are investments whose revenues do not depend on the choice of the project. For example, building a highway or a dam. When comparing service projects, the selection favors the one with the lowest production cost. In other words, the alternative to be chosen is the one with the highest (i.e., least negative) present value over the service life.

Revenue Projects

They are investments whose revenues do depend on the choice of project. When comparing revenue projects, usually the selection process favors the project which is to deliver the highest NPV.

Feasibility Study

This concept covers an engineering economic appraisal of the commercial viability of any project, including a prospective producing property.

The feasibility study is a procedure for assessing the various relationships that exist between factors which directly or indirectly affect a project. The objective is to identify the basic factors which control the probability of project success. Once all factors have been defined and reviewed, an attempt is made to quantify as many of the variables as possible to arrive at a "potential value or worth of the hydrocarbon property or project".

Those projects that appear as economically marginal or borderline situations, where no obvious decision is apparent, the feasibility study is of particular importance. Under these circumstances, all variables affecting the project that can be assessed must be incorporated in the decision-making process.

Value

It refers to a measure of the desirability or interest to undertake the project or acquire ownership of the property under consideration. Following are some examples of indicators of such well supported interest assessment:

Assessed Value:
This is the value of a property entered on the official assessor's records as the value of the property applicable in determining the amount of ad- valorem taxes to be paid by the property.

Book Value:
It is the original investment in the asset carried on the corporate books minus or subtracting any cumulative allowance for depreciation or amortization entered on the books.

Capitalized Value:
The capitalized value of a property or project is the sum of all discounted future annual net earnings expected to be generated by a property or project.

Full Cash Value:
It is usually the synonymous of market value, which means the estimate of value that is derived annually by using standard appraisal methods and techniques.

Insured Value:
It is the value at which a property has been insured against loss or disaster.

Market Value:
This is the value (price) established in a public market by exchanges between a willing buyer and a willing seller, when neither is under constraints to complete the transaction.

Replacement Value:

It refers to the current value of a property or asset as determined based on what it would cost to replace the property or its service with at least equally satisfactory and comparable property or service.

Salvage Value:

This is the amount of money for which any asset could be sold after its service to the project has been rendered or the dollar measure of its remaining usefulness.

Terminal Cash Surplus or Terminal Value (TCS):

The simplest measure of profit relates to the concept of a surplus. TCS is the sum of all project revenues, minus the sum of all CAPEX, OPEX and taxes. It is the end point of the cumulative curve, not the highest value achieved during the life of the project. Projects have an abandonment time. Up to that time, there are expenditures that cannot be avoided and must not be ignored in the project`s evaluation.

The Time-Value of Money Basic Formulas

The first issue to be resolved, in the process of discussing the available yardsticks for economic analysis, is the standard procedure to compare value of net income received at different times in each project or to compare different net cash flows for different projects. This is referred to, generically, as the time-value of money.

The cash flows occurring at different points in time can be handled using at least six basic interest formulas and five variables, as follows:

- **P** = A present sum of money. This is the initial amount of money in transactions of loans (debt) or investment (financed)
- **i** = Effective interest rate per period, which measures the cost or price of money, expressed as percentage per period
- **n** = Number of interest periods. The specific length of time that marks the duration of the transaction and thereby establishes a certain number of interest periods
- **F** (NFW or NFV) = A future sum of money that results from the cumulative effects of the interest rate over several interest periods
- $\mathbf{A_n}$ = A payment in a series of n equal payments, made at the end of each period of interest. In different words, discrete payments or receipts occurring at the end of some interest period

The six basic interest equations are summarized below:

Single payment, Present worth

$$P = F / (1 + i)^n$$

Single payment compound amount (Future Value)

$$F = P\ (1 + i)^n$$

UNIFORM SERIES, PRESENT WORTH

$$P = A \ \{[(1 + i)^n - 1] / (i (1 + i)^n)\}$$

Or

$$P = (A/i) (1 - 1/(1+i)^n)$$

UNIFORM SERIES, CAPITAL RECOVERY

$$A = P \ [(i (1 + i)^n / ((1 + i)^n - 1)]$$

UNIFORM SERIES, SINKING FUNDS

$$A = F \ \{i / [(1 + i)^n - 1]\}$$

UNIFORM SERIES, COMPOUND AMOUNT

$$F = A \ \{[(1 + i)^n - 1] / i\}$$

Strengths of Discounted Cash-Flow (DCF) Criteria

- DCFs tie control over the expenditure of funds to the condition under which funds must be obtained
- Cash-flows can be related directly to the objective of maximizing the wealth of the corporation and or shareholders

The discount rate to be used is the company's cost of capital, which should represent all financial charges associated with the investment.

The acceptance of projects according to cash-flow criteria guarantees that returns are proportionate with the cost of funding new investments, and that shareholder will receive the expected return they request when making funds available to the company.

Cash flows are used to calculate the net present value (NPV) that is a measure of the residual gain over and above normal financing charges. This gain accrues to the company´s owners. It represents the value that management has obtained for shareholders through its investment decision.

Limitations of Cash-Flow Criteria

- DCFs do not consider the potentially negative impact that investments may have on financial statements
  - A project with a positive NPV may well have negative effects on reported earnings in the early periods, because of high start-up CAPEX. However, management would accept the early decline in profits recognizing that they would be adequately compensated through favorable cash flows earnings in later periods. This is because new investments are considered only if they bring improvement to the company

- Cash flows ignore non-economic aspects potentially important
    - Managers of major corporations use additional criteria in the decision process of major investments

Capital Budgeting

This chapter also reviewed capital budgeting and showed how companies should approach their long-term investment decisions. Essentially, it evaluated projects by computing the NPV of all net cash flows that investments generated, discounting at the rate that reflected the company's cost of financing. Because the NPV measures the economic gain that accrues to shareholders, this ensures the maximization of shareholders wealth.

Capital budgeting also refers to the process of selecting among alternative projects for funds allocation. The process includes the activities of:

- Planning
- Evaluation an assessing a single project
    - Estimates of costs
    - Benefits
    - Expected returns
    - Associated risks
- Selection
- Implementation
- Control
- Continuous reevaluation
- Auditing of results

Here the term evaluation is used for each project or investment alternative available to the company, entity or financial institution.

Procedurally, above activities can be organized as follows:

1. Generation of project ideas
    a. Good project ideas and the proper estimation of cash flows are crucial to the success of any capital budgeting program
    b. To achieve positive NPV, a company needs to differentiate itself from its competitors by either being first or more efficient
2. Estimation of cash flows
    a. The proper estimation of cash flows includes the choice of an appropriate time horizon
    b. The discount rate to be applied in investment evaluations is generally the company's weighted average cost of capital
    c. The discount rate represents the average cost of various sources of funds that the corporation draws on in funding new investments

   d. If new investments are to make an economic contribution to the company, they must provide a yield that exceeds the cost of capital
3. Economic evaluation of projects through application of appropriate selection criteria
   a. The main criteria for the economic evaluation of investments are the NPV, the PIR, and IRR. They rely on the concept of discounting and explicitly recognize the time value of money
   b. NPV is a method of analysis in which a project's cash flows are discounted to a single present value. It is perhaps the most efficient analysis method analysts can use for determining project acceptability on an economic basis.
   c. The NPV of a project is defined as the present value of all its cash flows, discounted at the company's costs of capital
   d. NPV measures the economic gain to be derived over and above the costs of financing a project
   e. The Minimum Attractive Rate of Return (MARR) is an interest rate at which the firm can always earn or borrow money. It is generally dictated by management and is the rate at which NPV analysis should be conducted
   f. The IRR measures the effective yield of a project, which is then compared to the cost of funding the investment
   g. NPV and IRR are basically equivalent, and represent two alternative ways of looking at the same problem
   h. Under most circumstances, a consistent application of either NPV or IRR should produce the same investment decisions. However, some situations exist where conflicts may occur
   i. The PIR is another discount cash-flow criterion. It is the relation between the net cash flow of the project and the initial investment required to generate such cash flow (P_0)
   j. Although discounted cash-flow criteria or economic hurdles are commonly used, several limitations need to be recognized. The purpose of discounted cash-flow yardsticks is only to measure economic gain
   k. Other non-economic objectives that a company may have should be evaluated using additional criteria
4. Implementation of investment decisions, including controls and periodic re-evaluations
   a. This is a critical aspect of capital budgeting and is a common source of problems. All ongoing projects should be re-evaluated periodically to ensure that they still meet the criteria of investment success set out by the company. Otherwise, abandonment may be indicated
   b. Consider all mutually exclusive projects. This means that when one of several options that meet the same need is selected, the others will be rejected